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Bondarenko, Vitalij M. [Bondarenko, Vitaliĭ M.] (UKR-AOS);
Petravchuk, Anatoliy P. [Petravchuk, Anatoliĭ P.] (UKR-KIEVM)

Wildness of the problem of classifying nilpotent Lie algebras of vector fields in four variables. (English summary)

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Let $\mathbb{F}$ be a field of characteristic zero. The vector space over $\mathbb{F}$ of all $\mathbb{F}$-derivations of $\mathbb{F}[x_1, \dots, x_n]$ is denoted by $W_n(\mathbb{F})$, which is clearly a Lie algebra with respect to the Lie bracket

$$[D_1, D_2] = D_1 D_2 - D_2 D_1$$

for $D_1, D_2 \in W_n(\mathbb{F})$. Finite-dimensional subalgebras of $W_1(\mathbb{R})$, $W_1(\mathbb{C})$ and $W_2(\mathbb{C})$ were classified by S. Lie [*Theorie der Transformationsgruppen*, Teubner, Leipzig, 1893; JFM 25.0623.01]. The classification of finite-dimensional subalgebras of $W_2(\mathbb{R})$ was completed in [A. González-López, N. Kamran and P. J. Olver, Proc. London Math. Soc. (3) **64** (1992), no. 2, 339–368; MR1143228]. The problem of classifying finite-dimensional subalgebras of $W_4(\mathbb{R})$ and $W_4(\mathbb{C})$ remains open. This paper shows that the problem of classifying finite-dimensional nilpotent Lie algebras of polynomial vector fields in four variables is wild.

Hirokazu Nishimura

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