A Vietoris theorem in shape theory. II

By

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This paper is a continuation of [11], and will establish a number of results related to those in [11] including some of the shape properties of LC^n paracompacta (=paracompact Hausdorff spaces).

§1. Review and supplements of the argument in [11]. Let A be a closed subset of a topological space X. J. Dugundji [4] defined A to be PC_X^n if for each neighborhood U of A there is a neighborhood V of A such that each continuous map $h: S^k \to V$ is null-homotopic in U for $0 \leqq k \leqq n$. Using this notion the arguments in [11] are summarized as follows.

Let $f_0: X_0 \to Y_0$ be a closed continuous map from a topological space X_0 onto a paracompactum Y_0 such that Y is a closed subset of Y_0, $X = f_0^{-1}(Y)$, $x_0 \in X$, and $f_0^{-1}(y)$ is $PC_{X_0}^n$ for each point y of Y. Let $\{\mathfrak{V}_\lambda | \lambda \in \Lambda\}$ be the set of all the locally finite collections of cozero-sets of Y_0 such that (a) $Y \subset H_\lambda = \cup \{V | V \in \mathfrak{V}_\lambda\}$, (b) the correspondence $V \to V \cap Y$ for $V \in \mathfrak{V}_\lambda$ defines an isomorphism of $N(\mathfrak{V}_\lambda)$ onto $N(\mathfrak{V}_\lambda \cap Y)$, and (c) exactly one member of $\mathfrak{V}_\lambda$ contains y_0, where $y_0 = f_0(x_0)$ and N means the operation of taking the nerve of a cover. For $\lambda, \mu \in \Lambda$ let us define $\lambda \leq \mu$ by requiring that $\mathfrak{V}_\mu$ refines $\mathfrak{V}_\lambda$. Let us put $G_\lambda = f_0^{-1}(H_\lambda)$, $K_\lambda = N(\mathfrak{V}_\lambda)$, and let $k_{0\lambda}$ be the vertex of K_λ corresponding to the member of $\mathfrak{V}_\lambda$ containing y_0. Let $\phi_\lambda: (H_\lambda, y_0) \to (K_\lambda, k_{0\lambda})$ be a canonical map with $\phi_\lambda^{-1}(\mathrm{St}(v: K_\lambda)) = V$ for $V \in \mathfrak{V}_\lambda$ and let $\phi_{\lambda\mu}: (K_\mu, k_{0\mu}) \to (K_\lambda, k_{0\lambda})$ be a canonical projection for $\lambda \leq \mu$. Let us put $f_\lambda = \phi_\lambda \circ (f_0 | G_\lambda): (G_\lambda, x_0) \to (K_\lambda, k_{0\lambda})$.

Let $\mathfrak{W}_0$ be the homotopy category of spaces having the homotopy type of a pointed CW complex. Then $\{(K_\lambda, k_{0\lambda}), [\phi_{\lambda\mu}], \Lambda\}$ is isomorphic to the Čech system of (Y, y_0) in pro-$\mathfrak{W}_0$, $\{(G_\lambda, x_0), [i_{\lambda\mu}], \Lambda\}$ is cofinal in $\mathfrak{U}(X, x_0; X_0) = \{(U, x_0), [i_{UU'}]; U$ nbds of X in $X_0\}$, and $\{1, f_\lambda, \Lambda\}$ is a special system map from $\{(G_\lambda, x_0\}, [i_{\lambda\mu}], \Lambda\}$ to $\{(K_\lambda, k_{0\lambda}), [\phi_{\lambda\mu}], \Lambda\}$. Let us put pro-$\pi_k(\mathfrak{U}(X, x_0; X_0)) = \{\pi_k(U, x_0), \pi_k(i_{UU'}); U$ ubds of X in $X_0\}$ and denote by pro-$\pi_k(Y, y_0)$ the k-th homotopy pro-group of (Y, y_0). Then $\{1, f_\lambda, \Lambda\}$ induces a morphism of pro-groups

$$\text{pro-}\pi_k(f_0, X): \text{pro-}\pi_k(\mathfrak{U}(X, x_0; X_0)) \longrightarrow \text{pro-}\pi_k(Y, y_0),$$

and the argument in [11] yields

THEOREM 1.1. *The morphism* pro-$\pi_k(f_0, X)$ *is an isomorphism for* $0 \leqq k \leqq n$ *and an epimorphism for* $k = n+1$.

LEMMA 1.2. *For any* $\lambda \in \Lambda$ *there exist* $\mu \in \Lambda$ *with* $\lambda \leq \mu$ *and a continuous map* $g_{\lambda\mu} : (K_\mu^{n+1}, k_{0\mu}) \rightarrow (G_\lambda, x_0)$ *such that*
(1) $f_\lambda g_{\lambda\mu} \simeq \phi_{\lambda\mu} j_\mu$, *where* $j_\mu : (K_\mu^{n+1}, k_{0\mu}) \rightarrow (K_\mu, k_{0\mu})$ *is the inclusion*;
(2) *for any polyhedron* (P, p_0) *with* $\dim P \leqq n$, *the diagram*

$$\begin{tikzcd}
 & {[P, G_\mu]} \arrow[r, "(i_{\lambda\mu})_\#"] \arrow[d, "(f_\mu)_\#"'] & {[P, G_\lambda]} \arrow[d, "(f_\lambda)_\#"] \\
{[P, K_\mu^{n+1}]} \arrow[urr, "(g_{\lambda\mu})_\#"] & {[P, K_\mu]} \arrow[l, "(j_\mu)_\#^{-1}"] \arrow[r, "(\phi_{\lambda\mu})_\#"] & {[P, K_\lambda]}
\end{tikzcd}$$

is commutative, where $[P, Z]$ *means the set of all the homotopy classes of basepoint preserving continuous maps from* (P, p_0) *to a pointed space* (Z, z_0). *In such a case we write* $\lambda \ll \mu$.

PROOF. Let us use the notation in [11]. Let $\kappa \prec \lambda \prec \mu$ and let $\xi : (P, p_0) \rightarrow (G_\mu, x_0)$ be any continuous map and suppose that P is subdivided so fine that for each vertex p of P we have $\xi(\mathrm{St}\, p) \subset f_0^{-1}(V)$ with some $V \in \mathfrak{V}_\mu$. Then $f_\mu \xi(\mathrm{St}\, p) \subset \mathrm{St}\,(v\,; K_\mu)$. If we define $\eta : (P, p_0) \rightarrow (K_\mu, k_{0\mu})$ by $\eta(p) = v$, η is a simplicial approximation of $f_\mu \xi$. Since $\dim P \leqq n$, $\eta(\mathrm{St}\, p) \subset \mathrm{St}\,(\eta(p)\,; K_\mu^{n+1})$. Hence by [11, p. 699] we have $f_\lambda g_{\lambda\mu} \eta(\mathrm{St}\, p) \subset \mathrm{St}\,(\phi_{\lambda\mu}\eta(p)\,; K_\lambda)$. Thus, $\xi(\mathrm{St}\, p) \cup g_{\lambda\mu}\eta(\mathrm{St}\, p) \subset f_0^{-1}(V_{\lambda,\gamma})$, where $V_{\lambda,\gamma} \in \mathfrak{V}_\lambda$ corresponds to $\phi_{\lambda\mu}\eta(p)$. Hence $i_{\kappa\mu}\eta \simeq i_{\kappa\lambda} g_{\lambda\mu}\eta$. Replacing κ by λ and writing $g_{\lambda\mu}$ instead of $i_{\kappa\lambda} g_{\lambda\mu}$, we have Lemma 1.2.

LEMMA 1.3. *Let* (Z, z_0) *be any pointed topological space with* $\mathrm{ddim}\, Z \leqq n$. *Then, for any shape morphism* $h : (Z, z_0) \rightarrow (Y, y_0)$ *which is represented by* $\{[h_\lambda] \in [Z, K_\lambda] \mid \in \Lambda\}$ *with* $[h_\lambda] = [\phi_{\lambda\mu}][h_\mu]$ *for* $\lambda \leq \mu$, *there exists a set* $\{[g_\lambda] \in [Z, G_\lambda] \mid \lambda \in \Lambda\}$ *with* $[g_\lambda] = [i_{\lambda\mu}][g_\mu]$ *for* $\lambda \leq \mu$, *such that* $[h_\lambda] = [f_\lambda][g_\lambda]$ *for* $\lambda \in \Lambda$, *where* [ ] *denotes the homotopy class as usual.*

PROOF. Since $\mathrm{ddim}\, Z \leqq n$, we may, and will, assume that $h_\lambda(Z) \subset K_\lambda^n$ for $\lambda \in \Lambda$. Let $\lambda \ll \mu$ and $\mu \ll \nu$. Then we have the following homotopy commutative diagram (with base-points suppressed).

$$\begin{tikzcd}
 & K_\nu^n \arrow[r, "i_\nu"] \arrow[d, "\phi^n_{\mu\nu}"] & K_\nu^{n+1} \arrow[r, "j_\nu"] \arrow[drr, "g_{\mu\nu}"] & K_\nu \arrow[d, "\phi_{\mu\nu}"] & G_\nu \arrow[l, "f_\nu"'] \arrow[d, "i_{\mu\nu}"] \\
Z \arrow[ur, "h_\nu"] \arrow[r, "h_\mu"] \arrow[dr, "h_\lambda"'] & K_\mu^n \arrow[r, "i_\mu"] \arrow[d, "\phi^n_{\lambda\mu}"] & K_\mu^{n+1} \arrow[r, "j_\mu"] \arrow[drr, "g_{\lambda\mu}"] & K_\mu \arrow[d, "\phi_{\lambda\mu}"] & G_\mu \arrow[l, "f_\mu"] \arrow[d, "i_{\lambda\nu}"] \\
 & K_\lambda^n \arrow[r, "i_\lambda"] & K_\lambda^{n+1} \arrow[r, "j_\lambda"] & K_\lambda & G_\lambda \arrow[l, "f_\lambda"']
\end{tikzcd}$$

By (1) of Lemma 1.2, we have $f_\mu g_{\mu\nu} i_\nu \simeq \phi_{\mu\nu} j_\nu i_\nu \simeq j_\mu i_\mu \phi^n_{\mu\nu}$ and by (2) of Lemma 1.2 we have $g_{\lambda\mu} i_\mu \phi^n_{\mu\nu} \simeq i_{\lambda\mu} g_{\mu\nu} i_\nu$. Hence $g_{\lambda\mu} i_\mu h_\mu \simeq i_{\lambda\mu} g_{\mu\nu} i_\nu h_\nu$. If $\nu \ll \kappa$, then $g_{\lambda\mu} i_\mu h_\mu \simeq i_{\lambda\nu} g_{\nu\kappa} i_\kappa h_\kappa$. For $\lambda \in \Lambda$, let us put $g_\lambda = g_{\lambda\mu} i_\mu h_\mu$ by choosing any μ with $\lambda \ll \mu$. Then $[g_\lambda]$ is uniquely determined by λ and $[g_\lambda] = [i_{\lambda\mu}][g_\mu]$. Moreover, $[h_\lambda] = [f_\lambda][g_\lambda]$ for $\lambda \in \Lambda$.

LEMMA 1.4. *If, in Lemma* 1.3, *each continuous map from* (Z, z_0) *to* (G_λ, x_0) *is factored through a pointed polyhedron* $(P_\lambda, p_{0\lambda})$ *with* $\dim P_\lambda \leqq n (\lambda \in \Lambda)$, *then* $\{[g_\lambda] | \lambda \in \Lambda\}$ *in Lemma* 1.3 *is uniquely determined by h.*

PROOF. Suppose that $\{[g'_\lambda] | \lambda \in \Lambda\}$ satisfies $h_\lambda \simeq f_\lambda g'_\lambda$, $g'_\lambda \simeq i_{\lambda\mu} g'_\mu$ for $\lambda \leq \mu$, and $[g'_\mu] = [\beta_\mu][\alpha_\mu]$ with $[\alpha_\mu] \in [Z, P_\mu], [\beta_\mu] \in [P_\mu, G_\mu]$ and $\dim P_\mu \leqq n$. Then there is $[\xi_\mu] \in [P_\mu, K_\mu^n]$ such that $j_\mu i_\mu \xi_\mu \simeq f_\mu \beta_\mu$. By Lemma 1.2 we have $g_{\lambda\mu} i_\mu \xi_\mu \simeq i_{\lambda\mu} \beta_\mu$. Hence we have $g'_\lambda \simeq i_{\lambda\mu} g'_\mu \simeq i_{\lambda\mu} \beta_\mu \alpha_\mu \simeq g_{\lambda\mu} i_\mu \xi_\mu \alpha_\mu$. Since $j_\mu i_\mu \xi_\mu \alpha_\mu \simeq f_\mu g'_\mu \simeq j_\mu i_\mu h_\mu$, we have $i_\mu h_\mu \simeq i_\mu \xi_\mu \alpha_\mu$. Therefore $g'_\lambda \simeq g_{\lambda\mu} i_\mu h_\mu \simeq g_\lambda$.

Theorem 1.5 below is a direct consequence of Lemmas 1.3 and 1.4 and improves [11, Theorem 4.3]; it was announced by Bogatyi [2] for the case where X, Y, Z are metric compacta and $\dim Z \leqq n$.

THEOREM 1.5. *Let* $f : (X, x_0) \to (Y, y_0)$ *be a closed continuous surjective map such that X is metrizable and* pro-$\pi_k(f^{-1}(y), x) = 0$ *for each* $y \in Y$, *each* $x \in f^{-1}(y)$ *and each k with* $0 \leqq k \leqq n$. *Then, for a pointed topological space* (Z, z_0) *with* $\operatorname{ddim} Z \leqq n$ *(cf.* [5]) *the map* $f_\natural : \mathfrak{S}_0[Z, X] \to \mathfrak{S}_0[Z, Y]$ *induced by f is bijective.*

The following is a generalization of [11, Theorem 1.2].

THEOREM 1.6. *Let f be the same as in Theorem* 1.5. *If* $\operatorname{ddim} X \leqq n$ *and* $\operatorname{ddim} Y \leqq n+1$, *then f induces a shape equivalence.*

PROOF. Let $\lambda \ll \mu$. By [5] there exist $\nu \in \Lambda$ with $\mu \leq \nu$ and continuous maps $\psi_{\mu\nu} : (K_\nu, k_{0\nu}) \to (K_\mu^{n+1}, k_{0\mu})$, $\zeta : (G_\nu, x_0) \to (P, p_0)$, $\xi : (P, p_0) \to (G_\mu, x_0)$ with a polyhedron (P, p_0) of dimension $\leqq n$ such that $\varphi_{\mu\nu} \simeq j_\mu \psi_{\mu\nu}$, $i_{\mu\nu} \simeq \xi\zeta$. Then by Lemma 1.2 we have $\varphi_{\lambda\nu} \simeq f_\lambda g_{\lambda\mu} \psi_{\mu\nu}$, $i_{\lambda\nu} \simeq g_{\lambda\mu} \psi_{\mu\nu} f_\nu$.

§ 2. Another Vietoris-type theorem.

Let $\Phi_k(X, x_0) : \pi_k(X, x_0) \to \text{pro-}\pi_k(X, x_0)$ be the natural morphism.

THEOREM 2.1. *Let $f: X \to Y$ be a closed continuous map from a topological space X onto a paracompactum Y such that each fiber is PC_X^n. Then for $x_0 \in X$ and $y_0 = f(x_0)$, the composite of the morphisms*:

$$\pi_k(X, x_0) \xrightarrow{\pi_k(f)} \pi_k(Y, y_0) \xrightarrow{\Phi_k(Y, y_0)} \text{pro-}\pi_k(Y, y_0)$$

is an isomorphism for $0 \leqq k \leqq n$ and an epimorphism for $k = n+1$.

PROOF. Apply Theorem 1.1 to the case where $X = X_0$.

COROLLARY 2.2. *Let (X, x_0) be a pointed topological space which is homotopy dominated by a pointed LC^n paracompactum (Z, z_0). Then $\Phi_k(X, x_0)$ is an isomorphism for $0 \leqq k \leqq n$ and an epimorphism for $k = n+1$.*

PROOF. Apply Theorem 2.1 to the identity may $1_Z: Z \to Z$. Then $\Phi_k(Z, z_0)$ is an isomorphism for $0 \leqq k \leqq n$ and an epimorphism for $k = n+1$. We have then Corollary 2.2 easily.

REMARK. Dugundji's Vietoris-type theorems [4, Theorem 5.2 and the first part of Theorem 5.1] follow immediately from Theorem 2.1 in view of Corollary 2.2.

COROLLARY 2.3. *Let (X, x_0) be the same as in Corollary 2.2. If $\pi_{n+1}(X, x_0) = 0$, then* pro-$\pi_{n+1}(X, x_0) = 0$.

§ 3. Some properties of LC^n paracompacta.

THEOREM 3.1. *Let X be an LC^n paracompactum and A a closed subset of X with $a \in A$. Let $\mathfrak{U}(A, a; X)$ be the inverse system which is defined in § 1. Then the pro-group $\{\pi_k(U, a), \pi_k(i_{UV}) | U, V \in \mathfrak{U}(A, a; X)\}$ is isomorphic to pro-$\pi_k(A, a)$ for k with $0 \leqq k \leqq n$.*

PROOF. Let $\{U_\lambda | \lambda \in \Lambda\}$ be the totality of all the cozero-sets of X containing A. Then the inverse system $\{(U_\lambda, a), i_{\lambda\lambda'}, \Lambda\}$ is cofinal in $\mathfrak{U}(A, a; X)$; it is associated with (A, a) in the sense of [9] by [10, Theorem 5.1]. Let $\mathfrak{U}_\lambda = \{(U_{\lambda\mu}, \mu_{\lambda\mu}), \mu \in \Omega(\lambda)\}$ be the Čech system of the space (U_λ, a). Then $\mathfrak{U} = \{(U_{\lambda\mu}, u_{\lambda\mu}), \mu \in \Omega(\lambda), \lambda \in \Lambda\}$ becomes an inverse system defined on a filtered category (cf. [1, p. 165]). Since $\mathfrak{U}$ is associated with (A, a) in the sense of [9], $\mathfrak{U}$ is isomorphic to the Čech system of (A, a). Hence pro-$\pi_k(A, a) \simeq \{\pi_k(U_{\lambda\mu}, u_{\lambda\mu}), \mu \in \Omega(\lambda), \lambda \in \Lambda\}$. Since U_λ is an LC^n paracompactum, it follows from Corollary 2.2 that $\{\pi_k(U_{\lambda\mu}, u_{\lambda\mu}), \mu \in \Omega(\lambda), \lambda \in \Lambda\} \cong \{\pi_k(U_\lambda, a), \pi_k(i_{\lambda\lambda'}), \Lambda\}$ for $0 \leqq k \leqq n$. Hence pro-$\pi_k(A, a) \cong \{\pi_k(U_\lambda, a), \pi_k(i_{\lambda\lambda'}), \Lambda\}$.

COROLLARY 3.2. *Let A be a closed subset of an LC^n paracompactum X and let $0 \leqq m \leqq n$. Then A is PC_X^m iff* pro-$\pi_k(A, a) = 0$ *for each $a \in A$ and $0 \leqq k \leqq m$.*

By virtue of Corollary 3.2 we obtain from Theorem 1.1

THEOREM 3.3. *Let X be a paracompactum and let $f: X \to Y$ be a closed continuous surjective map such that pro-$\pi_k(f^{-1}(y), x)=0$ for each $y \in Y$, each $x \in f^{-1}(y)$ and each k with $0 \leqq k \leqq n$. In case there is an LC^{n-1} paracompactum X_0 which contains X as a closed subset, the induced morphism pro-$\pi_k(f)$: pro-$\pi_k(X, x) \to$ pro-$\pi_k(Y, f(x))$ is an isomorphism for $0 \leqq k \leqq n$ and an epimorphism for $k=n+1$.*

§ 4. Finite-dimensional LC^n paracompacta.

THEOREM 4.1. *Let $f: X \to Y$ be a closed continuous map from a topological sqace X onto a paracompactum Y such that each fiber is PC_X^n, and let $y_0=f(x_0)$, $x_0 \in X$. If* ddim $Y \leqq n+1$, *then Y is uniformly movable.*

PROOF. Let us use the notation in § 1; in the present case we have $X_0=G_\lambda=X$ for each $\lambda \in \Lambda$. Let $\lambda \in \Lambda$. Then by Lemma 1.2 there are $\mu \geq \lambda$ and $g_{\lambda\mu}:(K_\mu, k_{0\lambda_0}) \to (X, x_0)$ such that $f_\lambda g_{\lambda\mu} \simeq \phi_{\lambda\mu} j_\mu : (K_\mu^{n+1}, k_{0\mu}) \to (K_\lambda, k_{0\lambda})$. Since ddim $Y \leqq n+1$, there are $\nu \geq \mu$ and $\psi_{\mu\nu}:(K_\nu, k_{0\nu}) \to (K_\mu^{n+1}, k_{0\mu})$ such that $j_\mu \psi_{\mu\nu} \simeq \phi_{\mu\nu}$. Let us put $h=fg_{\lambda\mu}\psi_{\mu\nu}$: $(K_\nu, k_{0\nu}) \to (Y, y_0)$. Then we have $\phi_{\lambda\nu} \simeq \phi_{\lambda\mu} j_\mu \psi_{\mu\nu} \simeq f_\lambda g_{\lambda\mu}\psi_{\mu\nu} = \phi_\lambda f g_{\lambda\mu}\psi_{\mu\nu}$, that is, $\phi_{\lambda\nu} \simeq \phi_\lambda h$. This shows that (Y, y_0) is uniformly movable.

COROLLARY 4.2. *A pointed LC^n paracompactum (X, x_0) with* ddim $X \leqq n+1$ *is uniformly movable.*

Corollary 4.2 was announced by Kozlowski and Segal [7] for the case where dim $X \leqq n$.

Let $\mathfrak{H}_0$ and $\mathfrak{S}_0$ be the homotopy category and the shape category, of pointed topological spaces, respectively, and let $S: \mathfrak{H}_0 \to \mathfrak{S}_0$ be the shape functor.

Then the following is a direct consequence of Lemma 1.2.

THEOREM 4.3. *Let $f: X \to Y$ be the same as in Theorem 4.1. If P is a polyhedron with* dim $P \leqq n$ *and $p_0 \in P$, then f induces a bijective map $f_\sharp : [P, X] \to \mathfrak{S}_0[P, Y]$.*

COROLLARY 4.4. *Let X be an LC^n paracompactum. If P is a polyhedron with* dim $P \leqq n$ *and $p_0 \in P$, then $[P, X] \to \mathfrak{S}_0[P, X]$ induced by the shape functor S is bijective.*

Corollary 4.4 was proved by Kozlowski and Segal [7].

THEOREM 4.5. *Let f be the same as in Theorem 4.1. Then, for any shape morphism $h:(Z, z_0) \to (Y, y_0)$ with* ddim $Z \leqq n$, *there exist continuous maps $\xi:(Z, z_0) \to (P, p_0)$ and $\eta:(P, p_0) \to (X, x_0)$ such that P is a polyhedron with* dim $P \leqq n$ *and $h=S(f\eta\xi)$.*

PROOF. Apply the argument in the proof of Lemma 1.3 to the case where $X=X_0$.

THEOREM 4.6. *Let f be the same as in Theorem 4.1. If* ddim $Y \leqq n$, *then (Y, y_0) is shape dominated by a pointed polyhedron of dimension $\leqq n$.*

COROLLARY 4.7. *If X is an LC^n paracompactum with* ddim $X \leqq n$, *then (X, x_0) is shape dominated by a pointed polyhedron of dimension $\leqq n$.*

Corollary 4.7 was announced by Kozlowski and Segal [7] for the case where $\dim X \leqq n$.

THEOREM 4.8. *Let X be an LC^n metric space, Z a topological space with $\dim Z \leqq n$, and $x_0 \in X$, $z_0 \in Z$. Then the map $[Z, X] \to \mathfrak{S}_0[Z, X]$, induced by the shape functor S, is bijective.*

PROOF. Let $g: (Z, z_0) \to (X, x_0)$ be any continuous map. Then by [8, Lemma 2.2] g is factored as $g = g_1 g_0$, where $g_0: (Z, z_0) \to (T, t_0)$, $g_1: (T, t_0) \to (X, x_0)$ are continuous maps, T is a metric space and $\dim T \leqq n$. By a pointed version of [6, Chap. V, § 6] there is a pointed polyhedron (P, p_0) with $\dim P \leqq n$ such that any continuous map $\xi: (T, t_0) \to (X, x_0)$ can be factored through (P, p_0). Hence by Lemmas 1.3 and 1.4 we have Theorem 4.8.

ADDENDUM. After having completed this paper, the author received Dydak [12] and found that in [12] Dydak proved Theorems 1.5 and 1.6 for the case of Y being metrizable and Corollaries 2.2 and 2.3 for the case of X being LC^n metrizable. Moreover, Dydak [12] proved Theorem 4.8 under a weaker assumption that $\operatorname{ddim} Z \leqq n$; the proof there would be completed if one can prove a suitable factorization theorem.

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