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Growth and Shape of Transportation Networks

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Normally, network-style infrastructure is built up slowly over time. The shapes that networks form are thought to be related to the performance of networks during this process. Using reduced travel time as an evaluation criterion, this study examines road and rail transportation networks to determine the differences in growth patterns that are caused by various factors, including network speed, urban population distribution, and network shape.
Key words: Transportation Network, Growth, Speed, Performance

1. Introduction

Transportation networks have undergone rapid technological changes. However, the growth patterns of the transportation networks vary widely from one city to another. For example, Japan's bullet train (*Shinkansen*) network forms a tree structure, while the Japanese subway train network usually forms a grid structure. Similarly, the growth patterns of transportation networks in Tokyo and London are also quite different. These differences can be attributed to many different factors, such as network performance due to technological innovation, urban population distribution, and the shape of cities.

This study focuses on these types of transportation network growth patterns. In addition to simulating different growth patterns due to transportation network speed and population distribution, and observing the resultant network features, this study will determine the mechanism that can be used to describe these differences. As well, optimum network performance and amount of maintenance, taking into account the trade-off between network speed and construction cost, will be considered.

Batty and Xie (1997) simulated the areal expansion of a city and the linear development of a network using cellular automata (CA), which is a type of discrete model. They constructed a model of city growth generated by random numbers, in which cells are selected at random from neighboring cells, taking into account the hierarchical level of the network. It was found that the denser the network is, the higher its hierarchical level. They suggested that the growth of a transportation network can be evaluated using local indicators. However, for trunk networks in particular, which form the axis that shape the skeleton of national and city-wide networks, it is important to construct networks based on evaluation indicators that measure overall efficiency, rather than on simply improving local indicators.

Urban network is designed not only by considering local accessibility but also mobility in the whole area. Network design models allow us to investigate what type of network is superior when we are taking overall accessibility into account. Magnanti and Wong (1984), Minoux (1989) and Ahuja *et al*. (1993) provide many references on network design including applications. However network design in the real world transportation network is unsolvable because of its size. One of the possible alternatives is to consider evolution of network in which the optimal adding interval of transportation network is determined one by one. Hirayama *et al*. (2001) proposed a model for extending networks that minimize inconvenience. This method was able to capture the shape features of the growth process. However, they did not discuss the possibility of obtaining diverse shapes due to differences in initial conditions. Given these findings, this study will examine transportation networks that are designed to minimize mean required travel time as a whole to clarify the features of the shapes and the effects of networks.

Our model is advantageous in the sense that (1) network growth process, which is not determined by neighboring status but by the optimality of the whole region, is realized, and (2) configuration of the city and transportation demand distribution are easily controllable.

2. Transportation Network Growth Model

2.1 The ideal city

In this study, an ideal city was used as the model according to Watanabe (2008).

The ideal city is defined as a total of 169 demand points (points of access and egress on a high-speed transportation network), which are represented on a 13×13 square grid. The transportation demand between an arbitrary pair of demand points is proportional to the population at each of the two demand points.

A general link is already constructed enabling straight-line travel between any two points, making it possible to travel in a straight line between any two points at speed 1. The number of general links is 312.

A section that connects grid points vertically or horizontally is considered as a single high-speed link. A high-speed link enables travel in c units of time ($0 \leq c \leq 1$), where the time is normalized based on the time required using the

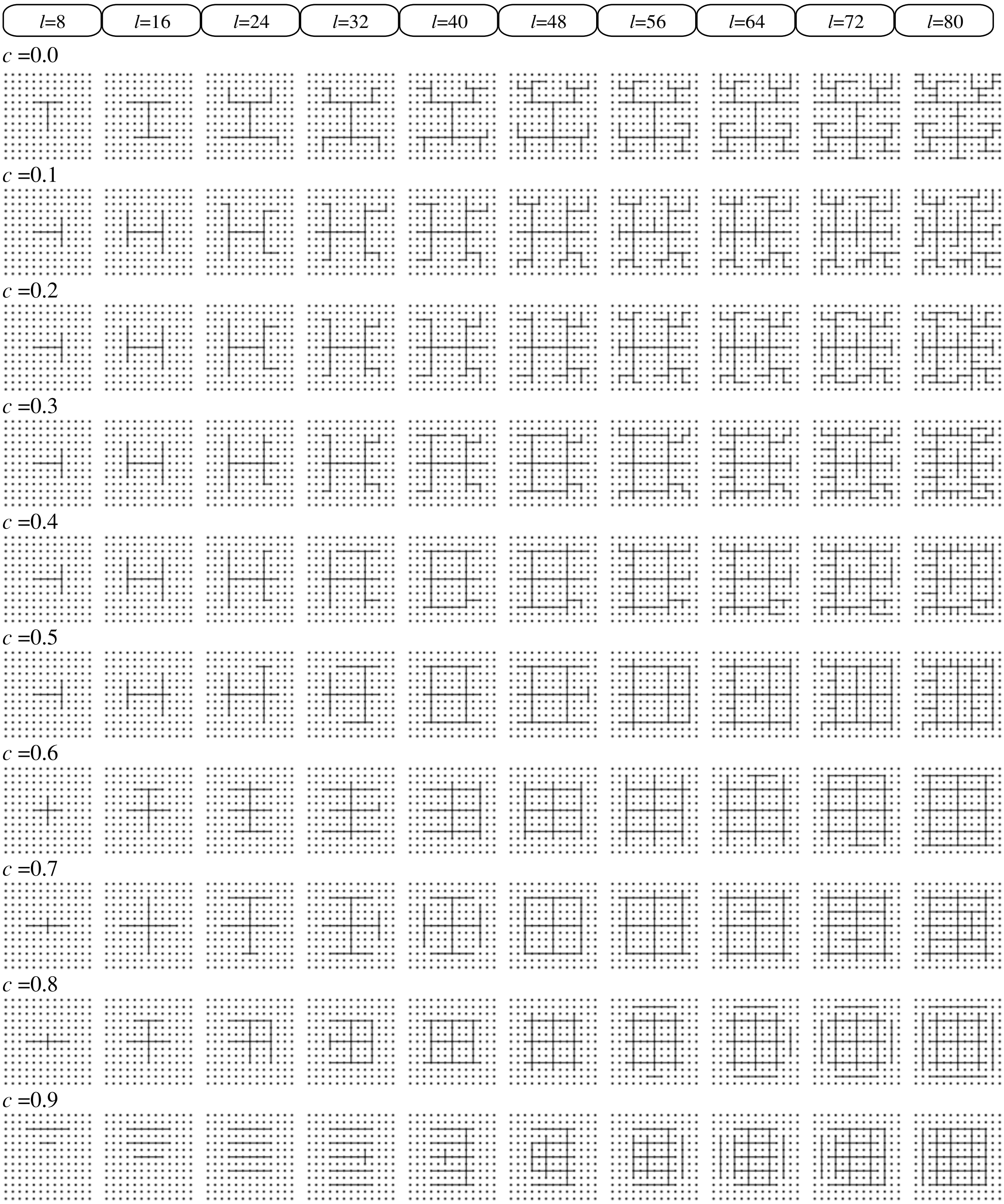

Fig. 1. Speed and shape of network growth with high-speed links.

main link. The number of high-speed links, l, varies from 1 to 312.

The travel route between demand points is either the route using only the general links, or the route via high-speed links, whichever results in the shortest travel time.

The shape of real cities is not regular, as implied by the above conditions. However, the aim of this study is to determine the fundamental characteristics of such a network. Thus, a simplified network in an ideal square city was used.

2.2 Rules of network construction order

In the ideal city, only a single high-speed link was created at a time. Assume that the high-speed links are constructed in order, one by one, and define the "period" as the length of time required to construct a single such link.

The "mean travel time" is defined as the mean travel time between all demand points. To determine the order of construction, a sequential optimum construction method, whereby high-speed links that minimize the value of mean demand time in each "period" are successively constructed, was used. Many transportation networks are, to a certain ex-

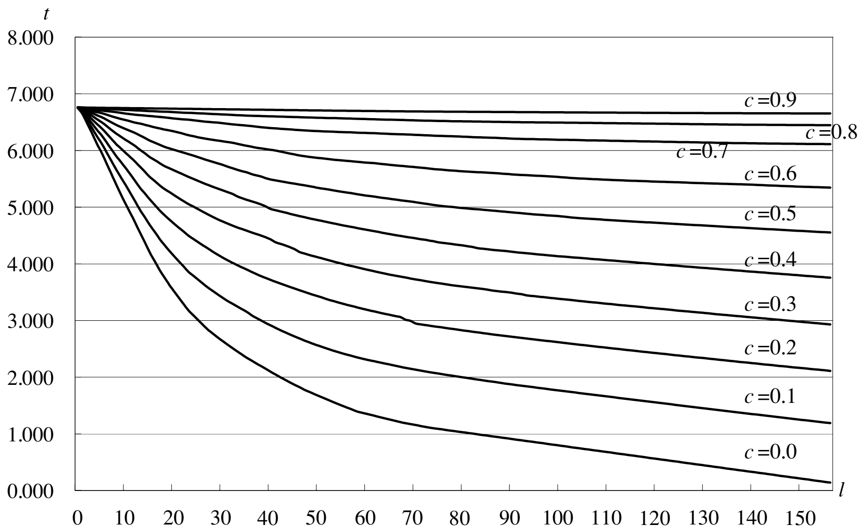

Fig. 2. Mean travel time reduction process.

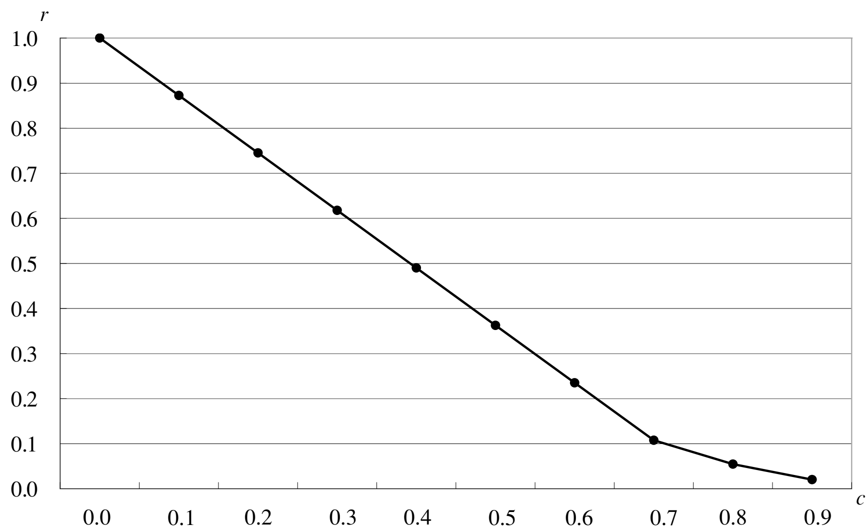

Fig. 3. Reduction rate, $r_3$12.

tent, constructed according to broad-ranging scheme, based on a prescribed plan. However, in this study, it was assumed that the transportation network is constructed one by one without any overarching plan.

2.3 Evaluation indicators

In addition to illustrating the construction order and shapes in relation to the transportation network growth patterns defined by the above rules, in order to clarify their features, three quantitative indicators were created to evaluate growth pattern processes.

The first indicator is the rate of reduction in mean travel time. If the mean travel time at demand point i during the time in which l high-speed links are constructed (l periods) is t_{il}, the mean travel time for the whole network can be expressed by the following expression.

$$T_l = \sum_{i=1}^{169} t_{il}/169. \quad (1)$$

Furthermore, the rate of reduction, r_l, in mean travel time when l high-speed links have been constructed (after l periods) relative to the mean travel time when there are no high-speed links (0 periods) is determined by

$$r_l = 1 - T_l/T_0. \quad (2)$$

The second indicator is the number of closed paths, p. When the number of high-speed links in the growing transportation network is e, and the number of demand points connected by these high-speed links is v, then the number of closed paths formed by high-speed links can be expressed as

$$p = e - v + 1. \quad (3)$$

The closed path capacity utilization factor, α, can be calculated as the number of closed paths divided by the maximum number of closed paths, as shown below (Okudaira, 1976; Honda, 2005).

$$\alpha = \frac{e - v + 1}{2v - 5}. \quad (4)$$

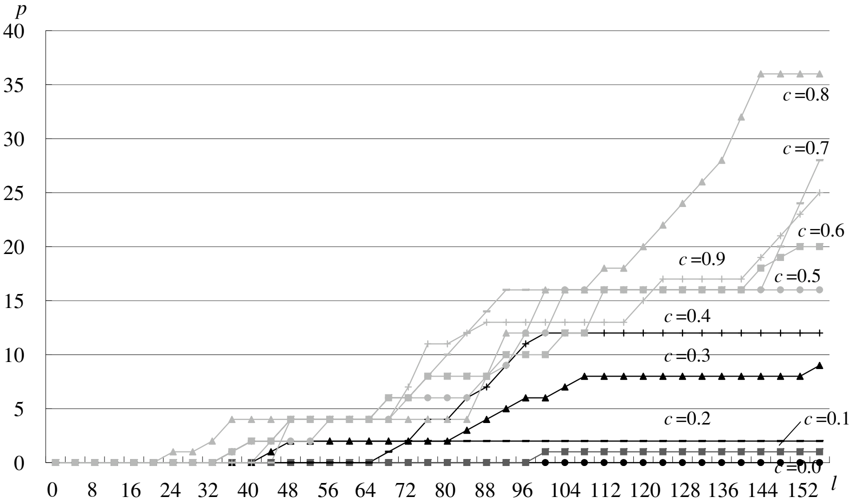

Fig. 4. Number of closed paths.

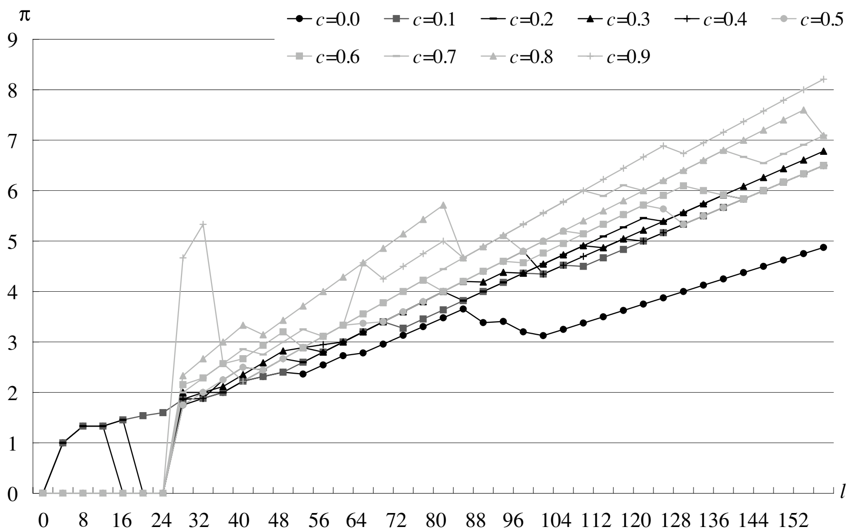

Fig. 5. Indicator π.

The third indicator is the π indicator. This indicates the degree to which the graph of the transportation network resembles a circle. It is defined as

$$\pi = \frac{l}{d} \tag{5}$$

where l is the number of links that equals to the overall length of the transportation network, and d is the diameter of the graph, that is, the required time between the two demand points that are furthest apart (Okudaira, 1976). Obviously π becomes one when the network is a chain of links. As the network is densely constructed with keeping its circular shape, π gets larger. However, if the network extends to a particular direction, π becomes small. If the lengths of networks are same, then the larger π means that the shape of the network resembles a circle.

These three indicators can be used to determine the characteristics of the network.

3. Transportation Network Growth Patterns

3.1 Speed and growth patterns of high-speed links

In the first case, it was assumed that the population is the same at each demand point. Based on this assumption, the speed of the high-speed links, c, was varied from 0 to 1 in increments of 0.1 to determine the shape of the network as a function of l. Figure 1 shows the results.

Networks where the speed is high (i.e., c is low) show a tendency to try to expand outward from the center of the city, with repeated branching. As a result, the network shape has essentially a radial form: the higher the speed of the links, the fewer the number of loops present. The network had a tendency at high speeds to extend rapidly to every corner of its periphery. On the other hand, networks where the speed is slow (i.e., c is high) are shaped like regular grids. Once the grid-shaped routes that form the trunk lines are created, the network has a tendency to grow in a manner such that the grids are further subdivided into smaller grids. Generally, the lower the speed of the links, the smaller the size of the grid and the lower the ef-

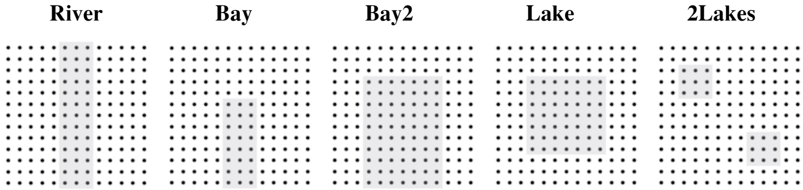

Fig. 6. Various virtual city shapes (gray square: non-residential area, black dot: demand point).

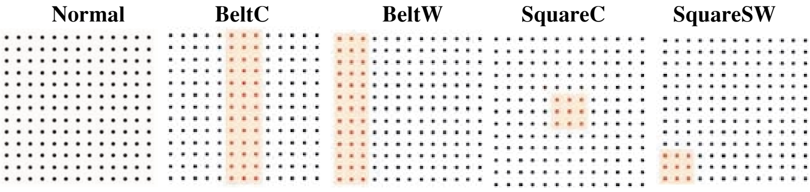

Fig. 7. Various population density shapes (orage square: high-density area, orange dot: demand point (high density), black dot: demand point (low density)).

Table 1. Population density at the demand points.

Shape	High density area		Low density area		Total population
	Population/node	# node	Population/node	# node	
Normal	—	—	13	312	4176
BeltC	100	39	1	276	4176
BeltW	100	39	1	276	4176
SquareC	430	9	1	303	4176
SquareSW	430	9	1	303	4176

fectiveness in reducing the mean travel time, even as the network grows by branching. Thus, growth by construction of shorter length routes from the center of the region is preferable to growth by branching.

Figure 2 illustrates the process of mean travel time reduction as a function of speed. For slow networks, having a speed of greater than 0.7, the reduction in mean travel time remains very small as high-speed links grow. In contrast, for fast networks, having a speed of less than 0.7, there is a large reduction in mean travel time as the number of high-speed links increases. Figure 3 shows that, for a reduction rate when $l = 312$, r_{312}, there is a linear improvement in the reduction rate for $c \leq 0.7$, while when $c \geq 0.7$, the reduction in mean travel time is small compared to when there are no high-speed links.

As well, the main features of the network shape, which are seen in Fig. 1, are confirmed using the number of closed paths and the indicator π. Figure 4 shows the number of closed paths. The number of closed paths for networks of high speed is small, and most of the networks with high-speed links have a tree structure, due to branching. On the other hand, the number of closed paths in networks of low speed is high, and a regular structure that includes closed paths can be seen. Given the above, the network can be expected to be useful between two distant points when the speed is fast. When the speed is slow, the effectiveness of each high-speed link in reducing mean travel time is low, so that the influence of the high-speed links occurs only in the vicinity of where the links are constructed. In addition, the indicator π in Fig. 5 reveals that, although the values vary according to the growth process, in general, networks of lower speed grow in a shape that is closest to round.

Similar trends can be observed in real networks. For example, most high-sped train networks, such as the *Shinkansen* and the TGV, have no closed paths and grow by means of branching. On the other hand, subway networks in cities are usually composed of numerous routes that intersect one another. The differences in patterns due to the speed in network growth models reflect the features of real transportation networks.

3.2 Growth patterns due to differences in population distribution

The shapes of real cities vary, and their populations are not evenly distributed. The shapes of transportation networks and the order of their construction naturally change, due to changes in population distribution. Thus, the effect of population distribution on the growth of the network was considered.

The first case considered a non-square city, where parts of the city are uninhabitable due to the presence of rivers, bays, lakes, and mountains. As shown in Fig. 6, the ideal cities were created using five types of shapes, including: River, Bay, Bay2, Lake, and Lake2. For each type, the growth of the transportation networks was examined. It was assumed that areas that are shaded in gray in Fig. 6 are uninhabitable, with zero traffic demand. We assume that general links exist

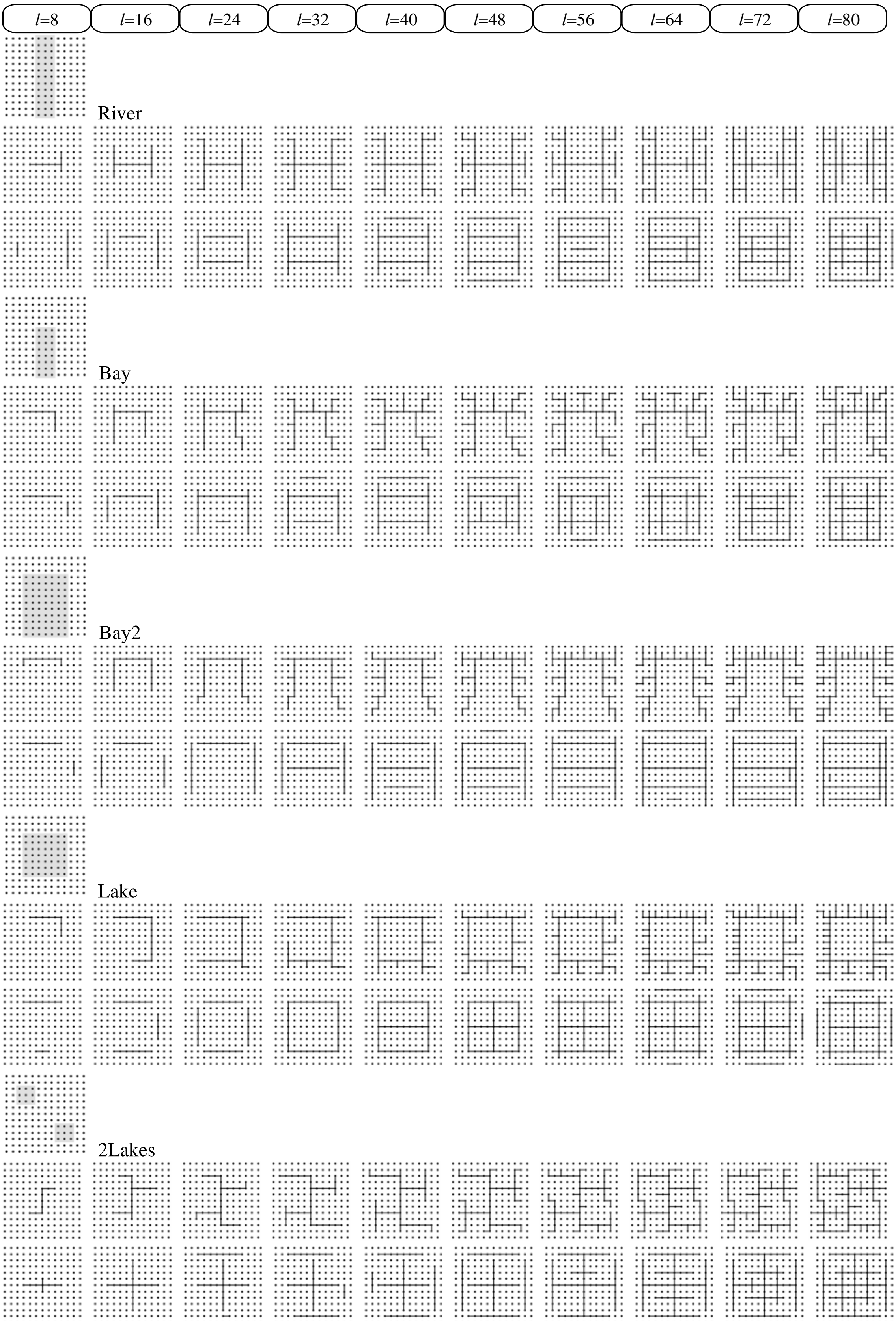

Fig. 8. Shapes of network growth for different city shapes (upper: $c = 0.2$, lower: $c = 0.8$).

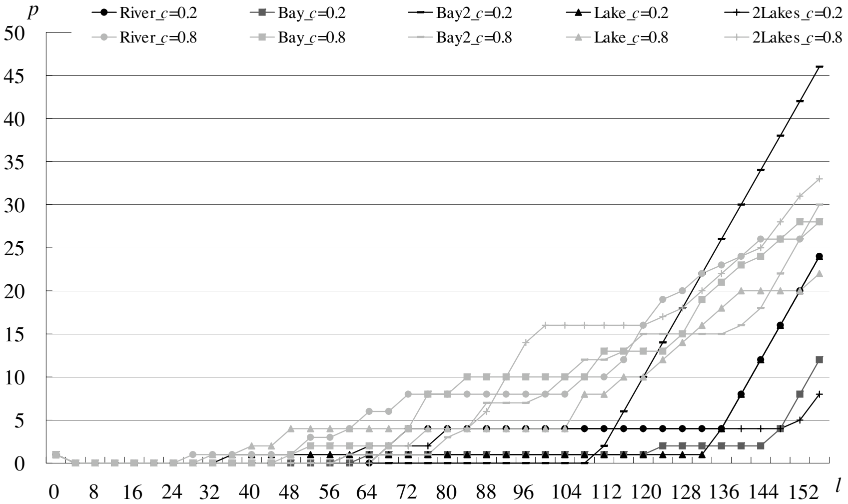

Fig. 9. Number of closed paths.

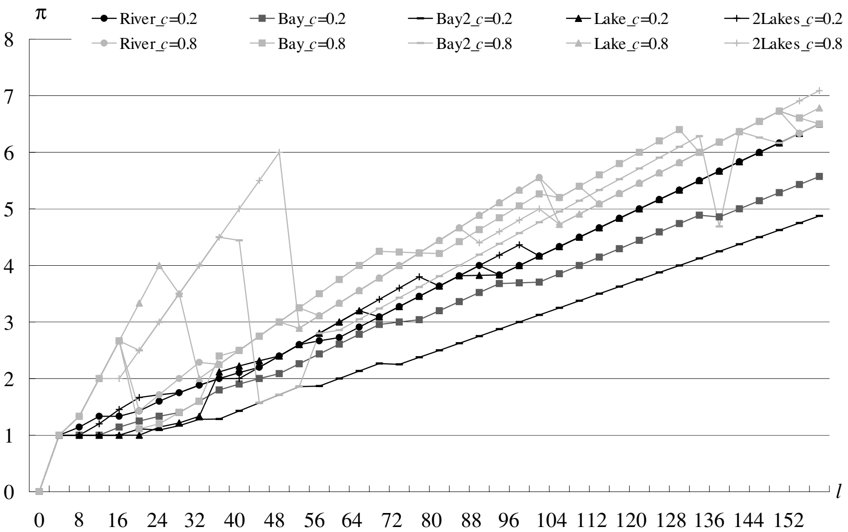

Fig. 10. Indicator π.

even in uninhabitable areas. So we can go through on the general links and also construct the high-speed links. The value of the speed ratio, c, was 0.2 for high-speed networks and 0.8 for low-speed networks.

As well, not only the shape of the city, but also the population density was considered. The population distribution was modeled using the following four types, which are shown in Fig. 7: BeltC, BeltW, SquareC, and SquareSW. It was assumed that the population density is higher at the demand points of the high-density areas than at other areas, as shown in Table 1. However, the total population of the city stayed constant in each of the runs.

3.3 Impact of the shapes of ideal cities on network growth

Figure 8 shows the network growth process as a function of the shape of the cities. Calculations were performed for two cases: $c = 0.2$ and $c = 0.8$.

It can be seen that the network shapes are different than those in the case of uniform population. However, the same basic features can be seen, such as the tendency of high-speed network links to grow by branching in a radial pattern, and the tendency of slow-speed network links to grow in one direction in a grid pattern.

It is interesting to consider whether a high-speed link can be created on vacant land. In a high-speed network, bridge links, which connect demand points, are not constructed on vacant land. On the other hand, in low-speed networks, many bridge links are constructed on vacant land. This is because in high-speed networks, one link alone is sufficient to serve the need for high-speed linking, even without constructing a bridge, as long as one route connects the demands points, even if it is a detour. On the other hand, in low-speed networks, there is insufficient time reduction for each link constructed. This leads to the construction of numerous routes to connect the demand points. This result can be applied to the construction planning of bridges over rivers and straits, roads crossing bays, and tunnels passing through mountain areas.

Figure 9 shows the effect of the number of closed roads on the shape of the city, while Fig. 10, shows the effect

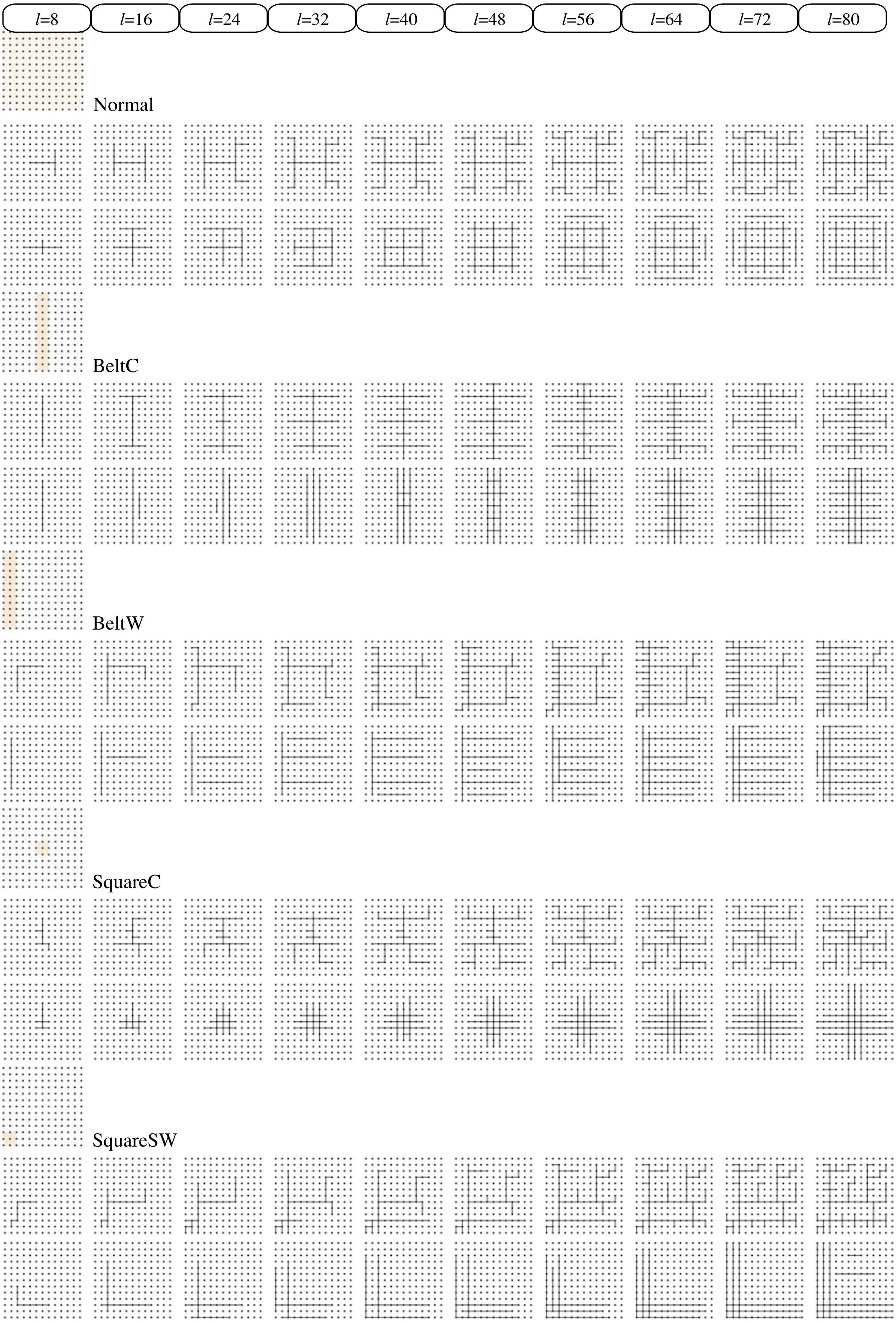

Fig. 11. The effect of population density on the shape of network growth (upper: $c = 0.2$, lower: $c = 0.8$).

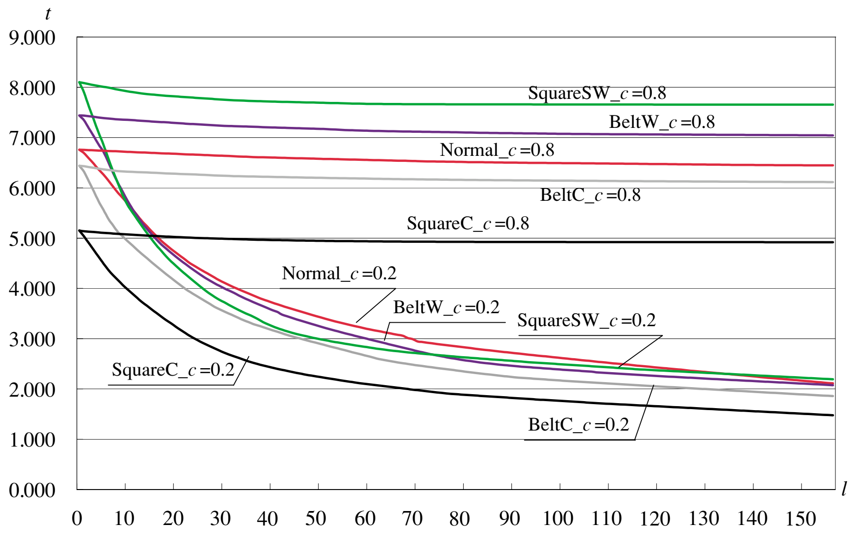

Fig. 12. Mean travel time reduction.

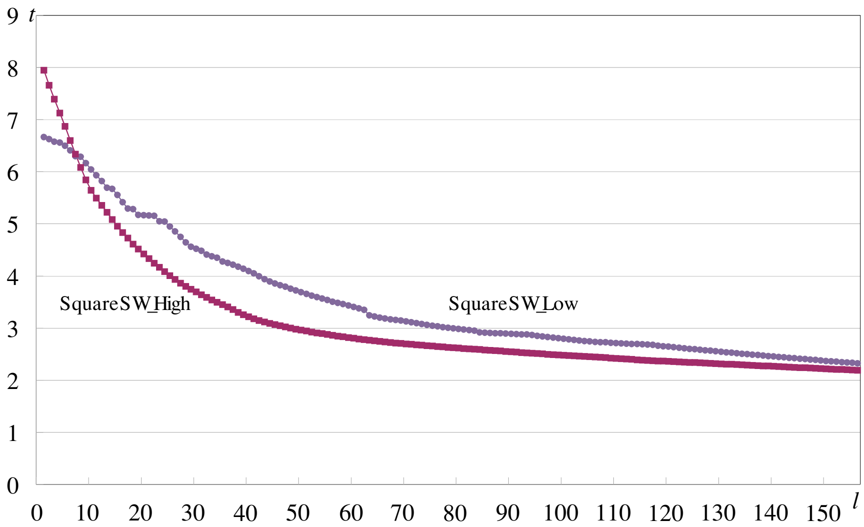

Fig. 13. Difference in mean travel time between high-density and low-density areas.

of the π indicator on the shape of the city. Both of the figures show that the smaller the number of closed paths, the greater the tendency for the network to adopt a tree structure, while the lower the network speed, the greater the number of closed paths, and the paths are more circular. For the low network speed case, the results are similar to the uniform population case.

3.4 Impact of population density on network growth

Figure 11 shows the growth process of the network for different population density patterns. As shown in Fig. 8, the calculation was performed for two cases: $c = 0.2$ and $c = 0.8$.

In every case, network growth starts from the high-density area. However, depending on the speed of the high-speed links, networks have different growth patterns after this period. That is, in the case of high-speed links, there is a tendency to give priority to links extending to low-density areas. On the other hand, in the case of low-speed links, the growth pattern develops such that links first sufficiently cover high-density areas before starting to extend to low-density areas. It should be noted that high-speed transportation prioritizes regional areas, while low-speed transportation prioritizes cities. In SquareC and SquareSW, a low-speed network grows by extending towards the east, west, south, and north directions, centered on the high-density areas. Networks are first constructed in cities, which have a high population density. Low-speed networks grow by branching out from high-density areas. These patterns are evident in the railway networks and road networks of real cities.

Figure 12 shows how mean travel time is reduced, and Fig. 13 shows the difference in the reduction process between high-density areas and low-density areas. In the case of SquareC, where the population is overly concentrated in one area, mean travel time is extremely short. This is because many people are living in the central area where moving is advantageous. If Belt and Normal cases are compared, in the initial stage, Normal, which has a uniform pop-

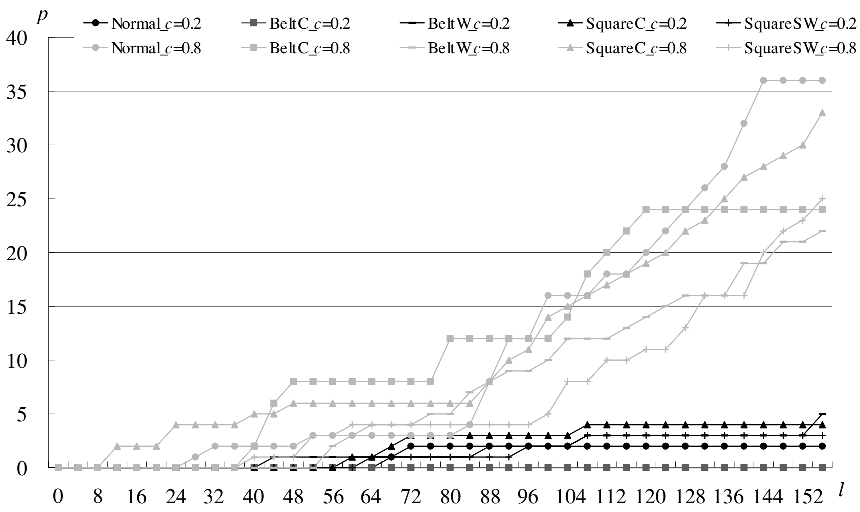

Fig. 14. Number of closed paths.

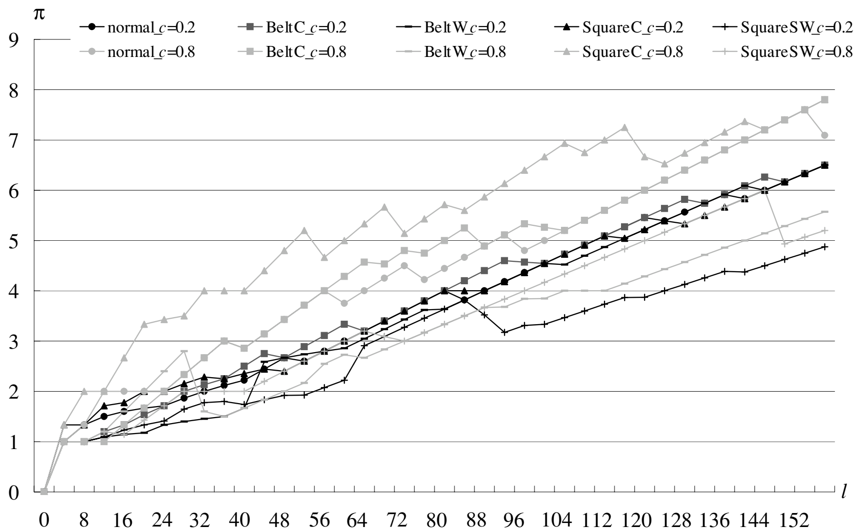

Fig. 15. Indicator π.

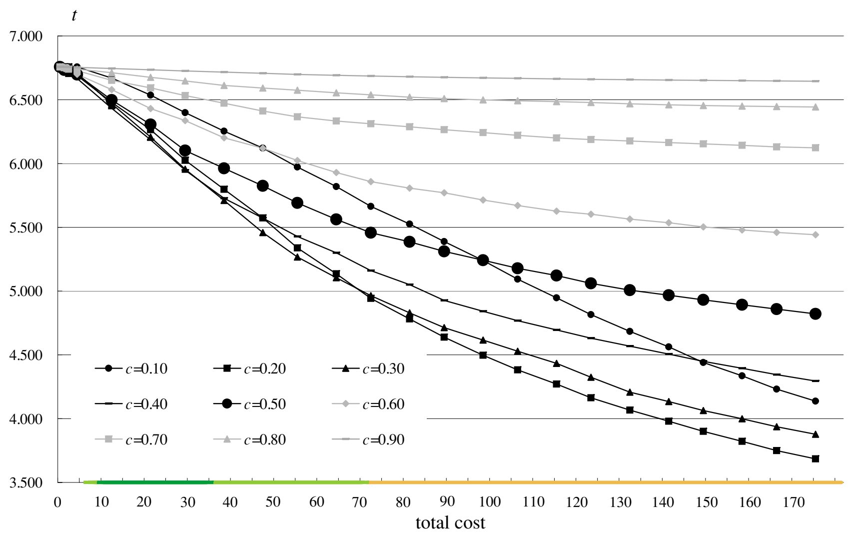

Fig. 16. Relationship between total cost and mean travel time for different speeds.

Table 2. Calculation results and indicator values for each case ($l = 156$).

Case	Shape	c	t	r	Node	p	α	d	π
C	Normal	0.0	0.140	97.9	157	0	0.00	31	5.03
		0.1	1.190	82.4	156	1	0.00	24	6.50
		0.2	2.112	68.8	155	2	0.01	24	6.50
		0.3	2.929	56.7	148	9	0.03	23	6.78
		0.4	3.756	44.4	145	12	0.04	24	6.50
		0.5	4.553	32.6	141	16	0.06	24	6.50
		0.6	5.346	20.9	137	20	0.07	24	6.50
		0.7	6.112	9.6	129	28	0.11	20	7.80
		0.8	6.446	4.6	121	36	0.15	20	7.80
		0.9	6.652	1.6	132	25	0.10	19	8.21
VC	River	0.2	2.061	71.4	133	24	0.09	24	6.50
		0.8	6.827	5.3	129	28	0.11	24	6.50
	Bay	0.2	2.091	70.3	145	12	0.04	29	5.38
		0.8	6.691	5.0	129	28	0.11	24	6.50
	Bay2	0.2	2.225	70.8	111	46	0.21	32	4.88
		0.8	7.174	6.0	127	30	0.12	24	6.50
	Lake	0.2	2.054	73.3	133	24	0.09	24	6.50
		0.8	7.274	5.3	135	22	0.08	23	6.78
	2Lakes	0.2	1.991	71.1	149	8	0.03	24	6.50
		0.8	6.562	4.6	124	33	0.14	20	7.80
PP	BeltC	0.2	1.859	71.1	157	0	0.00	24	6.50
		0.8	6.115	5.0	133	24	0.09	20	7.80
	BeltW	0.2	2.076	72.1	152	5	0.02	24	6.50
		0.8	7.043	5.3	135	22	0.08	28	5.57
	SquareC	0.2	1.478	71.3	153	4	0.01	24	6.50
		0.8	4.919	4.5	124	33	0.14	20	7.80
	SqurareSW	0.2	2.193	72.9	154	3	0.01	32	4.88
		0.8	7.654	5.5	132	25	0.10	30	5.20

ulation, has the lower mean travel time for the entire city. However, as the transportation network grows, Belt leads to a lower mean travel time. This means that the higher the population density, the greater the improvement in transport infrastructure. On the other hand, areas of lower population do not benefit substantially from reduced travel time due to transportation network growth.

Figures 14 and 15 show the changes in the number of closed paths and the indicator π. In the case of high population density, the lower the speed of the networks, the higher the number of closed paths, the higher the value of π, and the more circular the shape of the network. Comparing network shapes, it can be noted that, in the cases of BeltW and SquareSW, where high-density areas are located outside the cities, both the number of closed paths and the value of π are low. One explanation for this is that, since population is unbalanced, network growth patterns are also unbalanced.

Table 2 shows a list of calculation results for this section.

4. Trade-off between Speed and Cost

The construction of a road network or rail network is extremely expensive. As shown in Tables 3 and 4, the real costs of high-speed networks are generally more expensive than that of low-speed networks. Monorails and new transit systems, which are usually constructed in crowded urbanized areas, are relatively expensive comparing with bullet trains because of higher land price or complicated structures. If there is a fixed budget to construct railways, the following options are available: constructing a high-speed, short distance network or a low-speed, long distance network. Both options present trade-offs. Thus, this section will examine the relationship between construction cost and the speed of a transportation network. This will lead to the determination of the appropriate speed for the transportation network.

First, the number of possible networks that can be constructed given the construction costs is considered. As Table 3 shows, the relationship between cost and speed is based on an average speed for expressways and national highways of 76.7 km/h and 32.8 km/h, with construction costs totaling 6.37 billion yen and 2.99 billion yen, respectively. Taking an average, it can be seen that a road with a speed of 1.17 km/h will cost 0.1 billion yen. By dividing this value by the speed, the number of networks per 0.1 billion yen can be determined. Table 5 summarizes the relationship between the speed and number of networks that can be constructed. The mean travel times for networks that could be constructed at a certain cost were compared, as construction costs increased. In this way, a combination of the desirable speed and shape that results in the mini-

Table 3. Road construction costs (source: Road Bureau, Ministry of Land, Infrastructure, Transport and Tourism (2006)).

	Average velocity (km/h)	Cost (million yen/km)
Expressway	76.7	0.64
National highway	32.8	0.30
Prefecture highway	30.8	0.11

Table 4. Railway construction costs (source: Municipal Transportation Works Association (2008)).

	Average velocity (km/h)	Cost (million yen/km)
Bullet train	220	0.70
LRT	60–120	0.15–0.25
Tram	60–70	0.10–0.20
New transit system	50–60	0.70–1.20
Monorail	65–80	1.00–1.90
Subway	80–100	2.50–3.50
Guideway bus	60	0.30–0.40

Table 5. Construction cost by speed.

c	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Velocity	10.00	5.00	3.33	2.50	2.00	1.67	1.43	1.25	1.11
Link/cost	0.12	0.23	0.35	0.47	0.59	0.70	0.82	0.94	1.05

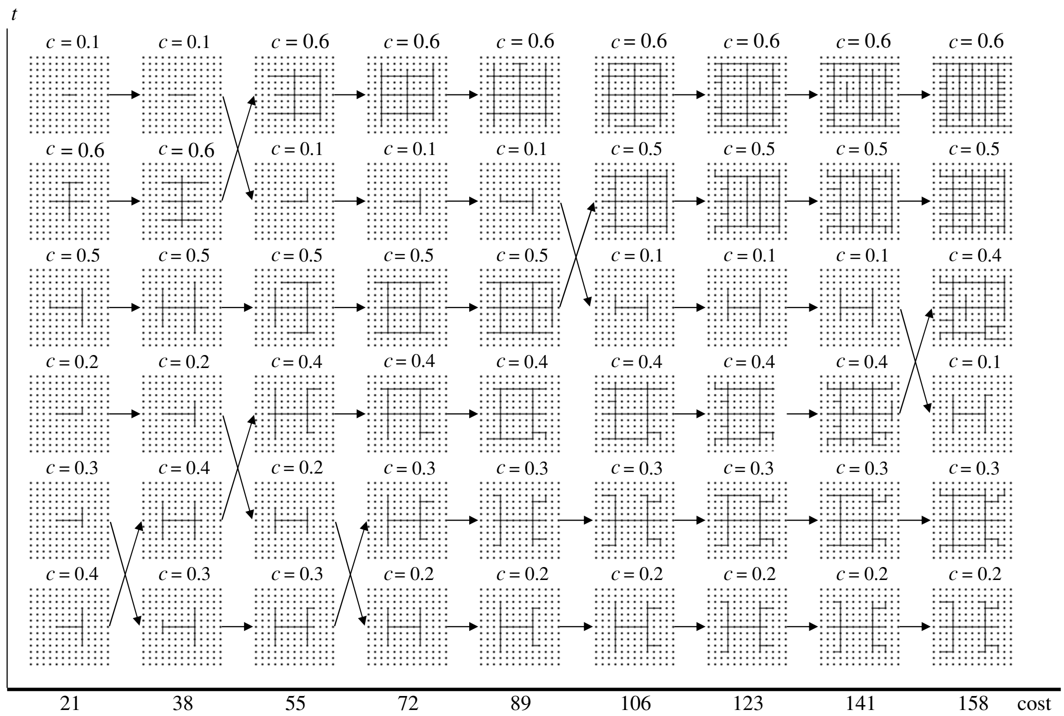

Fig. 17. Total cost as a function of desirable speed and shape.

mum mean travel time can be determined. Furthermore, by using a base unit for the number of networks that can be constructed, changes in the available budget can be easily taken into consideration to determine the desirable combination of speed and the number of networks.

Figure 16 shows the relationship between total cost and mean travel time of transportation networks by speed. The lower the mean travel time, the more desirable networks are. It can be seen that, in the initial stage, a network of $c = 0.3$ reduces required travel time significantly. Links slower than this cannot reduce the required time, even if constructed over longer distance networks. Furthermore, it can be seen that networks cannot be effective unless they can provide transportation at a certain speed. After this, as construction costs increase, the network with the minimum required travel time is the faster network of $c = 0.2$. This is because, as the number of links increases, the effectiveness of the network at speed $c = 0.3$ in reducing travel time reaches a limiting value. Also, the length of the network at speed $c = 0.2$ grows sufficiently to fully demonstrate its usefulness. Based on these results, it can be concluded that networks need a certain length in order to display their effectiveness. Since only a small number of networks of speed $c = 0.1$, the highest speed, can be constructed due to the high construction costs, mean travel time cannot be minimized.

Figure 17 shows the relationship between total cost and a combination of desirable speed and shape. The lower figures indicate networks having lower mean travel time. Since high-speed links are selected, there are no closed paths in the desirable network for any of the cost values, and the network grows with a tree structure. It can be seen that that given the current cost conditions, it is desirable to have networks with no closed paths and with a tree structure of high speed.

5. Conclusions

In this study, different models were constructed to describe the growth of transportation networks. Based on the network speed and population distribution, the growth processes and shapes of transportation networks were examined. Furthermore, the trade-offs between construction costs and travel speed were considered.

First, using the transportation network growth models, it was shown that the shape of networks depends on the speed of high-speed links. High-speed networks tend to grow by means of repeated branching from a central trunk. On the other hand, low-speed networks tend to grow in the shape of a grid, with straight lines overlapping each other, without branching. The effect of speed differences on growth patterns can be illustrated by the contrasting features of the *Shinkansen* network, which has a tree structure, with urban subway networks, which have many closed paths.

Second, it was demonstrated that different city shapes form different network shapes. When the speed of a trunk line link is high, bridge links that connect demand points sandwiched between vacant lands are hardly ever constructed. However, when the speed of a trunk line link is low, bridge links are more likely to be constructed.

Third, it was found that when the population is concentrated in one area, a significant reduction in mean travel time is achieved with only a small number of links. On the other hand, the gap in mean travel time between high-density areas and low-density areas gets larger in such situations. A more detailed examination is required to determine whether it is better for the long-term growth of the network to prioritize overall efficiency for the city or to prioritize equity between different areas.

Fourth, it was shown that, taking into consideration the trade-offs between speed and construction costs, a desirable network speed and shape are primarily constrained by cost.

The growth of networks changes the convenience and accessibility of places, and even changes the geographical distribution of human activities. In the future, it is expected that transportation network growth models will give consideration to factors such as the location of facilities and traffic demand variations, as well as to the time required to build the transportation networks. In addition, growth models will be expanded to deal with the study of network shapes where local optimization rather than overall optimization is used as a criterion and to deal with optimal growth processes. The results of this study can be used as base data to formulate plans for transportation network construction.

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