

Testing the importance of collective correlations in neutrinoless $\beta\beta$ decay

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We investigate the extent to which theories of collective motion can capture the physics that determines the nuclear matrix elements governing neutrinoless double- β decay. To that end we calculate the matrix elements for a series of isotopes in the full pf shell, omitting no spin-orbit partners. With the inclusion of isoscalar pairing, a separable collective Hamiltonian that is derived from the shell model effective interaction reproduces the full shell-model matrix elements with good accuracy. A version of the generator coordinate method that includes the isoscalar pairing amplitude as a coordinate also reproduces the shell model results well, an encouraging result for theories of collective motion, which can include more single-particle orbitals than the shell model. We briefly examine heavier nuclei relevant for experimental double- β decay searches, in which shell-model calculations with all spin-orbit partners are not feasible; our estimates suggest that isoscalar pairing also plays a significant role in these nuclei, though one we are less able to quantify precisely.

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I. INTRODUCTION

Neutrinoless double-beta ($0\nu\beta\beta$) decay, if observed, would tell us that neutrinos are their own antiparticles, i.e., Majorana particles. It has the potential to reveal the overall neutrino mass scale and hierarchy as well, but only to the extent that we know the nuclear matrix elements that, together with the mass scale, determine the decay rate [1]. The matrix elements, which must be calculated, are at present quite uncertain [2], and reducing that uncertainty is becoming increasingly urgent as decisions on planning and funding ton-scale $\beta\beta$ decay experiments draw near [3–5].

Theorists compute the $0\nu\beta\beta$ -decay nuclear matrix elements in a variety of models, including the shell model [6,7], the interacting boson model (IBM) [8], the quasiparticle random-phase approximation (QRPA) [9–11], and the generator coordinate method (GCM) [12–14], the last two of which can incorporate energy density functional (EDF) theory [9,12,13]. The GCM and the IBM pay particular attention to collective phenomena such as deformation and pairing, and neglect, for the most part, noncollective correlations. The prominence of these collective approaches makes it important to know how accurate they can be or, in other words, the extent to which collective correlations determine the $0\nu\beta\beta$ decay nuclear matrix elements. And among all possible collective correlations, which are the most relevant for $0\nu\beta\beta$ decay? These questions have never been addressed in a systematic way.

Although we will not provide conclusive answers, we can fully address the questions in nuclei for which shell model calculations in a full harmonic-oscillator shell are feasible. A full-shell valence space is useful because it guarantees spin-orbit partners for all orbitals. Spin-orbit pairs are important because the two-body matrix elements of the $0\nu\beta\beta$ decay operator between these orbitals are typically large. Spin-orbit partners are needed, furthermore, to fully capture the effects of isoscalar spin-one pairing between protons and neutrons. The pf shell is a prime example of a space in which each orbital has a spin-orbit partner, and shell model calculations there are very successful [15]. They can describe collective phenomena such as deformation despite neglecting cross-shell correlations, which are not very important for ground-state properties of lower pf -shell nuclei close to stability [15].

We thus explore the role of collective correlations in the pf shell, testing to see which collective degrees of freedom have the largest effect on $0\nu\beta\beta$ decay matrix elements and the degree to which the most important degrees of freedom determine the matrix elements. In larger valence spaces, which at present are still beyond what the shell model can treat well, the role of collective correlations is only expected to be greater, at least away from shell closures. Of course in most pf -shell nuclei $\beta\beta$ decay is either energetically forbidden or exceedingly slow compared to single- β decay; nevertheless, nuclear matrix elements can be calculated and their quality assessed. Some recent papers [16,17] have taken a similar approach, gaining insight into $\beta\beta$ decay through the systematic calculation of matrix elements in nuclei that could not themselves be used in $\beta\beta$ decay experiments.

Here, we conduct two kinds of tests. First, we extract from Ref. [18] the separable collective Hamiltonian that best approximates a full shell-model effective interaction in the pf

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shell. This Hamiltonian employs a monopole interaction and collective pieces: isovector $J = 0$ and isoscalar $J = 1$ pairing terms, a quadrupole-quadrupole term, and a spin-isospin term. We compare the $0\nu\beta\beta$ decay matrix elements that this interaction produces with those produced by the full shell model interaction in the Ca, Ti, and Cr isotopic chains (heavier elements are computationally more demanding, as well as more sensitive to orbitals beyond the pf shell), and identify the most relevant collective correlations for $\beta\beta$ decay. Second, we use the collective interaction within a GCM calculation that includes the isoscalar pairing amplitude and the quadrupole moment as generator coordinates, and compare the resulting $0\nu\beta\beta$ decay matrix elements to those of the shell model. Finally, we try to assess the degree to which our conclusions hold for the heavier nuclei in which $\beta\beta$ decay could be detected in next-generation experiments.

The rest of this paper is structured as follows. Section II describes the extraction of the separable collective interaction and discusses each of its components. Section III briefly presents the $0\nu\beta\beta$ decay operator and compares the matrix elements, calculated in the shell model with both the full and collective Hamiltonians, for isotopes of Ca, Ti, and Cr. It also shows GCM matrix elements for the same nuclei, calculated with the same collective interaction, and finally discusses the matrix elements for heavier nuclei that are of real interest for $0\nu\beta\beta$ decay experiments. Section IV is a conclusion.

II. SEPARABLE COLLECTIVE INTERACTION

We work in the pf -shell configuration space, comprising the $0f_{7/2}$, $1p_{3/2}$, $1p_{1/2}$, and $0f_{5/2}$ orbitals. As a reference Hamiltonian we use the shell model interaction KB3G [19], which has been extensively tested throughout the pf shell. This interaction provides a very good description of nuclear structure, including spectroscopy, electromagnetic and Gamow-Teller transitions, and deformation [15]. Then, following the work of Dufour and Zuker [18], we build the separable collective Hamiltonian that best approximates KB3G. Roughly speaking, Ref. [18] determines the structure of the lowest-lying collective states in the particle-hole and pairing representations with a given angular momentum J , isospin T , and parity π , and then constructs a series of separable terms, with appropriate strengths, that reproduce those states. Dufour and Zuker find that the most important terms in the particle-hole channel are the isoscalar quadrupole and spin-isospin ($\sigma\tau\sigma\tau$) interactions, and in the pairing channel the isovector $J^\pi = 0^+$ and isoscalar $J^\pi = 1^+$ interactions.

The separable collective Hamiltonian, H_{coll} , that includes the full monopole piece of the KB3G interaction and the dominant collective terms found by Dufour and Zuker has the form

$$H_{\text{coll}} = H_M + g^{T=1} \sum_{n=-1}^1 S_n^\dagger S_n + g^{T=0} \sum_{m=-1}^1 P_m^\dagger P_m + g_{ph} \sum_{m,n=-1}^1 : \mathcal{F}_{mn}^\dagger \mathcal{F}_{mn} : + \chi \sum_{\mu=-2}^2 : Q_\mu^\dagger Q_\mu : , \quad (1)$$

TABLE I. Strengths (in MeV) of the isovector pairing ($g^{T=1}$), isoscalar pairing ($g^{T=0}$), spin-isospin (g_{ph}), and quadrupole (χ) interactions in the separable collective Hamiltonian H_{coll} [Eq. (1)]. The values are taken from Ref. [18] and scaled to nucleon number $A = 42$. For heavier isotopes the strengths are multiplied by $(42/A)^{1/3}$.

$g^{T=1}$	$g^{T=0}$	g_{ph}	χ
-0.377	-0.587	0.057	-0.141

where the colons indicate normal ordering. The monopole Hamiltonian H_M includes two-body terms and one-body (single-particle) energies, both taken from KB3G. In addition

$$\begin{aligned} S_n^\dagger &= \frac{1}{\sqrt{2}} \sum_{\alpha} \sqrt{2l_{\alpha} + 1} (a_{\alpha}^\dagger a_{\alpha}^\dagger)_{0,0,n}^{0,0,1}, \\ P_m^\dagger &= \frac{1}{\sqrt{2}} \sum_{\alpha} \sqrt{2l_{\alpha} + 1} (a_{\alpha}^\dagger a_{\alpha}^\dagger)_{0,m,0}^{0,1,0}, \\ \mathcal{F}_{mn} &= 2 \sum_{\alpha} \sqrt{2l_{\alpha} + 1} (a_{\alpha}^\dagger \tilde{a}_{\alpha})_{0,m,n}^{0,1,1}, \\ Q_{\mu} &= \frac{1}{\sqrt{5}} \sum_{\alpha,\beta} \langle n_{\alpha} l_{\alpha} \| r^2 Y_2 / b^2 \| n_{\beta} l_{\beta} \rangle (a_{\alpha}^\dagger \tilde{a}_{\beta})_{\mu,0,0}^{2,0,0}, \end{aligned} \quad (2)$$

where \mathcal{F}_{mn} , written in first quantization, is just $\sum_i \sigma_m(i) \tau_n(i)$, b is the usual oscillator parameter, a_{α}^\dagger creates a nucleon in a single-particle orbital with principal quantum number n_{α} and orbital angular momentum l_{α} , and \tilde{a}_{α} destroys a nucleon in the time-reversed orbital [more precisely, $\tilde{a}_{\alpha} \equiv (-1)^{l_{\alpha} + 1 - m_{\alpha} - s_{\alpha} - \tau_{\alpha}} a_{l_{\alpha}, -m_{\alpha}, s_{\alpha}, -\tau_{\alpha}}$, where m_{α} is the z component of the orbital angular momentum, s_{α} is the z component of the spin, and τ_{α} is the z component of the isospin]. The superscripts following the parentheses stand for the two-particle orbital angular momentum, spin, and isospin, and the subscripts for their z components. The strengths of the various terms, $g^{T=1}$, $g^{T=0}$, g_{ph} , and χ , are taken from Ref. [18] and appear in Table I for mass $A = 42$ (they scale with $A^{-1/3}$). Note that the pairing and quadrupole-quadrupole terms are attractive, as expected. Reference [20] uses a similar collective Hamiltonian, also based on the decomposition in Ref. [18], but without the spin-isospin term, to study the competition between isovector and isoscalar pairing in pf -shell nuclei.

The significance of the various terms in H_{coll} is as follows: The monopole Hamiltonian H_M adds effective neutron- and proton-number-dependent effective single-particle energies to the bare energies. The remaining terms are collective—an isovector spin-0 pairing interaction, an isoscalar spin-1 pairing interaction, a quadrupole-quadrupole interaction, and a Landau-Migdal-style spin-isospin interaction. Many studies of nuclear collectivity (e.g., [21–23]) include only isovector pairing (usually without the proton-neutron part) and quadrupole-quadrupole terms. And isoscalar pairing is frequently downplayed. Among the models studying $0\nu\beta\beta$ decay matrix elements, the EDF-based GCM and the IBM have not yet included isoscalar pairing explicitly.

According to Ref. [18], the terms included in H_{coll} are the most important for pf -shell nuclei (we could also have included, for example, an isovector quadrupole-quadrupole

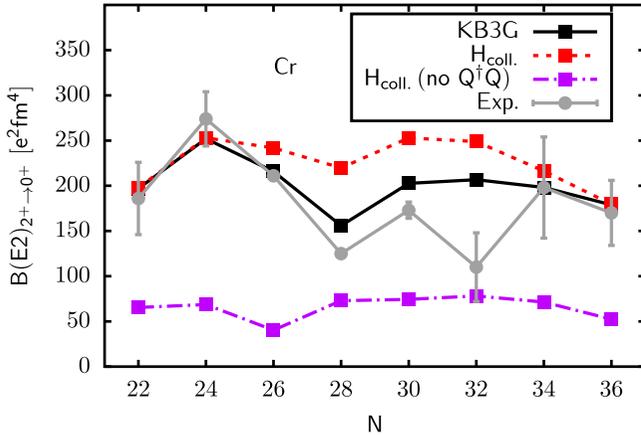


FIG. 1. $B(E2)$ values for transitions from the first 2^+ state to the ground state in Cr isotopes, as a function of mass number A . Experiment [24] (gray circles with error bars, connected by solid line) is compared to calculations with several interactions. Black squares (solid line) show results produced by the KB3G interaction, red squares (dashed line) by the full collective interaction H_{coll} , and purple squares (dotted line) by H_{coll} without the quadrupole-quadrupole part. Protons have an effective charge of $1.5e$ and neutrons of $0.5e$.

interaction, a hexadecupole-hexadecupole piece, or an isovector $J = 2$ pairing term). We can test the adequacy of H_{coll} by comparing its results to those of the KB3G interaction. Figure 1 shows calculated and experimental $B(E2)$ strengths from the lowest 2^+ state to the ground state in even-mass Cr isotopes, which have enough valence-space nucleons to exhibit collective behavior. The strengths produced by H_{coll} and KB3G agree nicely, both with each other and experiment (however KB3G describes the data better). A similar picture emerges from a comparison of $B(E2)$ strengths for Ca, Ti, and Fe isotopes. The agreement suggests that H_{coll} captures the correlations necessary for a good description of low-energy states in the pf -shell nuclei.

One advantage of the separable Hamiltonian is that we can selectively include or exclude particular terms to test their importance for $0\nu\beta\beta$ decay or for any other observable. It is well known that individual terms play unique roles in nuclear structure spectroscopy. The quadrupole-quadrupole interaction, for instance, is crucial for generating deformation and shape vibrations and thus for a good description of $B(E2)$ strengths. Figure 1 illustrates this fact; in the Cr isotopes the transition strengths are drastically suppressed when the quadrupole-quadrupole term is removed from the interaction. By contrast, the removal of other terms, such as isoscalar pairing, has only a small effect on the $B(E2)$'s.

III. RESULTS FOR DOUBLE-BETA DECAY

Here we examine the importance of the various parts of the separable collective Hamiltonian for $0\nu\beta\beta$ decay and the usefulness of models based on a collective picture, such as the GCM, for calculating the associated nuclear matrix elements.

These can be written in the form

$$M_{0\nu} = M_{0\nu}^{GT} - \left(\frac{g_V}{g_A}\right)^2 M_{0\nu}^F + M_{0\nu}^T, \quad (3)$$

with $g_V = 1.0$ and $g_A = 1.27$ the vector and axial coupling constants, respectively. A detailed definition of each term above appears in Ref. [25]. The Gamow-Teller (GT) matrix element, $M_{0\nu}^{GT}$, is the largest; in the nuclei studied here it accounts for about 85% of the total matrix element and for the sake of convenience we omit the other terms from the discussion to come. In addition, we use the closure approximation (with closure energy parameter $\langle E \rangle = 7.72$ MeV), which introduces an error of at most 10% in both shell model and QRPA calculations [26,27]. We also ignore the effects of two-body weak currents, studied in Refs. [28,29], and assume simple Argonne-type short-range correlations [30]. None of these choices and approximations affect any of our conclusions. Finally, we restrict our study to isotopic chains in the lower part of the pf shell, where exact shell-model calculations are possible and higher-lying single-particle orbitals are not relevant for ground-state properties.

A. $0\nu\beta\beta$ decay of pf -shell nuclei in the shell model

First we turn to the ability of the collective interaction H_{coll} to reproduce the shell model GT matrix elements in Eq. (3) that are obtained with the full KB3G shell model interaction [6,31]. In addition, to explore the roles of the various collective pieces in H_{coll} , we perform additional calculations, each time removing some combination of the isovector/isoscalar pairing, quadrupole-quadrupole, and spin-isospin terms. We never remove the monopole part, however, because its role is simply to fix the energies of single-particle orbitals.

Figure 2 shows that the matrix elements obtained from the full H_{coll} are very close to those obtained from the KB3G interaction. That a simple separable collective Hamiltonian can capture the main features of the matrix element suggests that fine details of the nuclear interaction affect $0\nu\beta\beta$ decay only moderately. The result confirms and significantly extends the findings of Refs. [32,33], which contained relatively small variations—between 10% and 20%—in the matrix elements obtained from different shell-model effective interactions.

Figure 2 also shows that the matrix elements change relatively little when the quadrupole-quadrupole interaction is excluded. This result holds not only for the mostly spherical Ca and Ti isotopes involved in the decay matrix elements of Ca, but also for the Ti and Cr isotopes (not shown), which involve deformed nuclei, as Fig. 1 indicates. This apparent lack of sensitivity to quadrupole correlations contrasts with the results of several studies in the shell model, EDF-based GCM, and QRPA that point to a reduction of the $0\nu\beta\beta$ decay matrix elements between deformed nuclei [9,34–36]. The reason is probably the moderate deformation of the pf -shell nuclei considered here. If the quadrupole correlations are increased by doubling the strength of the quadrupole-quadrupole interaction, the corresponding matrix elements shrink, as expected, by 30%–40%. In addition, when the quadrupole-quadrupole interaction is excluded from the calculation of the parent nuclei (but not the daughter nuclei), the matrix elements decrease by

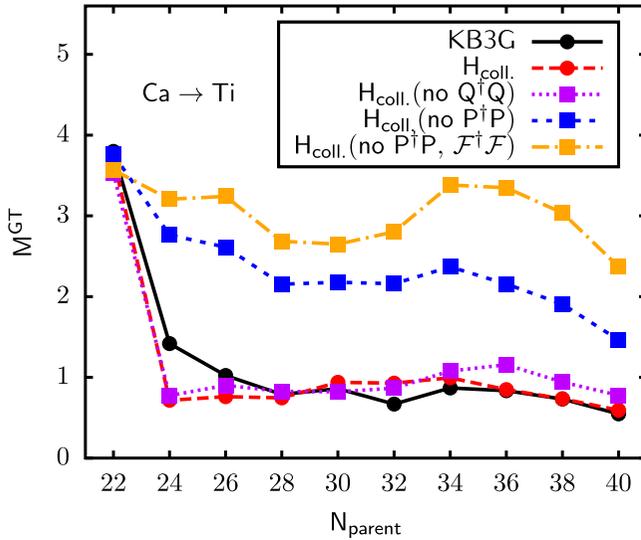


FIG. 2. Gamow-Teller part of the $0\nu\beta\beta$ decay matrix elements $M_{0\nu}^{GT}$, for the decay of Ca isotopes into Ti as a function of the neutron number N_{parent} in the parent nucleus. Results are shown for the KB3G interaction (black circles, solid line), the full collective interaction H_{coll} (red circles, dashed line), H_{coll} with the quadrupole-quadrupole term removed (purple squares, dotted line), H_{coll} with the isoscalar pairing term removed (blue squares, short-dashed line), and H_{coll} with both the isoscalar-pairing and spin-isospin pieces removed (orange squares, dot-dashed line).

15%–20%. That result is consistent with those of previous studies [9,34–36] that note a small matrix element when the parent and daughter have different quadrupole properties.

Perhaps the most striking feature of Fig. 2 is the suppression of the matrix elements by isoscalar pairing. Removing that term from the Hamiltonian increases the matrix elements by more than a factor of two (closer to three in many isotopes), or between 1 and 2 units. When, in addition, the spin-isospin term is removed, the matrix elements grow even further. As Fig. 3 shows, the large effect of isoscalar pairing is common to the matrix elements of all the Ca, Ti, and Cr isotopes we study, from those with $N \sim Z$ to very neutron-rich nuclei. For the matrix elements of the most isospin-asymmetric nuclei (^{58}Ca and ^{60}Ca) the effect of isoscalar pairing is somewhat milder but still important. The sensitivity to isoscalar (proton-neutron) pairing is familiar from QRPA [37,38] and GCM studies [14] and makes it clear that a good description of proton-neutron correlations is crucial to obtain accurate $0\nu\beta\beta$ decay nuclear matrix elements.

The significance of isoscalar pairing is not quite as straightforward as it first appears, however. The matrix elements vary just about 10% when only the spin-isospin interaction is omitted from H_{coll} . As Fig. 2 shows, when the spin-isospin term is included in the separable collective Hamiltonian, the impact of omitting isoscalar pairing, though still significant, is smaller than with the spin-isospin term excluded. This result suggests that the missing isoscalar-pairing correlations can to some extent be compensated for, or captured, by other collective interactions. In that sense, we can consider the dramatic changes in the matrix elements shown in Figs. 2

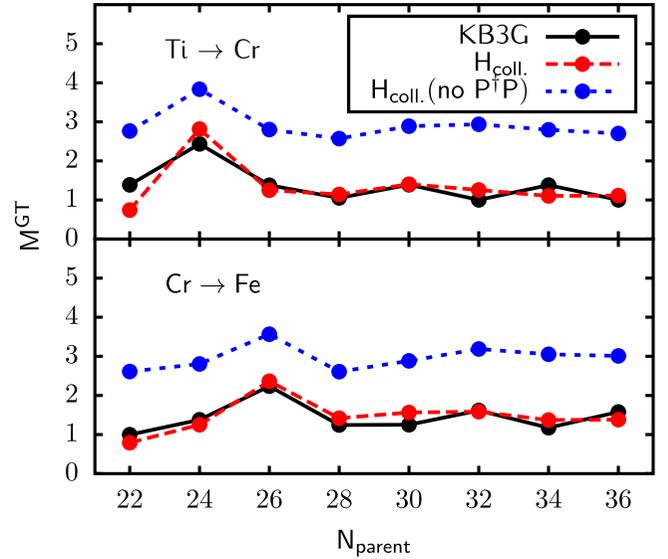


FIG. 3. Gamow-Teller part of the $0\nu\beta\beta$ decay matrix elements, $M_{0\nu}^{GT}$, for the decay of Ti isotopes into Cr (top panel), and Cr isotopes into Fe (bottom), as a function of the neutron number N_{parent} of the parent nucleus. Results are shown for the KB3G interaction (black, solid line), the collective interaction H_{coll} (red, dashed line), and H_{coll} without the isoscalar pairing term (blue, short-dashed line).

and 3 to be an upper bound for the effects of isoscalar pairing. Pieces of the nuclear Hamiltonian, both collective and noncollective, that are not included in H_{coll} might soften the impact of omitting isoscalar-pairing, in the same way that the spin-isospin interaction does.

The impact of isoscalar pairing correlations in $0\nu\beta\beta$ decay is undeniable. One way to understand it is through spin-isospin SU(4) symmetry. The GT operator, if we neglect the neutrino potential, is invariant under SU(4) transformations, implying that only states belonging to the same irreducible representations (irreps) of SU(4) can be connected by the operator; the matrix elements between states in different irreps vanish. Furthermore, in the absence of spin-orbit splitting in the H_M piece, the collective Hamiltonian H_{coll} is invariant under SU(4) if the isovector and isoscalar pairing terms have the same strength, $g^{T=1} = g^{T=0}$. The situation resembles that associated with the $\beta\beta$ decay Fermi operator, which because of isospin symmetry has vanishing matrix elements between states belonging to different isospin-SU(2) irreps, i.e., having different total isospin [39]. In $0\nu\beta\beta$ decay the neutrino potential breaks the SU(2) invariance of the operator and the matrix elements, $M_{0\nu}^F$, do not vanish, but they are nevertheless suppressed [6,8,10,17].

In pf -shell nuclei the spin-orbit splitting is sizable, and nuclear states are in general a combination of several different SU(4) irreps [40]. However, since $g^{T=0}$ is only slightly larger than $g^{T=1}$, and the spin-isospin interaction, which conserves the SU(4) symmetry, effectively increases the energy separation among SU(4) irreps, the fraction of irreps shared between the parent and daughter nuclei is small. This fact is illustrated in the top part of Fig. 4, which shows the percentage of the ground state in each Ti isotope (daughter nucleus) belonging in irreps

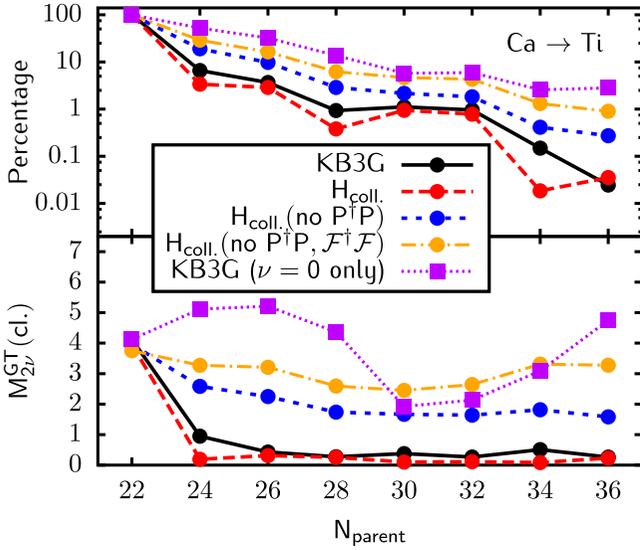


FIG. 4. Top panel: Percentage of ground state in daughter nuclei (Ti isotopes) belonging to SU(4) irreducible representations (irreps) that are also present in the corresponding parent nuclei (Ca isotopes), as a function of the neutron number N_{parent} of the parent nucleus. Results are shown for the KB3G effective interaction (black circles, solid line), the collective Hamiltonian H_{coll} (red circles, dashed line), H_{coll} without the isoscalar pairing term (blue circles, short-dashed line), H_{coll} without both the isoscalar pairing and the spin-isospin terms (orange circles, dot-dashed line), and the KB3G interaction diagonalized in a basis of a seniority-zero states (purple squares, dotted line). Bottom panel: $M_{2\nu}^{GT}(\text{cl.})$ (see text) as a function of N_{parent} . Correspondence between results and symbols/lines is the same as in the top panel.

that are also present in the ground state of the corresponding Ca isotope (parent). The small percentages mean that in the approximation that the neutrino potential is replaced by a constant, i.e., with the $0\nu\beta\beta$ decay operator replaced by the closure version of the $2\nu\beta\beta$ decay operator $M_{2\nu}^{GT}(\text{cl.})$, the matrix elements are tiny (see the bottom panel of Fig. 4). The result explains why M^{GT} for $0\nu\beta\beta$ decay, which reflects mild SU(4) breaking by the neutrino potential, is generally small rather than either tiny or large. The only exception is in mirror nuclei, where the irreps in the parent and daughter are identical. There the matrix elements are larger than others in the same isotopic chain, as both shell-model and GCM calculations show [6,16,17].

Little remains of SU(4) symmetry when the isoscalar pairing and the spin-isospin terms are removed from the Hamiltonian. As Fig. 4 shows, setting $g^{T=0} = 0$ causes the percentage of the ground states in parent and daughter nuclei belonging to shared SU(4) irreps to increase substantially, which in turn increases $M_{2\nu}^{GT}(\text{cl.})$. The effect is even stronger when the spin-isospin interaction is removed as well. And as Figs. 2 and 3 show, the $M_{0\nu}^{GT}$ matrix elements also increase dramatically. (The percentage of common irreps in the parent and daughter nuclei actually decreases faster with $N - Z$ than the matrix elements. The reason is that the matrix elements between states in the same irrep are proportional to $N - Z$ [41]).

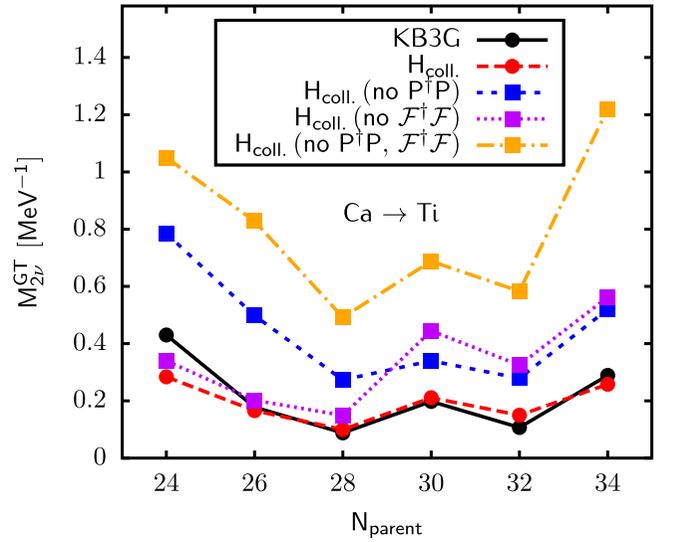


FIG. 5. Two-neutrino $\beta\beta$ decay matrix elements $M_{2\nu}^{GT}$, for the decay of Ca isotopes into Ti as a function of the neutron number N_{parent} in the parent nucleus. Results are shown for the KB3G interaction (black circles, solid line), the full collective interaction H_{coll} (red circles, dashed line), H_{coll} with the isoscalar pairing term removed (blue squares, short-dashed line), and H_{coll} with both isoscalar-pairing and spin-isospin parts removed (orange squares, dot-dashed line).

The same kind of SU(4) breaking is at play when ground states are forced to have seniority zero, that is, states consisting entirely of like-particle $J = 0$ pairs. By construction, seniority-zero states have no proton-neutron pairs or spin-isospin correlations and thus break SU(4) strongly. As a result, the percentage of the ground states in the parent and daughter nuclei belonging to shared irrep increases and both $M_{2\nu}^{GT}(\text{cl.})$ and $M_{0\nu}^{GT}$ grow (see Refs. [17,31]).

In addition, we study the impact of isoscalar pairing in $2\nu\beta\beta$ decay. The lower part of Fig. 4 suggests that spin-isospin and isoscalar pairing correlations are relevant for $2\nu\beta\beta$ decay, but for a detailed study the matrix elements need to be calculated beyond the closure approximation, because of the small momentum transfers involved in $2\nu\beta\beta$ transitions [2].

Figure 5 shows nonclosure $2\nu\beta\beta$ decay matrix elements calculated with the shell-model KB3G interaction, the collective Hamiltonian H_{coll} , and with the same Hamiltonian but excluding the isoscalar pairing and/or spin-isospin parts in H_{coll} . As in $0\nu\beta\beta$ decay, the results obtained with the full collective Hamiltonian H_{coll} are in very good agreement with the full shell-model results, suggesting that the collective Hamiltonian includes all the interaction components relevant for $2\nu\beta\beta$ decay and that fine details of the shell-model interaction only affect this decay moderately.

The impact of isoscalar pairing and spin-isospin correlations is sizable and, like in $0\nu\beta\beta$ decay, excluding both collective terms (or only the isoscalar pairing part) leads to significantly overestimated $2\nu\beta\beta$ decay matrix elements. Figure 5 also shows that for $2\nu\beta\beta$ decay, excluding only the spin-isospin interaction leads to overestimated matrix elements in neutron-rich nuclei as well (in $0\nu\beta\beta$ decay all matrix elements vary just by about 10%). In general, the effect of

excluding both isoscalar pairing and spin-isospin terms is larger than the sum of the matrix element increases resulting from not including each term individually. Overall the impact of the isoscalar pairing and spin-isospin terms are qualitatively similar but quantitatively different in the neutrinoless and two-neutrino $\beta\beta$ decay modes.

B. $0\nu\beta\beta$ decay of pf -shell nuclei in the GCM

The strength of the GCM, QRPA (based on EDF), and IBM (based on bosons) is their treatment of collectivity. Although these methods sacrifice some of the complex valence-space correlations captured by the shell model, they can effectively include larger single-particle spaces, which are frequently required to capture collective correlations. Here we test the ability of the GCM, with the same collective interaction discussed in Sec. II, H_{coll} , to reproduce shell-model $M_{0\nu}^{GT}$ matrix elements.

The GCM is an extension of mean-field theory that supplements the lowest-energy quasiparticle vacuum with other quasiparticle vacua that are constrained to have different expectation values for the operators representing collective coordinates. The method is used most commonly to allow vacua with a range of values for the axial quadrupole moment (Q_0) to appear in low-lying collective states; in such applications the quantum states are obtained by diagonalizing the Hamiltonian in the space of nonorthogonal vacua with different quadrupole moments, or equivalently, different values of the deformation parameter β .

The generator coordinates, the collective degrees of freedom in the GCM, are chosen at the beginning of the calculation, and it is crucial to include all collective degrees of freedom that are important for the phenomena being studied. Nonaxial quadrupole coordinates are ostensibly important but because they affect ground states less than excited states (and because they make angular-momentum projection quite time consuming), we restrict ourselves to axially deformed shapes. And we neglect like-particle pairing fluctuations because they change $M_{0\nu}^{GT}$ by 30% or less in the pf shell [12]. Isoscalar pairing is another story, however. Beginning with the QRPA work of Ref. [38], it has been apparent that dynamical isoscalar pairing correlations have a significant effect on $M_{0\nu}^{GT}$ (the static correlations vanish). Reference [14] showed how to add their effects by using $\langle P_0 + P_0^\dagger \rangle$ as a generator coordinate. This isoscalar pairing amplitude breaks the particle number and rotational symmetries but preserves axial symmetry, so we project the HFB states onto states with good particle number and angular momentum to restore the broken symmetries. The other components of the isoscalar pairing amplitude (related to $P_{\pm 1}^\dagger$) are included through the angular-momentum projection. Isospin symmetry is broken and not restored in our calculation.

Figure 6 shows our GCM results for the $0\nu\beta\beta$ decay of Ti and Cr isotopes with the interaction H_{coll} . We compare them to the shell-model values obtained from the diagonalization of H_{coll} . Both the “1d” version of the GCM, which treats only the isoscalar pairing amplitude as a coordinate, and the “2d” version, which adds a coordinate corresponding to axial deformation, agree well with the full shell-model results with H_{coll} . (The two GCMs agree with each other

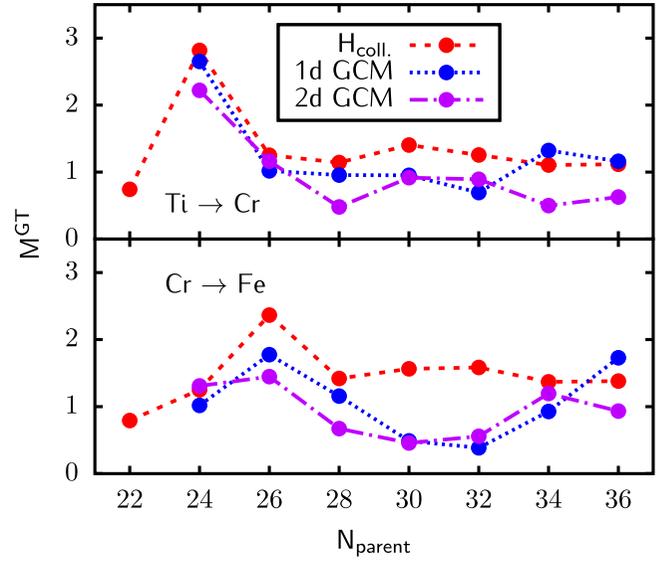


FIG. 6. Gamow-Teller part of $0\nu\beta\beta$ decay matrix elements, $M_{0\nu}^{GT}$, for the decay of Ti isotopes into Cr (top panel) and Cr isotopes into Fe (bottom panel), as a function of the neutron number N_{parent} in the parent nucleus. Results are shown for the shell model with the collective Hamiltonian H_{coll} (red, dashed line), the GCM with the same H_{coll} but without quadrupole-quadrupole interaction, and with the isoscalar pairing amplitude as only coordinate (blue, short-dashed line), and the GCM with the quadrupole-quadrupole interaction and with the axial quadrupole deformation parameter β as second coordinate (purple, dotted line).

because the addition of the quadrupole interaction, as we have already seen, does not have a large effect on the matrix elements.) Together with the demonstrated adequacy of H_{coll} , the agreement suggests that theories of collective motion, which can be extended to several shells, can provide reliable matrix elements in heavier nuclei, where a single valence shell may not be sufficient.

In the isotopes with neutron numbers in the range $N = 28 - 32$ the GCM results deviate from those of the shell-model. For these transitions either the parent or daughter nucleus contains a closed shell at $N = 28$ or $N = 32$, and collectivity plays a smaller role. In addition, at present our GCM calculation excludes vacua without pairing to avoid numerical instability, so that we omit the most important states in closed-shell systems. The inclusion of individual particle-hole excitations across shells in the GCM basis will improve the present results.

C. $0\nu\beta\beta$ decay in important nuclei near $A = 80$ and $A = 130$

The results presented so far illustrate the importance of collective correlations for the $0\nu\beta\beta$ decay matrix elements of nuclei in the lower part of the pf shell. Of all these isotopes, however, only ^{48}Ca actually has even a chance to be used in a $\beta\beta$ experiment. All other relevant nuclei are too heavy for shell-model calculations in complete oscillator shells, so that an analysis like that in Sec. III A is not possible. Nevertheless, we try to estimate the importance of isoscalar pairing for the $\beta\beta$ decay of these isotopes.

We consider two different valence spaces in calculating the matrix elements for the $0\nu\beta\beta$ decay candidates heavier than ^{48}Ca . In ^{76}Ge and ^{82}Se , the valence space comprises the $1p_{3/2}$, $1p_{1/2}$, $0f_{5/2}$, and $0g_{9/2}$ orbitals, while for ^{124}Sn , ^{130}Te , and ^{136}Xe , it comprises the $1d_{5/2}$, $2s_{1/2}$, $0g_{7/2}$, $0d_{3/2}$, and $0h_{11/2}$ orbitals. In each case, two spin-orbit partners are missing from the shell-model space. We base our calculations on the shell model GCN2850 effective interaction in the first space and the GCN5082 interaction in the second [6].

One drawback of incomplete oscillator shells and missing spin-orbit partners is that isoscalar pairing is inhibited. The single-particle orbitals included in the shell model calculations are bounded by large gaps, so that the inhibition is not artificial, but the ability of pairing correlations to transcend the shell gaps cannot be fully evaluated without explicitly including all spin-orbit partners. In addition, it is not appropriate in such valence spaces to use a separable collective Hamiltonian such as H_{coll} , which is designed for complete oscillator shells. In the absence of a good collective interaction, we proceed along two rather extreme paths, with the idea that the size of isoscalar pairing effects will be somewhere between what the two paths yield. Our first procedure is to set all $J^\pi = 1^+, T = 0$ two-body matrix elements equal to zero in our shell model Hamiltonian. Because these interaction matrix elements receive contributions not only from isoscalar pairing but also from other modes, removing them from the shell-model Hamiltonian may overestimate the effects of isoscalar pairing. Our second procedure is to subtract from the shell model interaction the isoscalar pairing interaction from H_{coll} , even though the incomplete oscillator shells limit its effectiveness. Because of the limitation, the resulting interaction probably underestimates the effects of isoscalar pairing. Following the recommendation of Refs. [18,42], we use the same isoscalar pairing strength as in Table I in these heavier nuclei.

The results appear in Fig. 7. In ^{48}Ca , for which we include all spin-orbit partners, the two prescriptions give very similar matrix elements, larger by about a factor of two than the KB3G matrix element, but smaller than the matrix element obtained with H_{coll} and excluding isoscalar pairing. This last result suggests (again) that additional perhaps non-collective correlations, present in the full shell-model interaction but not included in H_{coll} , may in part make up for the removal of isoscalar pairing correlations.

For heavier isotopes the two prescriptions produce different results. Setting all $J^\pi = 1^+, T = 0$ interaction matrix elements equal to zero increases the $0\nu\beta\beta$ decay matrix elements by 1 to 2 units, or between 60% and 80%. The effect is smaller in nuclei with larger isospin. On the other hand, subtracting the isoscalar pairing interaction in H_{coll} from the shell-model interactions leads to much smaller increases, of 25% or less. We conclude that isoscalar pairing in heavier $\beta\beta$ decay candidates is important, but probably less so than in the lighter isotopes studied in Sec. III A, as suggested by the range covered by the two prescriptions considered. This conclusion is tentative, however, because the effects of single-particle orbitals beyond the valence space cannot be completely assessed without including them explicitly.

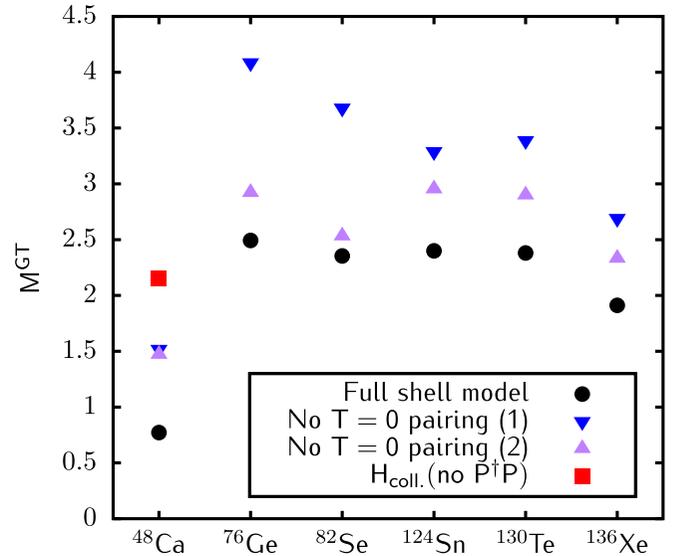


FIG. 7. Estimates of the effect of isoscalar pairing on $0\nu\beta\beta$ decay for nuclei used in or considered for experiments. The Gamow-Teller matrix elements, $M_{0\nu}^{GT}$, are shown for the full shell model effective interaction (black circles), the effective interaction with all $J^\pi = 1^+, T = 0$ two-body matrix elements set to zero (blue upside-down triangles), and the effective interaction with the isoscalar pairing interaction from H_{coll} subtracted (purple triangles). For ^{48}Ca the result from Fig. 2, obtained from H_{coll} without the isoscalar pairing term, is also shown (red square).

IV. SUMMARY

We have explored the role of collective correlations in $0\nu\beta\beta$ decay. To be able to study all relevant collective terms within the shell model we have mostly limited ourselves to nuclei in the pf shell, with $A = 60$ at most. We have found that a separable collective Hamiltonian that includes a monopole term, isovector and isoscalar pairing, a quadrupole-quadrupole interaction, and a spin-isospin term reproduces the matrix elements obtained with the full shell model interaction quite well. Among the collective terms, isoscalar pairing has a particularly strong effect, one that can be related to the approximate $SU(4)$ symmetry obeyed by the $0\nu\beta\beta$ decay operator and the nuclear interaction. In heavier nuclei, the effects of isoscalar pairing are harder to estimate and probably smaller than in the pf shell, but almost certainly still important.

GCM calculations with the same collective Hamiltonian for pf -shell nuclei agree well with full shell model calculations provided that the $T = 0$ pairing amplitude is one of the generator coordinates. The agreement suggests that theories of collective motion such as the GCM with a few generator coordinates may provide accurate matrix elements in heavier nuclei. If collective correlations from beyond the valence space turn out to be important, such models will be particularly useful because shell-model calculations in several oscillator shells are still computationally prohibitive.

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