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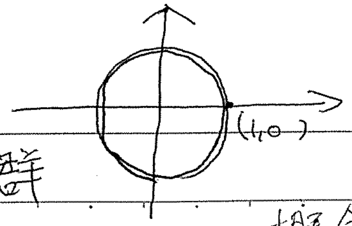
第4回 数理科学IIIA



$\pi(S^1,(1,0))$の決定　無限巡回群

概念

前回　cyclic group (巡回群)

位数　$\mathbb{Z}_2$　$x+x=0$

degree　　closed path

$S^1 \subseteq \mathbb{C}$ (複素平面)

z: 複素数

$a(z)$　angle 偏角　$\theta+2k\pi$

$\left.\begin{array}{ll}\theta_1 & a(z_1)\\ \theta_2 & a(z_2)\end{array}\right\} \Rightarrow \begin{array}{ll}\theta_1+\theta_2 & a(z_1 z_2)\\ \theta_1-\theta_2 & a(z_1/z_2)\end{array}$

$h: I \to S^1$ closed path $h(0)=h(1)=1$

$0=t_0<t_1<\cdots<t_n=1$

such that $t, t' \in [t_{i-1}, t_i] \Rightarrow |h(t')-h(t)|<1$

Lebesgue 数

θ_i　偏角　$h(t_i)/h(t_{i-1})$

such that $-\frac{\pi}{2}<\theta_i<\frac{\pi}{2}$

$|h(t_i)-h(t_{i-1})|<1$

degree of $h = \frac{1}{2\pi}\sum_{i=1}^{n}\theta_i$

整数　for $\prod_{i=1}^{n}\frac{h(t_i)}{h(t_{i-1})} = \frac{h(t_n)}{h(t_0)} = \frac{1}{1} = 1$

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independent of the choice of the subdivision of I.

two subdivisions have a common refinement

$[t_{i-1}, t_i]$　$t_{i-1}<s<t_i$　細分(refine)

θ_i

θ_i'　θ_i''

$\theta_i' = a\left(\frac{h(s)}{h(t_{i-1})}\right)$　$\theta_i'' = a\left(\frac{h(t_i)}{h(s)}\right)$　$|\theta_i'|<\frac{\pi}{2}$　$|\theta_i''|<\frac{\pi}{2}$

$\left.\begin{array}{l}\theta_i\\ \theta_i'+\theta_i''\end{array}\right\} h(t_i)/h(t_{i-1})$

$\theta_i-(\theta_i'+\theta_i'') = 2\pi m$　$|\ |<\frac{\pi}{2}$

$m=0$

$h \sim g$ (homotope) $\Rightarrow$ $\boxed{\text{degree of } h = \text{degree of } g}$

$F: I\times I \to S^1$ ~~set~~ such that

$F(t,0)=h(t)$

$F(t,1)=g(t)$

$F(0,s)=F(1,s)=1$

$t_0=0<t_1<\cdots<t_n=1$

$s_0=0<s_1<\cdots<s_m=1$

S^1　diameter <1

F　$[t_{i-1},t_i]\times[s_{j-1},s_j]$

$(t,s),(t',s') \in [t_{i-1},t_i]\times[s_{j-1},s_j]$

$\Rightarrow |F(t,s)-F(t',s')|<1$

$\theta_i' = a\left(\dfrac{F(t_i,s_{j-1})}{F(t_{i-1},s_{j-1})}\right)$ $|\theta_i'| \le \frac{\pi}{2}$

$\theta_i'' = a\left(\dfrac{F(t_i,s_j)}{F(t_{i-1},s_j)}\right)$ $|\theta_i''| < \frac{\pi}{2}$

$j=1,2,\dots,m$

以下を証明したい。

$$\boxed{\sum_{i=1}^{n}\theta_i' = \sum_{i=1}^{n}\theta_i''}$$

$\varphi_i = a\left(\dfrac{F(t_i,s_j)}{F(t_i,s_{j-1})}\right)$ $|\varphi_i| < \frac{\pi}{2}$ for $i=0,1,\dots,n$

$\theta_i''-\theta_i'$ $\varphi_i-\varphi_{i-1}$

偏角 $2\pi m$

$\dfrac{F(t_i,s_j)F(t_{i-1},s_{j-1})}{F(t_{i-1},s_j)F(t_i,s_{j-1})}$ $\theta_i''-\theta_i'=\varphi_i-\varphi_{i-1}$

$\sum\theta_i'' - \sum\theta_i' = \sum(\varphi_i-\varphi_{i-1}) = \varphi_n-\varphi_0$

$\varphi_0-\varphi_n=0$ $\beta\in\pi(S^1)$

$E^n=\{x\in\mathbb{R}^n \mid |x|\le 1\}$

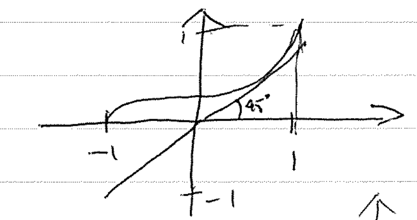
$n\le 2$

不動点定理

Theorem Any continuous map f of E^n into itself has at least one fixed point

$n=1$

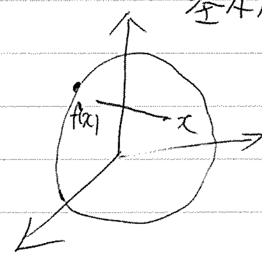
$E^1=[-1,1]$



$\pi_1(S^1)\to\pi_1(E^2)\to\pi_1(S^1)$ 基本群

$\mathbb{Z}$ 球

$S^1\to E^2\to S^1$



S^1はE^2のretract