No.
Date 6/13(火). 第9回 数理科学IIIA

Category (圏) 代数

$\mathbb{R}$上の線形空間　準同型 線形写像　$x \in A$　$a \in \mathbb{R}$

$F: A \to B$　$F(ax) = aF(x)$　演算を保つ.
$F(x+y) = F(x) + F(y)$

群 $F: A \to B$　$(x \cdot y) \cdot z = x \cdot (y \cdot z)$
$F(x \cdot y) = F(x) \cdot F(y)$　$F(e_A) = e_B$
$F(x^{-1}) = F(x)^{-1}$

$\mathbb{C}, \mathbb{D}$: categories (圏)

$F: \mathbb{C} \to \mathbb{D}$ functor (関手 かんしゅ)

$A \in \mathbb{C} \mapsto F(A) \in \mathbb{D}$　objects

$f \in \mathrm{Mor}\,\mathbb{C} \mapsto F(f) \in \mathrm{Mor}\,\mathbb{D}$

条件 $F(\mathrm{dom}(f)) = \mathrm{dom}(F(f))$　$f: A \to B$
$F(f): F(A) \to F(B)$
$F(\mathrm{cod}(f)) = \mathrm{cod}(F(f))$

$F(id_A) = id_{F(A)}$

No.
Date

$A \xrightarrow{f} B \xrightarrow{g} C$ in $\mathbb{C}$　$F(g \circ f) = F(g) \circ F(f)$

$\mathbb{C}^{op}$　opposite category (dual category)　双対圏 そうついけん

$F: \mathbb{C} \to \mathbb{D}$ covariant functor

$F: \mathbb{C}^{op} \to \mathbb{D}$ functor　Contravariant functor from $\mathbb{C}$ to $\mathbb{D}$

$A \xrightarrow{f} B$ in $\mathbb{C}$
$F(A) \xleftarrow{F(f)} F(B)$ in $\mathbb{D}$

$A \xrightarrow{f} B \xrightarrow{g} C$　$F(g \circ f) = F(f) \circ F(g)$

$\mathbb{C} \underset{G}{\overset{F}{\rightrightarrows}} \mathbb{D}$　$\mathbb{C}, \mathbb{D}$: categories
F, G : functors from $\mathbb{C}$ to $\mathbb{D}$

natural transformation (自然変換)

$A \in \mathbb{C} \longmapsto \alpha_A : F(A) \to G(A)$ in $\mathbb{D}$

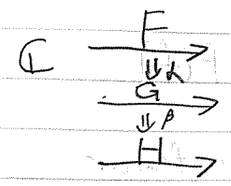
条件 $A \xrightarrow{f} B$ in $\mathbb{C}$

$$\begin{array}{ccc} F(A) & \xrightarrow{\alpha_A} & G(A) \\ {\scriptstyle F(f)}\downarrow & \circlearrowright & \downarrow{\scriptstyle G(f)} \\ F(B) & \xrightarrow[\alpha_B]{} & G(B) \end{array}$$

No.

Date . .

vertical composition



$\mathbb{C}, \mathbb{D}$: categories

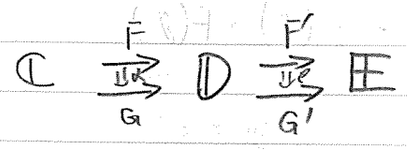
H, F, G : functors from $\mathbb{C}$ to $\mathbb{D}$

$\alpha : F \Rightarrow G$

$\beta : G \Rightarrow H$

$(\beta \circ \alpha)_A = \beta_A \circ \alpha_A$

$$\begin{array}{ccccc} F(A) & \xrightarrow{\alpha_A} & G(A) & \xrightarrow{\beta_A} & H(A) \\ {\scriptstyle F(f)}\downarrow & \circlearrowright & {\scriptstyle G(f)}\downarrow & \circlearrowright & {\scriptstyle H(f)}\downarrow \\ F(B) & \xrightarrow[\alpha_B]{} & G(B) & \xrightarrow[\beta_B]{} & H(\beta) \end{array}$$



$\mathbb{C}, \mathbb{D}, \mathbb{E}$: categories

$F, G : \mathbb{C} \to \mathbb{D}$ functors

$F', G' : \mathbb{D} \to \mathbb{E}$ functors

$\alpha : F \Rightarrow G$ natural trans

$\beta : F' \Rightarrow G'$

$\beta \cdot \alpha : F' \cdot F \Rightarrow G' \cdot G$

horizontal composition

$(\beta \cdot \alpha)_A : F'(F(A)) \to G'(G(A))$

$$\begin{array}{ccc} F(A) & \xrightarrow{\alpha_A} & G(A) \\ {\scriptstyle F'}\downarrow & \circlearrowright & \downarrow{\scriptstyle \beta_{G(A)}} \\ F'(F(A)) & \xrightarrow{F'(\alpha_A)} & F'(G(A)) \\ {\scriptstyle \beta_{F(A)}}\downarrow & \circlearrowright & \downarrow{\scriptstyle \beta_{G(A)}} \\ G'(F(A)) & \xrightarrow{G'(\alpha_A)} & G'(G(A)) \end{array}$$

$\beta_{G(A)} \circ F(\alpha_A)$

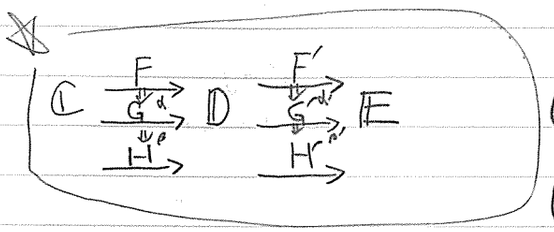
AIII 宇宙基礎 四の巣

No. (火)2

Date . .

$A \xrightarrow{f} B$ in $\mathbb{C}$ F'を掛ける.

$$\begin{array}{ccc} F(A) & \xrightarrow{\alpha_A} & G(A) \\ {\scriptstyle F(f)}\downarrow & \circlearrowright & \downarrow{\scriptstyle G(f)} \\ F(B) & \xrightarrow{\alpha_B} & G(B) \end{array} \qquad \begin{array}{ccc} F'(F(A)) & \xrightarrow{F'(\alpha_A)} & F'(G(A)) \\ {\scriptstyle F'(F(f))}\downarrow & \circlearrowright & \downarrow{\scriptstyle F'(G(f))} \\ F'(F(B)) & \xrightarrow[F'(\alpha_B)]{} & F'(G(B)) \end{array}$$



$(\beta' \circ \alpha') \circ (\beta \circ \alpha) =$

$(\beta' \circ \beta) \circ (\alpha' \circ \alpha)$

証明せよ.