PRODUCING THEORIES FOR MATHEMATICS EDUCATION THROUGH COLLABORATION: A HISTORICAL DEVELOPMENT OF JAPANESE LESSON STUDY

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This paper briefly sketches the Japanese theories for designing and reproducing better lessons to share and transfer the challenges and experiments of lesson study. Lesson study was initiated in 1873. The basic philosophy for developing children who learn mathematics by and for themselves and the basic manner of lesson study appeared in 1883. The major theories of mathematics education for designing and reproducing sciences were developed on the elaboration of theories proposed by various lesson study groups. Currently, it can be summarized as the theories for: clarify the objectives; distinguish teaching concept; establish the task sequence; and teaching approaches which includes assessments. This paper illustrates their historical development through lesson study.

Lesson study has been recognized as the collaborative activity of teachers and researchers on the lesson study processes to share better practice. In so many cases, it focused on the school based approach to establish the school as learning community. On the other hand, the Japanese lesson study was usually lead by the mathematics lesson study community which focused on specific content and lesson study theme. It produced the theories for mathematics education as for designing and reproducing better lessons to develop students who learn mathematics by and for themselves. And their achievements are embedded into national curriculum and textbooks. The objective of this paper is to illustrate how Japanese developed thier theories for designing and reproducing science in mathematics education.

Brief History of Japanese Lesson Study for Collaborations

An Origin of Lesson Study

The History of Japanese Lesson Study began with the establishment of the Elementary School at Normal School (Predecessor of Higher Normal School, and an origin of the University of Tsukuba) in Tokyo (1873) under the Code of School (1872) which intended to introduce French Style School System. From the beginning, observing other teachers' classes was an ordinal activity because it changed the teaching style from tutorial manner of Temple Schools for heterogeneous students to a whole class teaching system based on grades. Temple schools used a textbook up to Pythagorean Theorem and extraction of the square root using abacus. Additionally, they also alternated it to the adapted edition of imported textbooks. The first lesson study national workshop, a kind of training, set by the scholars of the School under the Ministry of Education was published as the book "Improvement of Teaching" (Wakabayashi and Shirai, 1883). The authors were Elementary School teachers at the time and it was the product of a 10 year experiment of lesson study. Some of them were scholars who taught at the normal school and made a survey of Pestalozzi method in USA. The book became a bible to guide the lesson study because it included the principles of teaching under the Pestalozzi method, dialectical questioning under Zen/Confucian style, and tasks for inquiry, objective-based lesson plan, and ways of critical discussion after observation of the class. Critical discussion points were the preferred teaching materials and methods, and the observed activities between teacher and students. These basic components had an influence through normal schools for practicum, and it became basic format of guidebooks published by teachers. Pestalozzi recognized the nature of students as active learners and Japanese teachers tried to develop students through the tasks for inquiry. These philosophy provided a ground for Japanese educational policy to develop students who learn the subject (mathematics) by and for themselves and with the objective to develop (mathematical) thinking and attitude. Tasks designing and the necessary components of lesson plans were the platform to develop the design science of lesson study. The guidebooks also included the model/sample protocols between teachers and students for every subject which was the origin for lesson study as a reproducible science.

Establishment of Journals and Societies for lesson study group

Elementary education became free in 1900. The Journal for Educational Research has published since 1904 by the Elementary School. These journals for teachers functioned for sharing the theories and themes of lesson study and supported lesson study groups nationwide. Various experimental/demonstration schools proposed their methods under their studies of German didactics and so on. One innovative approach by Jingo Shimizu (1924) was that students pose problems by and for themselves. Japan Society of Mathematical Education (JSME) was established for secondary (Junior High) school mathematics teachers (1919) and enhanced nationwide movements to introduce calculus and perspective-production geometry under the influence of Kline movements. Until Worldwide Financial Crisis (1929), lesson studies at elementary schools were oriented to develop and share the new methods of teaching under the exercise oriented national textbooks. Such lesson studies produced the necessary tasks and innovative task sequence for new national textbooks (1934-1939) which focused on problem solving to develop mathematical thinking. On the other hand, because various textbooks were preferable, secondary school lesson study was oriented to experiment the task sequence of the innovative unit for mathematics curriculum reform under Klein movement. The Secondary School textbooks category I and II (1943 and 1944) up to Calculus were the products which set the mathematization as for editing principle.

Emergent of Various Lesson Study Groups which Propose Curricula and Theories

After World War II, the government under the USA changed the national curriculum as a recommendation, so curriculum development became the role of every teacher and school. Compulsory education was extended until junior high schools, the secondary school became senior high schools and more teachers were hired.

Until World War II, lesson study to show practice for other school teachers used to be the custom of demonstration schools with support of scholars and inspectors. On the issues of democracy, Ministry of Education enhanced progressivism and the school board of education, under each prefecture and city, enhanced to demonstrate their challenges for other school teachers and parents. Teachers' union was established and every group began their lesson study against progressivism. At this era, lesson study became cultural activity of every school. Various researchers such as mathematicians began to collaborate. The Association of Mathematical Instruction (AMI) was established (1951) against the national curriculum under progressivism and the Soviet Union began to support it. JSME included the elementary school mathematics (1952) and the government began to support JSME financially. After the re-independence (1953), the national curriculum system came back and the withdraw group of AMI established new societies such as the Mathematics Education Society of Japan (MESJ, 1959). On their background, AMI enhanced specific theory based on materialism. MESJ was oriented to pure mathematics and the school curriculum up to World War II. From this era to the modernization (Japanese New Math), Japanese basic mathematics education theories for lesson study had been produced by various groups with support of math-educators and mathematicians.

Mathematicians provided various visions in relation to pure mathematics at the era of modernization. Some of them were supportive to introduce New Math with set and structure: Akitsugu Kawaguchi supported teachers' group in Sapporo for introducing dynamic geometry with transformation and proposed the experimental curriculum and textbook for their lesson study. Some of them did not agreed: Toyama group (AMI) infiltrated their curriculum and textbook based on their own deductive sequence and materialism theory by using their square-tile model, and so on. Toyama's textbook was never accepted by the Ministry but some commune used it as a textbook, illegally. Yasuo Akizuki, a notable mathematician, who was a chair of High School National Committee supported to put in mathematization and modeling process for making mathematical thinking process an aim, as well as put in set and structure for content. Odaka was a leading educator who came out from AMI and established the Study Group of Secondary School Mathematics under the Junior Secondary School at the University of Tsukuba. He published a number of books for teachers and students under the deep surveys of US and UK projects and lesson study experiments. His group proposed their original junior high school curriculum under their experiments which enhanced integration of different domain (subject) on the context of principles used before the occupation but had content that included New Math. He was a member of National Committee at second reform for modernization; however, he was discharged in the middle of his term because he proposed to put in other content instead of set and structure because of time restraints. Kiyoshi Yokochi, president of MESJ, also published a number of new math books with teachers. The above were only a part of the discussion, and in the era more than two hundred books of different teaching methods were published by various study groups to secure the modernizations and its implementations.

Various challenges and fruitful discussions were done amongst groups everywhere. Unfortunately, mass media participated in those discussion and teachers' union engaged in negative campaigning parents whom had no knowledge of Set and Structures. Set and Structures lost the position at the next reform. However there were fruitful practical theories being extended into textbooks. Their ideas got the position of major representations to teach number and operations in the textbooks under further national curriculum. Their ideas were elaborated as significant tools by teachers to develop students' mathematical thinking by enabling them to develop mathematics by and for themselves.

The institutes at every prefecture level renamed their teacher education centers to implement national training for every teacher since1985, which includes lesson study as a component. And lesson study became more oriented to the implementation of curriculum instead of curriculum development.

Theories for Lesson Study Produced by Lesson Study Groups

The theories for lesson study were developed by teachers, math-educators and mathematicians. In the world, theories are usually developed by researchers to contribute in Journals. On the other hand, in the Japanese tradition since the initiation of lesson study, educators also show their challenges at the classroom. Thus, lesson study researchers should be teachers as well as teachers should be researchers. In this context, the nature of Japanese lesson study theories are design and reproducible science.

Most of current known theories were established after World War II under the democracy. The Japanese theories as for the school subject specific theories (Herbst&Chazan, 2016) can be explained from four perspectives. The first perspective is the theories to explain why we teach it. It's clarify the aims and objectives of lesson and formative assesment. The national curriculum standards is an authorized document that explains the objectives. To clarify the objective of teaching, math educators have prepared related theories such as theories for mathematical thinking. The second perspective is the terminologies used to distinguish conceptual differences in teaching content. The third perspective is the theories used to establish the curriculum sequence and task sequence. The fourth perspective is the theories used to manage lessons and assessment for teaching such as a manner of questioning. These theories have been prepared by math educators and teachers through several lesson studies.

Theories to Explain Aims and Objectives

The Japanese aims of education have been described as three pillars: human character formation (such as values and attitudes), general thinking skills (such as mathematical thinking and ideas), and specific knowledge and skills (such as mathematical knowledge and skills) since 1956. Currently, these aims are special not only for Japan but also for other countries such as the Southeast Asian countries (Mangao et al., 2017). The first two aims are usually explained as higher-order thinking skills in Southeast Asia as the learning content for learning how to learn. According to the Japanese principle of the national curriculum, these aims are symbolized by a single concept: "Developing students who learn mathematics by and for themselves" (Shimizu, 1984). In Japanese mathematics education, this has been recognized in relation to mathematical activities as for reorganization of life in living (Ministry of Education, 1947). The activity has been re-explained as mathematical thinking and attitude (Ministry of Education, 1956) by Japanese math educators, who have tried to explain it further. Shigeo Katagiri (see Katagiri, Sakurai, and Takahasi, 1969 and Katagiri, Sakurai, Takahasi, and Oshima, 1971), who was a curriculum specialist in primary school mathematics in the Ministry of Education, established the framework for mathematical thinking with teachers (Isoda and Katagiri, 2012). Katagiri's Mathematical Thinking is categorized into three: Mathematical Ideas, Mathematical Ways of Thinking and Mathematical Attitude. Mathematical Ideas are the ideas for activities which are embedded in students' activities and later sophisticate. For example, the idea of set is generally embedded in various activities and later it represented by set as the technical term in the mathematics curriculum. Mathematical Ways of Thinking such as inductive thinking which uses various occasions, and Mathematical Attitude (mind set) as the representation of value. After re-independence, Japanese national curriculum standards became a brief as for the law. It included general aims and contents. Thus, theories to explain the objectives of the lesson were necessary: Theories of mathematical thinking provide the analytical terminologies to explain mathematical thinking has been used for the objective of lesson deepen. In Japanese lesson study, it is used for clarifying the curriculum, task sequence, teaching materials, and methods of teaching and to analyize teaching and learnig process. Acturally, Katagiri provided the list of questioning in the lesson to promote students' mathematical thinking. It is not only a list for problem solving strategies but is used for precise descriptions of objectives which explains why we should teach in the curriculum and how.

Terminology to Explain Sequences and Theories for Extension and Integration

The terminology to explain content distinguishes conceptual differences in curriculum. It is necessary to explain the process of reorganization of mathematical concepts in teaching sequence. The Japanese established most of it up to1960s (Japan Society of Mathematical Education, 2010). Japanese teachers need to learn the terminology for developing students who learn mathematics by and for themselves because the school curriculum sequence cannot exist as a system deduced from the set and axioms such as pure mathematics (see Freudenthal (1973)). The sequence in the Japanese curriculum standards has been explained by the principle of "extension and integration" since 1968, which is oriented toward enhancing mathematical activities and developing mathematical thinking. It corresponds to the principle of reinvention by Freudenthal (1973) who proposed the reorganization of mathematical experience as mathematization. Under this principle, the school mathematics curriculum can be seen as a set of partially ordered local mathematics theories, like a net.

Such inconsistencies through the extension and integration of local theories in relation to multiplication are explained by the theory (Isoda&Olfos, in printing) that is adaptation of the conceptual and procedural knowledge to meaning and procedure in Fig. 1.

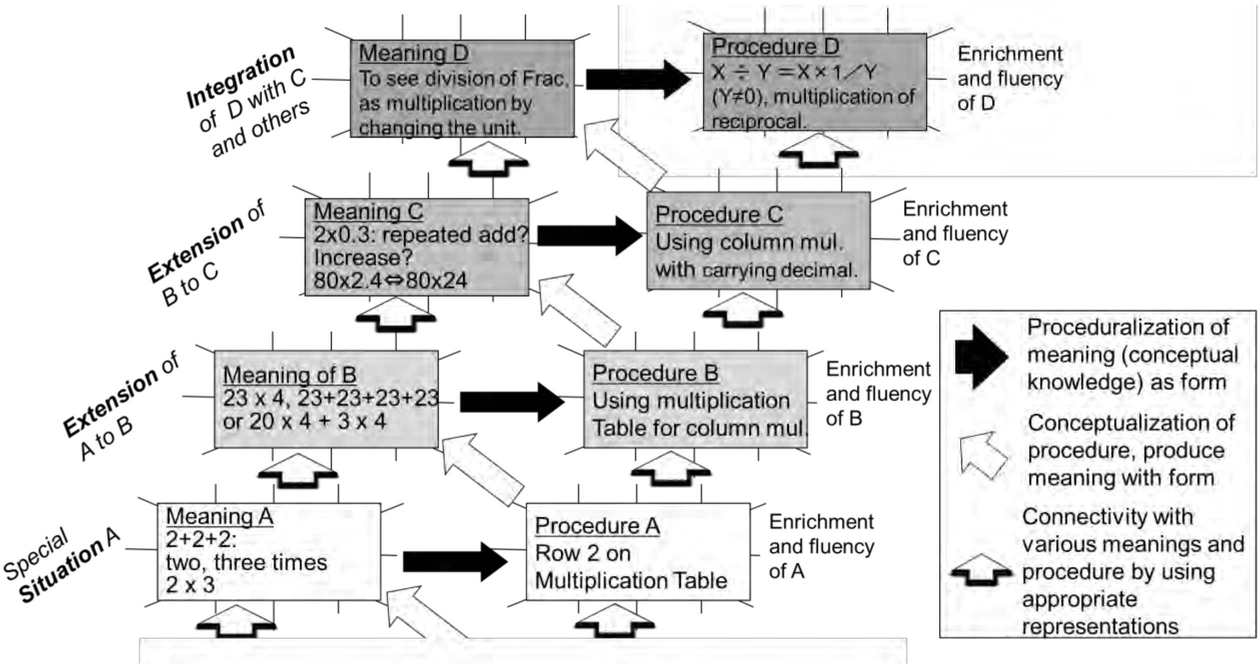

Figure 1: Extension and Integration Task Sequence for Multiplication

Conceptual and procedural knowledge (Hiebert, 1986) are used to explain the development of personal knowledge; however, in Fig. 1, Isoda (1992) used them to design and explain task sequences in the curriculum. In the curriculum sequence, as in textbooks, these are not discussed at the same time. Conceptual knowledge is usually taught to provide the meaning: it needs to use some known form of procedure. After introducing the meaning of multiplication as a binary operation (expression), the multiplication table is proceduralized from repeated addition; otherwise, students cannot distinguish it from addition as a new operation. In the process of extension and integration, inconsistencies usually appear. For example, for doing multidigit multiplication, students need to see the multidigit numbers under the base ten system for applying the multiplication table instead of just repeated addition. For the extension of multiplication to multidigit numbers with column methods, multiplication as repeated addition should be integrated with the base ten system. If we extend multiplication from whole numbers to decimals, the product of multiplication becomes small. It cannot be explained as repeated addition.

In the Japanese textbooks and Japanese teachers' lesson design these processes are discussed more precisely in relation to the task sequence. Conceptual and procedural knowledge are not seen as two different sides of the same coin in the textbooks. On the task sequence on extension and integration principle in the textbooks, it might be clearly distinguished. One of the reasons is that it is possible for students to learn the procedure without knowing when the procedure should be used. "If A, then B" is the format of the procedure. In the exercise in every chapter, students do only exercise B for solving given tasks. The condition, a part of A, is not necessary to consider in the exercise.

Before the extension of multiplication to decimal numbers, the product of multiplication only increases: "If it is multiplication of whole numbers, then the products become large." However, until extension of whole number to decimals, whole numbers meant numbers, so it looks correct to say, "If it is multiplication of numbers, then the products become large." This is possible learning content for students through the exercise in the textbook chapter. The necessity for all students to think about conditions in relation to "A" will be provided when students learn multiplication of decimals. Actually, at this stage, students do not know about decimals; they know only about whole numbers. Students are able to learn "A" when they encounter multiplication of decimal numbers. Another reason is related to the shortage of the capacity of working memory. If we limit working memory, some procedures are very convenient and faster for doing multiplication. Students do not need to consider the meaning of "A", because the numbers given in the exercise are not decimals. They have already established a convenient procedure that can be used without considering the original meaning of "A". After students attain fluency in the procedure, many students do not feel the necessity to go back to and interpret the original meaning of the situations. They may lose it because they do not need to think about the condition of "A" as long as they are applying it to learned situations. The opportunity for extension and integration is a chance to reorganize their mathematics by using their developed mathematical ideas. At the moment of extension and integration into the task sequence, students need to reconsider the real meaning through the overgeneralization of their ideas.

Problem-Solving Approach: Beyond a Teaching Method

If you have a chance to observe a lesson in a Japanese elementary school, the Japanese problem-solving approach looks the same as an open-ended approach, which involves posing an unknown task, solving it in various ways, comparing solutions with the whole class, and summarizing.

However, the Japanese problem-solving approach is prepared in the following way: aims and objectives for developing students who learn mathematics by and for themselves, terminologies to explain the content that students should learn, the curriculum and task sequence, and the teaching materials which embedded objective. On the other hand, an open-ended approach is characterized by an open-ended task. Consequently, the teaching materials used in the Japanese problem-solving approach are not the same as those in an independent task, topic, or content of mathematics because they exist under the aims, objectives, and task sequences under the curriculum. In the Japanese problem-solving approach, the task given by the teacher has the objective to produce the problematic which is necessary for students to recognize unknown. When teachers read a textbook without considering the theories, they cannot recognize the students difficulty because they know "A" already. On this condition, the key to the Japanese problem-solving approach is that the students reinvent the objective of the class set by their teacher as problematic, which should be solved because the problematic was planned by the teacher to encourage them to think mathematically. The contradiction in the planned sequence provides necessary dialectic-argumentation in this context. Given this limitation, the following exemplar (Fig. 2) on how the Japanese use the board in the lesson is meaningful. The Japanese approach means all those consequences and does not imply just a method of teaching like the scaffolding used to construct a building. Because the methods include the content of teaching. Every component of Fig 2 is explained by the theories for designing the classroom.

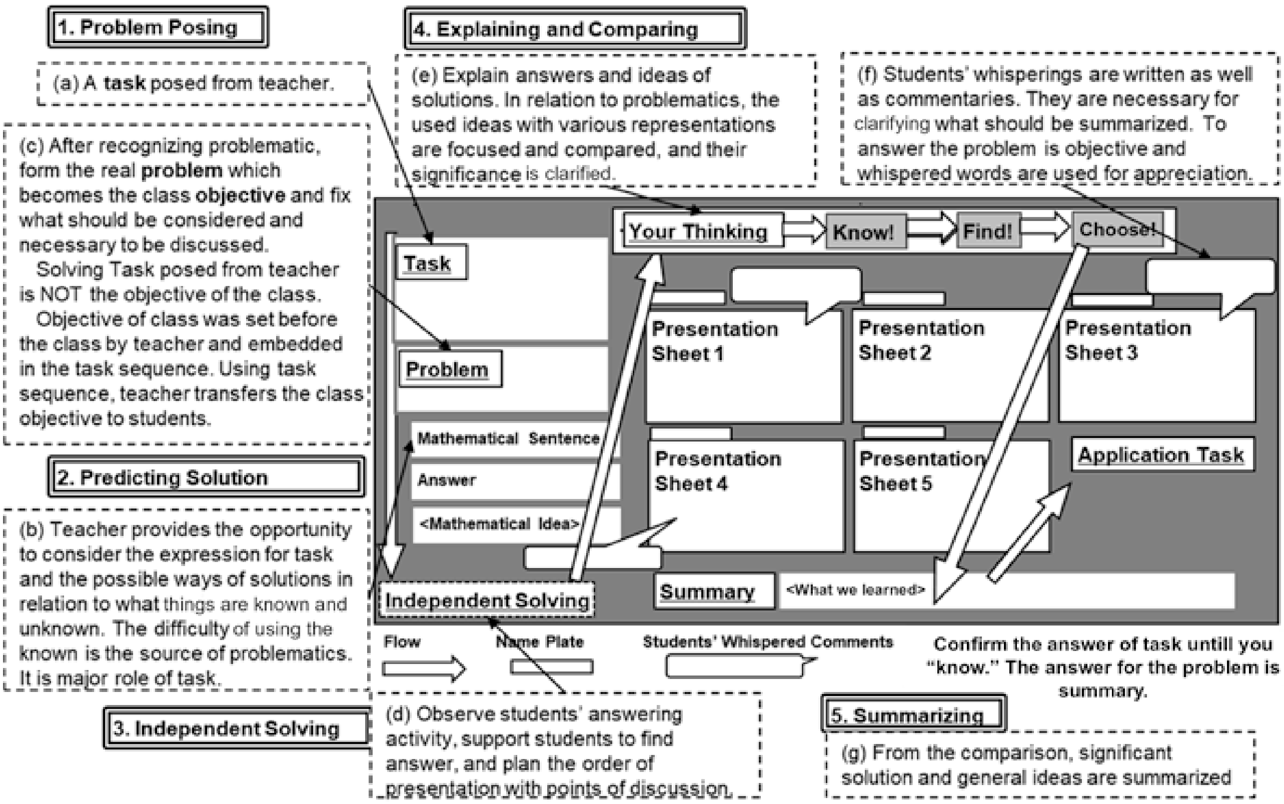

Figure 2: A Board Using Plan for Problem–Solving Approach (Isoda&Katagiri, 2012)

There are several other theories in Japan, and many of them have been proposed through critical discussions such as curriculum sequences. For example, the extension and integration principle provided provides task sequences that go against the general-to-specific principle proposed by Toyama group (AMI) since the 1950s which was called the water supply method (see Kobayasi, 1989). Toyama group criticized the National Curriculum and the certificated textbooks because they might produce misconceptions. They believed Toyama's theory never produce misconceptions because it avoided conceptual changes of Fig.1. Thus, the government side had to set the principle of extension and integration as the fallibilism nature of mathematics under modernization. Problem solving approach is the approach beyond misconceptions amongst task sequence. At that time, against the Toyama's critique, several counter theories were proposed to support extension and integration (on demands to develop mathematical thinking), such as Ito's theory (1961, 1962a, 1962b, 1962c, 1963a, 1963b, 1963c). Ito's theory to mediate ideas by models such as proportional number lines (Ito, 1968) was named "discovery methods" by Ito (published in English by Ito (1971)). He proposed the methods by preparing various representations and models for overcoming misconception by overgeneralization, and engaged lesson study with his lesson study groups. Odaka (1969, 1970, 1971, 1972, 1975, 1979, 1980, and 1982) established a schema theory with set tasks for the problem-solving approach, inspired by the idea of Piaget and the perspectives of mathematization in the 1943 textbook. Odaka produced a counter theory to explain an appropriate curriculum and task sequence—called the "exemplar approach"—against Toyama's sequence and schema theory based on materialism. Tadao Kaneko, written by Sakai and Hasegawa (1989), also theorized a task sequence for specific-to-general and general-to-special exercises (1987). Shigeru Shimada proposed the open-ended approach (1997; 1977 in Japan) based on his experience of the 1943 textbook under the mathematization principle, and Nobuhiko Nohda (1983) retheorized it as the open approach with perspective of German Didactics. There were discussions about embedding open-ended tasks into textbooks in the 1980s. Their theory for open-ended tasks itself did not indicate the task sequence for conceptual development directly. Odaka's, Kaneko's, and Isoda's theories were proposed for the ways to establish task sequence as for conceptual development on their theoretical bases. They published a number of books with several exemplar written by their lesson study groups. Their exemplar included the model protocols, not actual protocol itself, under the manner of design and reproducible science of lesson.

This paper illustrated how Japanese developed theories for designing and reproducing science.

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