No. 6/6(火)
Date

第8回 数理科学III A

Category theory (圏論)

$\Uparrow$

代数的位相幾何学 (Algebraic Topology) 幾 数学

Einlenberg - MacLane

集合入門

直積 $A\times B=\{(a,b)\mid a\in A, b\in B\}$ 可換 commutative commute

projection (射影)

$(a,b) \to a$

$$\begin{tikzcd} & A\times B \arrow[dl, "pr_1"'] \arrow[dr, "pr_2"] & \\ A \arrow[r, phantom, "\circlearrowleft"] & C \arrow[u, dashed, "\exists! h"] \arrow[l, "f_1"] \arrow[r, "f_2"'] \arrow[r, phantom, "\circlearrowleft", shift left=3] & B \end{tikzcd}$$

$\exists!\, h: C\to A\times B$

such that

$pr_1\circ h=f_1$

$pr_2\circ h=f_2$

直和

$$\begin{tikzcd} & C & \\ A \arrow[ur, bend left, "g_1"] \arrow[r, "in_1"] & A\sqcup B \arrow[u, dashed, "h"] & B \arrow[ul, bend right, "g_2"'] \arrow[l, "in_2"'] \end{tikzcd}$$

埋め込み injection

$\exists!\, h: A\sqcup B\to C$ such that

$g_1=h\circ in_1$

$g_2=h\circ in_2$

No.
Date

Category (定義) $\mathbb{C}$

objects : A, B, C $\quad ob(\mathbb{C})$

morphism (arrows) : f, g $\quad Mor(\mathbb{C})$

dom : $Mor(\mathbb{C})\to Ob(\mathbb{C})$ $\quad dom(f)=A$

cod : 〃 $\quad cod(f)=B$

$A\xrightarrow{f}B$

id : $Ob(\mathbb{C})\to Mor(\mathbb{C})$ $\quad id_A: A\to A$ 恒等写像

$dom(id_A)=A$

$cod(id_A)=A$

f, g $\quad \underline{cod(f)=dom(g)}$ (のとき) $\quad g\circ f$

$dom(g\circ f)=dom(f)$

$cod(g\circ f)=cod(g)$

$A\xrightarrow{f}B\xrightarrow{g}C\xrightarrow{h}D$

$h\circ(g\circ f)=(h\circ g)\circ f$ (結合律) $\quad id_A$

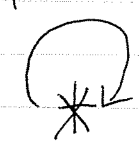
$f\circ id_A=f \quad id_B\circ f=f$

例 (examples)

I 集合 関数

III 群 準同型

V $\cancel{*}$ $*$ 単一の G



II 線型空間 ― 写像

IV 位相空間 連続写像

$e=id_*$

M monoid

preorder 擬順序

VI 順序集合 Partially ordered set $(A, \leq)$

objects morphism $a\leq b$ $\quad a\xrightarrow{f_{ab}}b$

$\boxed{a\leq a}$ (反射律)

$a\leq b$ & $b\leq a \Rightarrow a=b$ (反対称律)

$a\leq b$ & $b\leq c \Rightarrow a\leq c$ (推移律)

product $\mathbb{C}$

$$
\begin{tikzcd}
 & C \arrow[dl, bend right, "f_1"'] \arrow[d, dashed, "h"] \arrow[dr, bend left, "f_2"] & \\
A & A \times B \arrow[l, "pr_1"'] \arrow[r, "pr_2"] & B
\end{tikzcd}
$$

(universality)
普遍性による定義

coproduct 矢印を逆転

$\mathbb{C}$

$$A \xrightarrow{f} B$$

opposite
dual category

$\mathbb{C}^{op}$

$$A \xleftarrow{f^{op}} B$$

物の見方

$$
\begin{tikzcd}
 & C \arrow[d] \arrow[dl, bend right] \arrow[dr, bend left] & \\
A & A \times B \arrow[l] \arrow[r] & B
\end{tikzcd}
$$

conceptual

$A \times B \leq A$

$A \times B \leq B$

$C \leq A$

$C \leq B$

下限
$C \leq A \times B$

矢印を逆転させると
上限

$\mathbb{C}$

$$
\begin{tikzcd}
A \arrow[r, shift left, "f"] & B \arrow[l, shift left, "g"] \arrow[l, bend left=50, "h"]
\end{tikzcd}
$$

AとBは 同型
isomorphic

$g \circ f = id_A$

$f \circ g = id_B$

$h \circ f = id_A$

$f \circ h = id_B$

f : isomorphism

$g = f^{-1}$

$$g = (h \circ f) \circ g = h \circ \underbrace{(f \circ g)}_{id_B} = h$$

$$
\begin{tikzcd}
 & A \times B \arrow[dl, "pr_1"'] \arrow[dr, "pr_2"] & \\
A & & B
\end{tikzcd}
\qquad
\begin{tikzcd}
 & A \times' B \arrow[dl, "pr_1'"'] \arrow[dr, "pr_2'"] & \\
A & & B
\end{tikzcd}
$$

$$
\begin{tikzcd}
 & A \times' B \arrow[dl] \arrow[dr] \arrow[d, "h_1"] & \\
A & A \times B \arrow[l] \arrow[r] \arrow[d, "h_2"] & B \\
 & A \times' B \arrow[ul] \arrow[ur] &
\end{tikzcd}
$$

$id_{A \times' B} = h_2 \circ h_1$

$id_{A \times B} = h_1 \circ h_2$