No. 10/31(火)
Date

第3回 数理~~物質科~~科学IIIB

monoidal category

monoid

M：集合　$(x\cdot y)\cdot z = x\cdot(y\cdot z)$ 結合律

・2項演算　単位元 $e\cdot x = x\cdot e = e$

ex) 集合と写像の category Set

自然な形

$$(X\times Y)\times Z \cong X\times(Y\times Z)$$

bifunctor

$(X,Y)\mapsto X\times Y$

$Set\times Set \to Set$

自然

$$X\times\{*\}\cong X$$

形式的定義

monoidal category

$$\mathcal{B}=\langle B,\square,e,\alpha,\lambda,\rho\rangle$$

B : category

$\square : B\times B \to B$　bifunctor

$e\in B$: object

$\alpha = \alpha_{a,b,c} : a\square(b\square c)\cong(a\square b)\square c$

natural isomorphism

$\lambda_a : e\square a\cong a$

$\rho_a : a\square e\cong a$

① Pentagon diagram

$$\begin{tikzcd}
a\square(b\square(c\square d)) \arrow[r, "\alpha_{a,b,c\square d}"] \arrow[d, "1_a\square\alpha_{b,c,d}"'] & (a\square b)\square(c\square d) \arrow[r, "\alpha_{a\square b,c,d}"] & ((a\square b)\square c)\square d \\
a\square((b\square c)\square d) \arrow[rr, "\alpha_{a,b\square c,d}"'] & & (a\square(b\square c))\square d \arrow[u, "\alpha_{a,b,c}\square 1_d"']
\end{tikzcd}$$

② triangular diagram

$$\begin{tikzcd}
a\square(e\square c) \arrow[r, "\alpha_{a,e,c}"] \arrow[dr, "1_a\square\lambda_c"'] & (a\square e)\square c \arrow[d, "\rho_a\square 1_c"] \\
& a\square c
\end{tikzcd}$$

③ $\lambda_e=\rho_e : e\square e\to e$

$$\begin{tikzcd}
e\square(b\square c) \arrow[r, "\alpha_{e,b,c}"] \arrow[d, "\lambda_{b\square c}"'] & (e\square b)\square c \arrow[dl, "\lambda_b\square 1_c"] \\
b\square c
\end{tikzcd}$$

$$\begin{tikzcd}
a\square(b\square e) \arrow[r, "\alpha_{a,b,e}"] \arrow[dr, "1_a\square\rho_b"'] & (a\square b)\square e \arrow[d, "\rho_{a\square b}"] \\
& a\square b
\end{tikzcd}$$

↑ レポート

$e\square(b\square c)$
≅

$$\begin{tikzcd}
a\square(e\square(b\square c)) \arrow[r] \arrow[d] & (a\square e)\square(b\square c) \arrow[r] & ((a\square e)\square b)\square c \\
a\square((e\square b)\square c) \arrow[rr] & & (a\square(e\square b))\square c \arrow[u]
\end{tikzcd}$$

≅ $(e\square b)\square c$ ≅

$(a\square b)\square c$

e　≅　$b\square c$

M　□ 2項

$(a\square b)\square c = a\square(b\square c)$

$a\square b = b\square a$