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Every sufficiently cohesive topos is infinitesimally generated. (English. French summary)

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A topos \mathcal{E} is said to be *weakly generated* by a full subcategory \mathcal{C} of \mathcal{E} if the largest subtopos \mathcal{E} is the smallest subtopos of \mathcal{E} containing \mathcal{C} . It is well known that every topos is weakly generated by its subobject classifier. This paper, consisting of 8 sections, aims to clarify the basic observation that certain gros toposes are weakly generated by an object of exactly one point. *F. W. Lawvere* [“Toposes of laws of motion”, <http://www.acsu.buffalo.edu/~wlawvere/ToposMotion.pdf>] has claimed that “the basic program of infinitesimal calculus, continuum mechanics, and differential geometry is that all the world can be reconstructed from the infinitesimally small” and then proposes a mathematical formulation of the idea: a topos may be generated by a single object T “which in some of several senses is infinitely small. Of course, T is not just a single point, but it may have only a single point, or more generally the set of components functor may agree with the functor represented by 1 on T and its products and sums”. This proposal was refined in §7 of [*F. W. Lawvere*, *Theory Appl. Categ.* 19, 41–49 (2007; Zbl 1123.18001)], and this paper aims to elaborate on it.

The organization of the paper goes as follows. In §2 the author introduces the notion of substantial object and establishes a sufficient condition for such an object to weakly generate. In §3 consideration is restricted to the case where the substantial object is the classifier of dense monos determined by a subtopos, the main result being that, for the double negation topology, substantiality is enough to weakly generate. §4 is devoted to characterizing the quasi-closed topologies whose associated sheafification functors preserve the subobject classifier. In §5 the author incorporates, into the general context of a subtopos, a left adjoint to the sheafification, in which a sufficient condition for substantiality is obtained. It is shown in §6 that every sufficiently cohesive topos over a Boolean base is weakly generated by its subcategory of Leibniz objects. In §7 a characterization of substantial objects in toposes is presented. §8 addresses the author’s planned work on the monoid J^J and its submonoid of Euler reals, suggesting that the object J is the $\llbracket \rrbracket$ -closure of a point in a rig, just as the object of infinitesimals considered in [Zbl 1123.18001].

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

18B25 Topoi

18F20 Presheaves and sheaves, stacks, descent conditions (category-theoretic aspects)

03B99 None of the above, but in this section

Keywords:

topos theory; axiomatic cohesion

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