No.
Date 6/13(火).

第9回 数理科学IIIA

Category(圏)　代数

$\mathbb{R}$上の線形空間　準同型　$x \in A$

線形写像　$a \in \mathbb{R}$

$F: A \to B$　$F(\alpha x) = \alpha F(x)$　演算を保つ.

$F(x+y) = F(x) + F(y)$

群

$F: A \to B$　$(x \cdot y) \cdot z = x \cdot (y \cdot z)$

$F(x \cdot y) = F(x) \cdot F(y)$　$F(e_A) = e_B$

$F(x^{-1}) = F(x)^{-1}$

$\mathbb{C}, \mathbb{D}$: categories (圏)

$F: \mathbb{C} \to \mathbb{D}$ functor (関手)
かんしゅ

$A \in \mathbb{C} \mapsto F(A) \in \mathbb{D}$　objects

$f \in \mathrm{Mor}\,\mathbb{C} \mapsto F(f) \in \mathrm{Mor}\,\mathbb{D}$

条件 $F(\mathrm{dom}(f)) = \mathrm{dom}(F(f))$　$f: A \to B$

$F(\mathrm{cod}(f)) = \mathrm{cod}(F(f))$　$F(f): F(A) \to F(B)$

$F(id_A) = id_{F(A)}$

Zorn　No.　Date

$A \xrightarrow{f} B \xrightarrow{g} C$ in $\mathbb{C}$　$F(g \circ f) = F(g) \circ F(f)$

$\mathbb{C}^{op}$　opposite category
そうつい
双対圏
(dual category)

$F: \mathbb{C} \to \mathbb{D}$　covariant functor

$F: \mathbb{C}^{op} \to \mathbb{D}$　functor　Contravariant functor from $\mathbb{C}$ to $\mathbb{D}$

$A \xrightarrow{f} B$ in $\mathbb{C}$

$F(A) \xleftarrow{F(f)} F(B)$ in $\mathbb{D}$

$A \xrightarrow{f} B \xrightarrow{g} C$　$F(g \circ f) = F(f) \circ F(g)$

$$\begin{tikzcd} \mathbb{C} \arrow[r, shift left, "F"] \arrow[r, shift right, "G"'] & \mathbb{D} \end{tikzcd}$$

$\mathbb{C}, \mathbb{D}$: categories

F, G : functors from $\mathbb{C}$ to $\mathbb{D}$

natural transformation (自然変換)

$A \in \mathbb{C} \longmapsto \alpha_A : F(A) \to G(A)$ in $\mathbb{D}$

条件 $A \xrightarrow{f} B$ in $\mathbb{C}$

$$\begin{tikzcd} F(A) \arrow[r, "\alpha_A"] \arrow[d, "F(f)"'] \arrow[dr, phantom, "\circlearrowright"] & G(A) \arrow[d, "G(f)"] \\ F(B) \arrow[r, "\alpha_B"'] & G(B) \end{tikzcd}$$

vertical composition

$$\begin{tikzcd}
\mathbb{C} \arrow[r, bend left=60, "F"{name=F}] \arrow[r, "G" description, ""{name=G}] \arrow[r, bend right=60, "H"'{name=H}] & \mathbb{D}
\arrow[Rightarrow, from=F, to=G, "\alpha"] \arrow[Rightarrow, from=G, to=H, "\beta"]
\end{tikzcd}$$

$\mathbb{C}, \mathbb{D}$: categories

H, F, G : functors from $\mathbb{C}$ to $\mathbb{D}$

$\alpha : F \Rightarrow G$

$\beta : G \Rightarrow H$

$(\beta \circ \alpha)_A = \beta_A \circ \alpha_A$

$$\begin{tikzcd}
F(A) \arrow[r, "\alpha_A"] \arrow[d, "F(f)"'] \arrow[dr, phantom, "\circlearrowleft"] & G(A) \arrow[r, "\beta_A"] \arrow[d, "G(f)"'] \arrow[dr, phantom, "\circlearrowleft"] & H(A) \arrow[d, "H(f)"'] \\
F(B) \arrow[r, "\alpha_\beta"'] & G(B) \arrow[r, "\beta_B"'] & H(\beta)
\end{tikzcd}$$

$$\begin{tikzcd}
\mathbb{C} \arrow[r, bend left, "F"{name=F}] \arrow[r, bend right, "G"'{name=G}] & \mathbb{D} \arrow[r, bend left, "F'"{name=F2}] \arrow[r, bend right, "G'"'{name=G2}] & \mathbb{E}
\arrow[Rightarrow, from=F, to=G, "\alpha"] \arrow[Rightarrow, from=F2, to=G2, "\beta"]
\end{tikzcd}$$

$\mathbb{C}, \mathbb{D}, \mathbb{E}$: categories
$F, G : \mathbb{C} \to \mathbb{D}$ functors
$F', G' : \mathbb{D} \to \mathbb{E}$ functors
$\alpha : F \Rightarrow G$ natural trans
$\beta : F' \Rightarrow G'$

$\beta \bullet \alpha : F' \bullet F \Rightarrow G' \bullet G$

horizontal composition

$(\beta \bullet \alpha)_A : F'(F(A)) \to G'(G(A))$

F'

$F(A) \xrightarrow{\alpha_A} G(A)$

$\beta_{G(A)} \circ F(\alpha_A)$

$$\begin{tikzcd}
F'(F(A)) \arrow[r, "F'(\alpha_A)"] \arrow[d, "\beta_{F(A)}"'] \arrow[dr, phantom, "\circlearrowleft"] \arrow[dr, bend left] \arrow[dr, bend right=80] & F'(G(A)) \arrow[d, "\beta_{G(A)}"] \\
G'(F(A)) \arrow[r, "G'(\alpha_A)"] & G'(G(A))
\end{tikzcd}$$

$A \xrightarrow{f} B$ in $\mathbb{C}$　F'を掛ける.

$$\begin{tikzcd}
F(A) \arrow[r, "\alpha_A"] \arrow[d, "F(f)"'] \arrow[dr, phantom, "\circlearrowleft"] & G(A) \arrow[d, "G(f)"] \\
F(B) \arrow[r, "\alpha_B"] & G(B)
\end{tikzcd}$$

$$\begin{tikzcd}
F'(F(A)) \arrow[r, "F'(\alpha_A)"] \arrow[d, "F'(F(f))"'] \arrow[dr, phantom, "\circlearrowleft"] & F'(G(A)) \arrow[r, "\beta_{G(A)}"] \arrow[d, "F'(G(f))"] \arrow[dr, phantom, "\circlearrowleft"] & G'(G(A \arrow[d, "G'(G("] \\
F'(F(B)) \arrow[r, "F'(\alpha_B)"'] & F'(G(B)) \arrow[r, "\beta_{G(B)}"] & G'(G(B)
\end{tikzcd}$$

☆

$$\begin{tikzcd}
\mathbb{C} \arrow[r, bend left=60, "F"{name=F}] \arrow[r, "G" description, ""{name=G}] \arrow[r, bend right=60, "H"'{name=H}] & \mathbb{D} \arrow[r, bend left=60, "F'"{name=F2}] \arrow[r, "G'" description, ""{name=G2}] \arrow[r, bend right=60, "H'"'{name=H2}] & \mathbb{E}
\arrow[Rightarrow, from=F, to=G, "\alpha"] \arrow[Rightarrow, from=G, to=H, "\beta"]
\arrow[Rightarrow, from=F2, to=G2, "\alpha'"] \arrow[Rightarrow, from=G2, to=H2, "\beta'"]
\end{tikzcd}$$

$(\beta' \circ \alpha') \circ (\beta \circ \alpha) =$

$(\beta' \circ \beta) \circ (\alpha' \circ \alpha)$

証明せよ.