Multi-Step Factor Analysis; A New Procedure for Hierarchical Factor Solution and/or Hierarchical Classification of Variables

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Introduction

Hierarchical model of motor ability structure was proposed by author in his doctoral dissertation; "Factorial Structure of Motor Ability (1969)",*4 and he presented the hierarchical structure of motor ability with Schmid and Leiman technique. There are several approaches to the hierarchical factor structure at present; that is, Bi-factor solution developed by Holzinger and Swine (1937)*3, Subdivided factor developed by Burt (1949)*1, and Schmid and Leiman procedure (1957)*8, and ROTHIST technique by Zavala (1971)*10. The first three techniques, however, are based upon dividing variance and covariance into several factors which are hierarchically structured and arranged in terms of factorial complexity. And Bi-factor solution and Subdivided factor solution are classical one. It may be said that they are already out of date, unless they are applied to obtain the initial solution to reach at the final solution based upon some sort of strict analytical background; for instance, principal, image, Rao's canonical and so on. This may be because their mathematical backgrounds include some sort of arbitrary assumptions. But this is not proper criticism, because some arbitrary judgements or empirical ones are included in the processes of computation of such techniques as Principal, Image and Canonical solution. However they have been based upon some strict mathematical theories. ROTHIST technique is quite different in theoretical construction from the above mentioned techniques; Bi-factor, Subdivided, and Schmid and Leiman. Technically this is based on rotation technique. That is, if small number of factors are rotated, the complexity of the rotated factors is rather large, while the large number of factors are rotated, the rotated factors's complexity is rather small, because the number of variables showing significant loadings on the rotated factors are small. This is natural inference. Then, rotation of factors will be repeated on such way that the number of factors is increased one by one, and then the hierarchical arrangement of factors can be obtained in terms of factor complexity.

The Schmid and Leiman procedure is based on rather strict mathematical theory and oblique solution. Suppose R be correlation matrix of order n, A_1 be the factor pattern matrix, and H_1 be the correlation matrix of the extracted oblique factors. The correlation matrix R can be decomposed as follows;

$$R = A_1 H_1 A_1' \tag{1}$$

Here H_1 is the correlation matrix, so H_1 can be factored again, and decomposed as follows, too;

$$H_1 = A_2 H_2 A_2' \tag{2}$$

where A_2 is the factor pattern matrix and H_2 is the correlation matrix among the extracted factors from H_1. Therefore, in general, i th correlation matrix among the factors extracted by (i + 1)th application of factor analysis is described as follows;

$$H_i = A_{i+1} H_{i+1} A_{i+1}' , \tag{3}$$

and this decomposition can be repeated until H_j is equal to identity matrix I. Therefore, finally H_j can be decomposed as

as $\quad H_j = A_{j+1} A_{j+1}' ,$ (4)

and also $\quad H_{j-1} = A_j H_j A_j ,$ (5)

so $\quad H_{j-1} = A_j A_{j+1} A_{j+1}' A_j'$ (6)

Then, put the right side of (6) into H_{j-1} in decomposition of H_{j-2}, and repeat this process up to R's decomposition of (1). After all, the following decomposition formula is induced;

$$R = A_1 A_2 A_3 \ldots\ldots A_j A_{j+1} A_{j+1}' A_j' \ldots\ldots A_3' A_2' A_1' \tag{7}$$

Then the hierarchical factor pattern matrix is as follows;

$$F = A_1 A_2 A_3 \ldots\ldots A_j A_{j+1} \tag{8}$$

This is basic idea of herarchical factor solution eveloped by Schmid and Leiman, and actually, they took the uniqueness into consideration on the process to reach the final hierarchical factor pattern matrix F. F is the orthogonal factor pattern as shown in (7). This process is based on the mathematical background but actually only a few orders of factors can be extracted. For instance, seventeen factors; one first order factor, five second order, and elevan third order, were extracted from the (31 x 31) correlation matrix in Matsuura's factor analytic study (1969)*4, and significant factor loadings tended to be rather small. As long as Matsuura's results investigated, the significant loadings on second order factors were rather low. Although Schmid and Leiman procedures have mathematical rationale, it can not always produce an elegant solution which has the practical meanings on the actually given data, just as other factoring procedures even if they are based upon some strict mathematical background.

On the other hand, several kinds of cluster analytic procedures were developed for classfying many categorical items into small groups of items. Many of these methods will try to classify the items by relative proximity between items each other. The proximity is evaluated by correlation coefficient and/or distance between them. For example, B-coefficient is one criterion variable for cluster analysis in the case of correlation matrix given. B-coefficient is basically the ratio of mean of the correlation coeficients between the variables which are assumed to belong to the same group of variables to the mean of the correlation coeficients between the rest of variables. Then, judgement on whether the variable belongs to a certain group or not is determined by a certain arbitrary value of decrease of B-coefficient and also arbitrary amount of B-coefficient itself. This is one of classical way of variable classification.

Anyhow, cluster analysis is based on such kind of arbitrary judgement. The clustering procedure, however, is useful and practical in various areas of study. The cluster analysis may give some sort of qualitative classification of variables but does not give any clear-cut sketch on factorial structure or domain structure. Then, author devised another procedure for hierarchical classification of variables and also for giving some practical knowledge on ability structure from hierarchical structural point of view. Therefore, the rationale for this procedure and its application to the actual data of motor ability will be discussed in this paper.

Rationale and Algorithm of Multi-step Factor Solution

Suppose R be the correlation matrix of order n, A_1 be the rotated factor pattern matrix produced by the first application of factor analysis and orthogonal rotation; that is, they are as follows;

$$R = \begin{bmatrix} r_{11} r_{12} r_{13} \cdots\cdots r_{1n} \\ r_{21} r_{22} r_{23} \cdots\cdots r_{2n} \\ \\ \\ r_{n1} r_{n2} r_{n3} \cdots\cdots r_{nn} \end{bmatrix} \qquad (9)$$

where r_{ii} is communality or unity.

$$A_1 = \begin{bmatrix} a_{111} a_{112} a_{113} \cdots\cdots a_{11m} \\ a_{121} a_{122} a_{123} \cdots\cdots a_{12m} \\ \\ \\ a_{1n1} a_{1n2} a_{1n3} \cdots\cdots a_{1nm} \end{bmatrix} \qquad (10)$$

$$= (A_{11} A_{12} A_{13} \cdots\cdots A_{1m})$$

A_1 is (n x m) matrix; n stands for number of variables and m for number of rotated factors. Then, suppose $A'_{1i} = (a_{11i}\ a_{12i}\ a_{13i} \ldots\ldots a_{1ni})$; factor loading vector of i th factor, and let $\hat{A}'_{1i}$ be a vector whose elements are the significant and useful loadings for interpreting i th factor and the number of its elements be q_i ($q_i < n$); that is, usually q_i is much less than n. Then, if only significant and useful loadings are described as elements of $\hat{A}'_1$, $\hat{A}'_1$ will be written as follows;

$$\hat{A}_1 = \begin{bmatrix} * & & & & \\ * & & & & \\ * & & & & \\ * & & & & \\ * & & & & \\ * & & & & \\ & * & & & \\ & * & & & \\ & * & & & \\ & * & & & \\ & * & & & \\ & & * & & \\ & & * & & \\ & & * & & \\ & & * & \cdots\cdots & * \\ & & & & * \\ & & & & * \\ & & & & * \\ & & & & * \\ & & & & * \\ & & & & * \end{bmatrix}$$

where * stands for significant useful loading.

Then, the variables which show significant and useful loadings on the i th factor may be reasonably considered as one group of variables which may have a common characteristics or common domain of variance. Let R_{1i} be a correlation matrix whose elements are correlations between the variables of i th set in q sets of variables. Then, R_{1i} be factored and let A_{2i} $(q_i \times q_i')$ be the resulted rotated factor pattern matrix;

$$A_{2i} = \begin{array}{c} \begin{bmatrix} * & & & & \\ * & & & & \\ * & & & & \\ * & & & & \\ * & & & & \\ & * & & & \\ & * & & & \\ & * & & & \\ & * & & & \\ & & * & & \\ & & * & & \\ & & * & & \\ & & & \cdots\cdots & \\ & & & & * \\ & & & & * \\ & & & & * \\ & & & & * \end{bmatrix} \\ \longleftarrow \quad q_i' \quad \longrightarrow \end{array}$$

Then, the variables which show significant and useful loadings for interpretation of i th factor of R_{1i} may be reasonably considered as one group of variables whose elements may have a common domain of variance of q_i variables. Therefore, the q_i variables in n variables can be classified into q_i' sets of variables by the second application of factor analysis. Such repetition of factoring R_{ki} whose order decreases as the factor analysis is repeated. This process of factor analysis must be repeated until only one factor can be extracted or the number of variables of a certain set is one or two. Thus, each of q sets of variables must be factored up to the criterion mentioned above. Finally the following step-wise variable structure will be resulted. (Fig. 1)

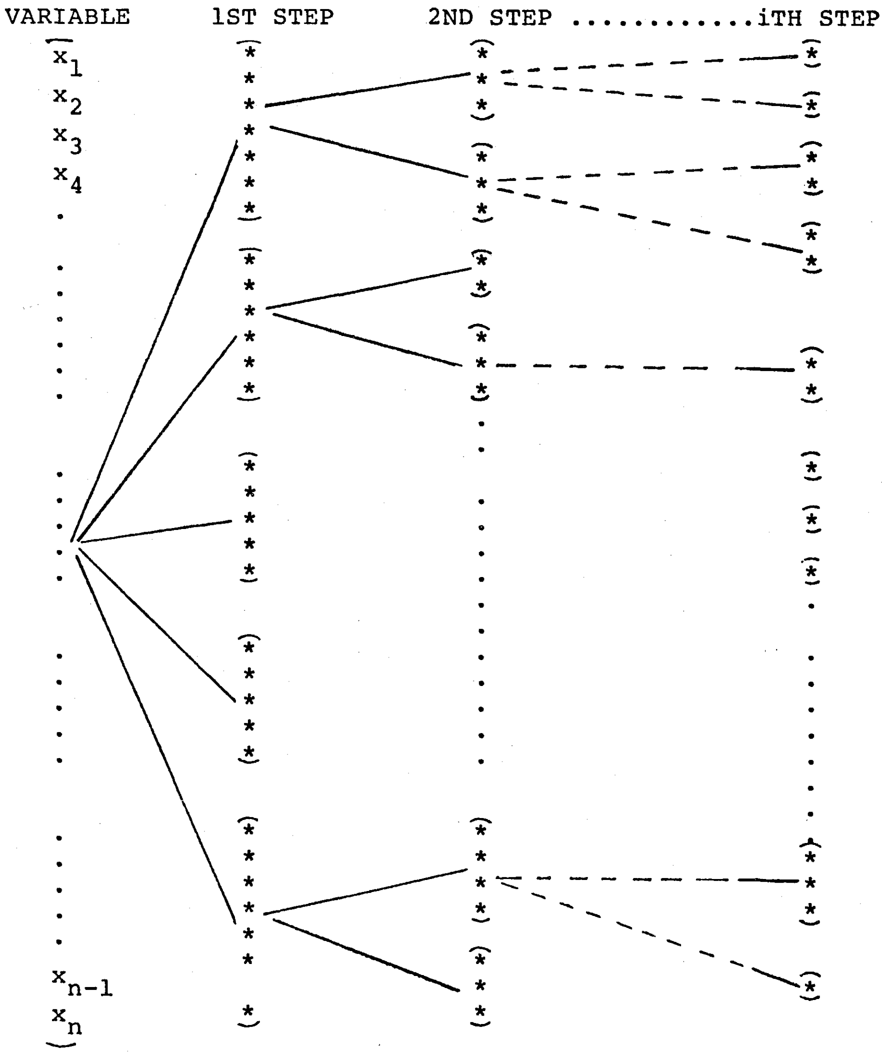


Fig. 1 Model of step-wise hierarchical arrangement of significant factor loadings
* stands for significant loadings.

The complexity of factors decreases as the factoring step goes up, because the number of variables which show significant and useful loadings on each factor decreases.

Application of Multi-step Factor Analysis to Investigate the Hierarchical Structure of Children's Ball Handling Skills

(1) Sample and method

275 kindergarten children were tested with 24 performance test items of ball handling skill. The sample size is shown in Table 1. These items were devised to investigage the children's ball handling skill with such premise as the motor ability of children, particularly, the pre-school aged, is quite different in its nature from that of children older than 9 or so, such that their motor ability should be measured by how safely and/or well they can perform the physical movement as test but not by making use of some metrical scale; e.g, CGS unit, on their performance results.

Table 1. Sample Size

Age	3	4	5	6	Total
Boy	16	60	46	10	132
Girl	18	58	54	13	143
Total	34	118	100	23	275

The used 24 test performances were as follows;

1. Catching the rolling ball with hand; ball is rolled from 5 m apart from the catching position and tennis ball (hard) used.
2. Catching the rolling ball with hand; same situation but softball used.
3. Catching the rolling ball with both hands; same situation and tennis ball (hard) used.
4. Catching the rolling ball with hands; same situation but softball used.
5. Rolling the ball for accuracy; roll the ball for an aim (50 cm x 50 cm square) 5 m apart from testee and tennis ball (hard) used.
6. Rolling the ball for accuracy; roll the ball for an aim (50 cm x 50 cm square) 5 m apart from testee and softball used.
7. Striking the rolling ball with dominant hand; 2 m apart and tennis ball used.
8. Striking the rolling ball with hand 2 m apart and Mari (20 cm in diameter) used.
9. Place kick for accuracy; kick the placed ball to an aim (50 cm x 50 cm square) 3 m apart from kicking position and dodge ball used.
10. Place kick for accuracy; kick the placed ball to an aim (50 cm x 50 cm square) 4 m apart from kicking position and dodge ball used.
11. Place kick for accuracy; kick the placed ball to an aim 5 m apart from kicking position and dodge ball used.
12. Kick the rolling ball; kick the ball rolling from front for an aim (50 cm x 50 cm square) 3 m apart from kicking position and dodge ball used.
13. Kick the rolling ball; kick the ball rolling from side (right) for an aim 3 m apart from kicking position and dodge ball used.
14. Catching the bouncing ball; catch the ball bouncing at 1.5 m ahead of testee and dodge ball used.
15. Throwing for accuracy; throw the ball to an aim (40 cm diameter-circle, altitude 100 cm from floor to center of circle) from throwing position and tennis ball (hard) used.

16. Throwing for accuracy; same situation except that the distance to aim was 3 m.
17. Throwing for accuracy; same situation except that the distance to aim was 4 m.
18. Throwing for accuracy; same situation except that the distance to aim was 5 m.

19, 20, 21, and 22 are same to 15, 16, 17, and 18 respectively but Mari (20 cm in diameter) used.

23. Catching the flying ball; catch the ball flying from the place 3 m apart from catching position and tennis ball used.
24. Catching the flying ball; same situation but Mari used.

The number written at the head of each item description corresponds to the item number appeared in several tables shown later. These items were tested on such way that 10 trials were permitted to every child and the number of successful trials was counted as score. Then, the means and standard deviations were computed and they are shown in Table 2. Then, the intercorrelations were approximated with

Table 2. Mean and Standard Deviation all ages pooled

Item	Boy		Girl	
	Mean	S.D.	Mean	S.D.
1	8.08	1.98	7.98	2.00
2	8.53	1.68	8.19	1.90
3	8.14	1.79	8.41	1.60
4	8.80	1.43	9.01	1.30
5	2.90	1.48	2.54	1.32
6	2.91	1.48	2.60	1.25
7	8.82	1.57	8.70	1.52
8	9.94	1.52	9.37	.99
9	3.08	1.64	2.69	1.32
10	2.49	1.30	2.24	1.15
11	2.30	1.32	1.97	1.12
12	2.15	1.51	2.03	1.63
13	1.88	1.10	1.91	1.31
14	8.00	1.98	8.15	1.98
15	3.58	1.90	2.65	1.63
16	2.23	1.65	1.59	.97
17	1.49	.79	1.32	.72
18	1.44	.80	1.00	.00
19	2.42	1.51	2.52	1.63
20	1.63	1.02	1.56	.87
21	1.94	1.48	1.54	.69
22	1.21	.43	1.60	.89
23	3.50	2.39	3.19	2.11
24	4.22	2.42	4.96	2.77
Sample Size (n)	132		143	

Pearson coefficients because most of Pearson correlation coefficients were larger than the ones which were computed with non-parametric method; contingency coefficient using (10 x 10) contingency table.

The correlation matrix of order 24 is shown in Table 3. According to the algorithm of multi-step factor

Table 3. The Initial Correlation Matrix of Order 24

Item No.																								
1	1.00000	0.40514	0.36229	0.30697	0.06931	0.08192	0.22383	0.24397	0.02972	-0.10690	0.02995	-0.09948	0.00454	0.31911	0.09242	0.16345	0.07606	0.11291	0.09146	0.10711	-0.17845	-0.13790	0.22875	0.13214
2	0.40514	1.00000	0.30529	0.39603	0.13005	0.12131	0.23679	0.29515	0.05806	-0.04919	0.14528	0.07245	0.21537	0.24016	0.17790	0.08811	0.21298	0.14908	0.09628	0.27121	0.18591	-0.04614	0.22710	0.20071
3	0.36229	0.30529	1.00000	0.44333	0.21302	0.18117	0.24103	0.36466	0.05593	-0.00008	0.01537	-0.11034	0.06735	0.28324	0.12388	-0.00745	0.01407	0.12347	0.12176	0.04236	-0.02824	-0.06348	0.06700	0.13027
4	0.30697	0.39603	0.44333	1.00000	0.19969	0.18593	0.25116	0.41339	0.13255	0.00824	0.03376	0.03445	-0.02541	0.23743	0.07620	0.04064	0.07756	0.02640	0.12759	0.14169	-0.02251	-0.26480	0.05371	0.11397
5	0.06931	0.13005	0.21302	0.19969	1.00000	0.34590	0.21457	0.19413	0.09872	0.06701	0.11583	-0.03920	0.15295	0.12756	0.12780	0.20261	0.34897	0.06924	0.17383	-0.05088	0.32970	-0.18733	0.22150	0.11445
6	0.08192	0.12131	0.18117	0.18593	0.34590	1.00000	0.04033	0.08134	0.17167	0.03637	0.06176	0.01294	0.15429	0.10930	0.16427	0.05014	0.32448	0.12036	0.07792	-0.12872	0.01481	-0.29322	0.16778	0.09806
7	0.22383	0.23679	0.24103	0.25116	0.21457	0.04033	1.00000	0.48825	0.15464	0.03472	0.05292	0.01331	0.01457	0.29923	0.16688	0.21443	0.13761	-0.01264	0.06536	-0.03705	0.03113	0.12088	0.09637	0.14991
8	0.24397	0.29515	0.36466	0.41339	0.19413	0.08134	0.48825	1.00000	0.17056	-0.00725	0.00713	0.04646	0.11873	0.31249	0.08458	0.02823	0.07915	-0.05759	0.10669	0.04060	0.13078	0.09226	0.08112	0.11676
9	0.02972	0.05806	0.05593	0.13255	0.09872	0.17167	0.15464	0.17056	1.00000	0.19680	0.14546	0.13612	0.02829	0.09800	0.06321	0.06575	-0.04505	0.15524	0.01733	0.13961	0.16756	0.15658	-0.03985	0.10290
10	-0.10690	-0.04919	-0.00008	0.00824	0.06701	0.03637	0.03472	-0.00725	0.19680	1.00000	0.16536	0.04305	0.02349	0.08385	0.07173	0.09575	0.04299	-0.01570	-0.01184	-0.05668	0.07626	0.30083	-0.05501	0.06105
11	0.02995	0.14528	0.01537	0.03376	0.11583	0.06176	0.05292	0.00713	0.14546	0.16536	1.000000	0.00168	0.02071	0.09907	0.15823	0.21339	0.11647	0.40480	0.07946	0.11766	0.02823	0.32556	0.12605	0.23880
12	-0.09948	0.07245	-0.11034	0.03445	-0.03920	0.01294	0.01331	0.04646	0.13612	0.04305	0.00168	1.00000	0.03906	0.04721	0.03301	-0.07672	-0.02149	0.00058	-0.00597	0.15333	0.35316	0.37587	0.07868	0.12296
13	0.00454	0.21537	0.06735	-0.02541	0.15295	0.15429	0.01457	0.11873	0.02829	0.02349	0.02071	0.03906	1.00000	0.10267	0.13311	0.03788	-0.18018	-0.06251	0.15417	0.13336	0.02383	-0.17461	0.05473	0.12122
14	0.31911	0.24016	0.28324	0.23743	0.12756	0.10930	0.29923	0.31249	0.09800	0.08385	0.09907	0.04721	0.10267	1.00000	0.12363	0.13185	-0.04174	0.09309	0.08081	0.13611	0.14666	0.07434	0.19338	0.26558
15	0.09242	0.17790	0.12388	0.07620	0.12730	0.16427	0.16688	0.08458	0.06321	0.07173	0.15823	0.03301	0.13311	0.12363	1.00000	0.42348	0.43243	0.13225	0.10069	0.20765	-0.03118	0.11849	0.20825	0.12965
16	0.16345	0.08811	-0.00745	0.04064	0.20261	0.05014	0.21443	0.02823	0.06575	0.09575	0.21339	-0.07672	0.03788	0.13185	0.42348	1.000000	0.39412	0.39549	0.07364	0.17895	0.06878	0.15437	0.41623	0.25386
17	0.07606	0.21298	0.01407	0.07756	0.34897	0.32448	0.13761	0.07915	-0.04505	0.05299	0.11647	-0.02149	-0.18018	-0.04174	0.43243	0.39412	1.00000	0.59231	0.34731	0.29063	0.73160	0.08223	0.24262	0.09844
18	0.11291	0.14908	0.12347	0.02640	0.06924	0.12036	-0.01264	-0.05759	0.15524	-0.01570	0.40480	0.00058	-0.06251	0.09309	0.13225	0.39549	0.59231	1.00000	0.14811	0.35170	0.34221	0.66404	0.44380	0.06848
19	0.09146	0.09628	0.12176	0.12759	0.17383	0.07792	0.06536	0.10669	0.01733	-0.01184	0.07946	-0.00597	0.15417	0.08081	0.10069	0.07364	0.34731	0.14811	1.00000	0.60430	0.15033	0.00467	0.55555	0.27462
20	0.10711	0.27121	0.04236	0.14169	-0.05088	-0.12872	-0.03705	0.04060	0.13961	-0.05668	0.11766	0.15333	0.1336	0.13611	0.20765	0.17895	0.29063	0.35170	0.60430	1.00000	0.74923	-0.05277	0.16615	0.24031
21	-0.17845	0.18591	-0.02824	-0.02251	0.3290	0.01481	0.03113	0.13078	0.16756	0.07626	0.02823	0.35316	0.02383	0.14666	-0.03118	0.06878	0.73160	0.34221	0.15033	0.74923	1.00000	0.31726	0.37409	0.14069
22	-0.13790	-0.04614	-0.06348	-0.26480	-0.18733	-0.29322	0.12088	0.09226	0.15658	0.30083	0.32556	0.37587	-0.17461	0.07434	0.11849	0.15437	0.08223	0.66404	0.00467	-0.05277	0.31726	1.00000	-0.03933	0.29786
23	0.22875	0.22710	0.06700	0.05371	0.22150	0.16778	0.09637	0.08112	-0.03985	-0.05501	0.12605	0.07868	0.05473	0.19336	0.20825	0.41623	0.24262	0.44380	0.35555	0.16615	0.37409	-0.03933	1.00000	0.46309
24	0.13214	0.20071	0.13027	0.11397	0.11445	0.09806	0.14991	0.11676	0.10290	0.06105	0.23880	0.12296	0.12122	0.26558	0.12965	0.25386	0.09844	0.06848	0.27462	0.24031	0.14069	0.29786	0.46309	1.00000

analysis mentioned in the preceding section, principal factor analysis with the estimated communalities of unity and the orthogonal rotation with Normal Varimax criterion were used on every step of factoring. The factor extraction was continued until 80% of total variance could be explained with the factors extracted and the criterion value of significancy and usability for factor interpretation was determined 0.40. These two values were determined empirically according to the comparison of the results that were induced with 9 pairs of values; 90%, 80% and 70%, and 0.3, 0.4 and 0.5 for degree of contribution and factor significancy criterion respectively. The judegement criterion for these comparisons was based upon which pair of these two values can explain the factorial structure more simplly and elegantly.

(2) Results

2-1.The first step of factor analysis.

Eleven factors were extracted and rotated, but 6 factors of them showed more than one of loading which was significant and usable for interpretation; larger than 0.4, and the·amount of contribution of these 6 factors is shown in Table 4. Thus, 75.03% of total variance can be explained by them. The 24

Table 4. Results by First Step of Factor Analysis

Factor	Amount of Cont.	Degree of Cont.	Item No. Showing Loading Over .4
F1	6.80857	28.36904 %	1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 13, 14
F2	5.75262	23.96925	15, 16, 17, 18, 19, 20, 21
F3	1.52004	6.33350	13, 22
F4	1.18867	4.95279	10, 11
F5	.87191	3.63295	23, 24
F6	1.86466	7.76941	15, 16, 17
Total	18.00647	75.02666 %	

test items were grouped into 6 by the first application of factor analysis. The 5 factors of 6 were interpreted as follows;

1. Rolling and/or bouncing ball handling skill.
2. General throwing skill for accuracy.
3. Uninterpretable.
4. Ball kicking skill.
5. Flying ball catching skill.
6. Throwing skill for accuracy, connecting with catching skill.

Thus, factor 1 and 2 and 6; that is, "rolling and/or bouncing ball handling skill", "general throwing skill for accuracy", and "throwing skill for accuracy, connecting with catching skill" seem to be able to be factored again, because the number of variables which show significant and usable loadings are more than two.

2-2. The multi-step of factor analysis of these three variable groups

Factor 1; Rolling and/or bouncing ball handling skill.

The correlation matrix among variables showing significant and usable for factor 1 is shown in Table 5. The principal factor technique and orthogonal rotation with Normal Varimax criterion were applied to this correlation matrix, and 7 factors were extracted and 5 factors showed more than one of loadings over 0.4. These factors are shown in Table 6. Thus, factor 1-2, 1-3, 1-4, and 1-5 can not be factored any more and then, the correlation matrix among variables showing significant loadings on factor 1-1; shown in Table 6, is shown in Table 7, and the results are shown in Table 8.

Table 5. Correlation Matrix of Variables Showing Significant Loadings on F1

1	1.00000	0.40514	0.36229	0.30697	0.06931	0.08192	0.22383	0.24397	0.02972	-0.09948	0.00454	0.31911
2	0.40514	1.00000	0.30529	0.39603	0.13005	0.12131	0.23679	0.29515	0.05706	0.07245	0.21537	0.24016
3	0.36229	0.30529	1.00000	0.44333	0.21302	0.18117	0.24103	0.36466	0.05593	-0.11034	0.06735	0.28324
4	0.30697	0.39603	0.44333	1.00000	0.19969	0.18593	0.25116	0.41339	0.13255	0.03445	-0.02541	0.23743
5	0.06931	0.13005	0.21302	0.19969	1.00000	0.34590	0.21457	0.19413	0.09872	-0.03920	0.15295	0.12756
6	0.08192	0.12131	0.18117	0.18593	0.34590	1.00000	0.04033	0.08134	0.17167	0.01294	0.15429	0.10930
7	0.22383	0.23679	0.24103	0.25116	0.21457	0.04033	1.00000	0.48825	0.15464	0.01331	0.01457	0.29923
8	0.24397	0.29515	0.36466	0.41339	0.19413	0.08134	0.48825	1.00000	0.17056	0.04646	0.11873	0.31249
9	0.02972	0.05806	0.05593	0.13255	0.09872	0.17167	0.15464	0.17056	1.00000	0.13612	0.02829	0.09600
12	-0.09948	0.07245	-0.11034	0.03445	-0.03920	0.01294	0.01331	0.04646	0.13612	1.00000	0.08906	0.04721
13	0.00454	0.21537	0.06735	-0.02541	0.15295	0.15429	0.01457	0.11873	0.02829	0.08906	1.00000	0.10267
14	0.31911	0.24016	0.28324	0.23743	0.12756	0.10930	0.29923	0.31249	0.09600	0.04721	0.10267	1.00000

Note: The used item number; 1, 2, 3, 4, 5, 6, 7, 8, 9 12, 13, 14

Table 6. Results by Second Step of Factor Analysis on F1

Factor	Amount of Cont.	Degree of Cont.	Item No. Showing Loading Over .4
F 1-1	3.82791	31.89925 %	1, 2, 3, 4, 7, 8, 14
F 1-2	1.32389	11.08341	13
F 1-3	.63434	5.28616	14
F 1-4	1.22099	10.74910	5, 6
F 1-5	2.09611	17.46758	9, 12
Total	9.10324	75.91189%	

Table 7. Correlation Matrix

1	1.00000	0.40514	0.36229	0.30697	0.22383	0.24397	0.31911
2	0.40514	1.00000	0.30529	0.39603	0.23679	0.29515	0.24016
3	0.36229	0.30529	1.00000	0.44333	0.24103	0.36466	0.28324
4	0.30697	0.39603	0.44333	1.00000	0.25116	0.41339	0.23743
7	0.22383	0.23679	0.24103	0.25116	1.00000	0.48825	0.29923
8	0.24397	0.29515	0.36466	0.41339	0.48825	1.00000	0.31249
14	0.31911	0.24016	0.28324	0.23743	0.29923	0.31249	1.00000

Note: The used variable No.; 1, 2, 3, 4, 7, 8, 14

Table 8. Results by Third Step of Factor Analysis on F 1-1

Factor	Amount of Cont.	Degree of Cont.	Item No. Showing Loading Over .4
F1-1-1	2.36955	33.85071 %	1, 2, 3, 4, 8
F1-1-2	1.75242	25.03457	4, 7, 8
F1-1-3	1.19049	17.00700	8, 14
Total	5.31246	75.89228%	

Table 9. Correlation Matrix

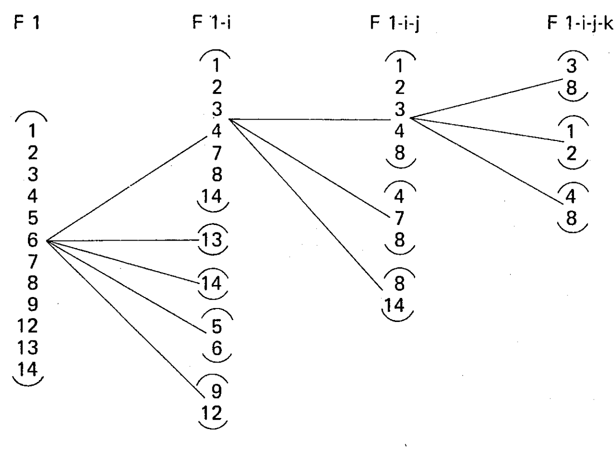
1	1.00000	0.40514	0.36229	0.30697	0.24397
2	0.40514	1.00000	0.30529	0.39603	0.29515
3	0.36229	0.30529	1.00000	0.44333	0.36466
4	0.30697	0.39603	0.44333	1.00000	0.41339
8	0.24397	0.29515	0.36466	0.41339	1.00000

Note: The used variable No.; 1, 2, 3, 4, 8

Table 10. Results by Fourth Step of Factor Analysis on F1-1-1

Factor	Amount of Cont.	Degree of Cont.	Item No. Showing Loading Over .4
F1-1-1-1	1.11302	22.26040 %	3, 8
F1-1-1-2	1.67253	33.45060	1, 2
F1-1-1-3	1.11922	22.38440	4, 8
Total	3.90477	78.09540 %	

Then, the factor 1-1-1 and 1-1-2 may be factored again, so the correlation matrix among variables 1, 2, 3, 4 and 8 is shown in Table 9 and the results are shown in Table 10. Thereafter, any factor does not have any possibility to be factored again, because each of the extracted and rotated factors has only two significant variables. It was found that the variables 4, 7 and 8 had only one common factor through application of factor analysis. Thus, the following hierarchical factorial structure may be resulted;



Then, these extracted factors can be interpreted as follows;
F1 ; Rolling and/or bouncing ball handling skill.
F1-1; Rolling ball handling skill.
F1-2; Kicking the rolling ball.
F1-3; Catching the bouncing ball.
F1-4; Rolling the ball for accuracy.
F1-5; Kicking the ball for accuracy; general.
F1-1-1; Catching the rolling ball.
F1-1-2; Striking the rolling ball.
F1-1-3; Catching the bouncing ball.
F1-1-1-1; Cathing the rolling ball with hands; tennis ball used.
F1-1-1-2; Catching the rolling ball with hand.
F1-1-1-3; Catching the rolling ball with hands; softball used.
F1-1-2-1; Striking the rolling ball; same to F1-1-2, because only one common factor could be extracted.

Therefore, the hierarchical structure system of rolling and/or bouncing ball handling skill may be described in Fig. 2. Then, with the factor 2 extracted at the first step of factor analysis, the same

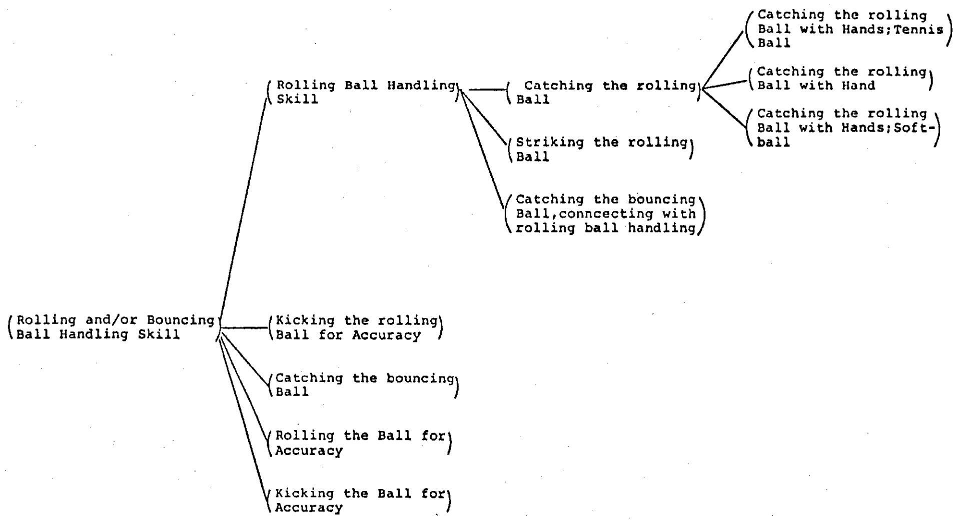


Fig. 2 The hierarchical structure of the rolling and/or bouncing ball handling skill domain.

procedures were carried out. The data for inference on factor 2; 2nd group of variables, are as follows; in Table 11 and 12. The Table 11 shows the correlation matrix among the variables which show significant and usable loadings on F2-factor, and Table 12 shows the results with the factor analysis of 2nd step. The correlation matrix factored at the third step of factor analysis on factor F2-1 is shown in Table 13 and the results are shown in Table 14.

It was found that both groups of variable 18, 20 and 21, and 15, 16 and 17 had not more than one common factor, so no more factoring continued. According to the similar procedure as worked out on the factor 1, the following hierarchical structure system of factor 2 was resulted.

Table 11. Correlation Matrix

15	1.00000	0.42348	0.43243	0.13225	0.10069	0.20765	-0.03118
16	0.42348	1.00000	0.39412	0.39549	0.07364	0.17895	0.06878
17	0.43243	0.39412	1.00000	0.59231	0.34731	0.29063	0.73160
18	0.13225	0.39549	0.59231	1.00000	0.14811	0.35170	0.34221
19	0.10069	0.07364	0.34731	0.14811	1.00000	0.60430	0.15033
20	0.20765	0.17895	0.29063	0.35170	0.60430	1.00000	0.74923
21	-0.03118	0.06878	0.73160	0.34221	0.15033	0.74923	1.00000

Note: The used variable No.; 15, 16, 17, 18, 19, 20, 21

Table 12. Results by Second Step of Factor Analysis on F2

Factor	Amount of Cont.	Degree of Cont.	Item No. Showing Loading Over .4
F2-1	3.09363	44.19471%	15, 16, 17, 18, 20, 21
F2-2	2.33572	33.36750	18, 19, 20
Total	5.42935	77.56221%	

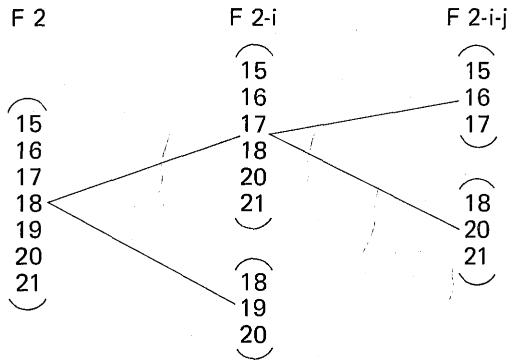
Table 13. Correlation Matrix

15	1.00000	0.42348	0.43243	0.13225	0.20765	-0.03118
16	0.42348	1.00000	0.39412	0.39549	0.17895	0.06878
17	0.43243	0.39412	1.00000	0.59231	0.29063	0.73160
18	0.13225	0.39549	0.59231	1.00000	0.35170	0.34221
20	0.20765	0.17895	0.29063	0.35170	1.00000	0.74923
21	-0.03118	0.06878	0.73160	0.34221	0.74923	1.00000

Note: The used variable No.; 15, 16, 17, 18, 20, 21

Table 14. Results by Third Step of Factor Analysis on F2-1

Factor	Amount of Cont.	Degree of Cont.	Item No. Showing Loading Over .4
F2-1-1	2.26851	37.80850%	18, 20, 21
F2-1-2	2.41594	40.26580	15, 16, 17
Total	4.68445	78.07430%	



F2 ; General throwing skill for accuracy
F2-1; Throwing skill for accuracy.
F2-2; Throwing skill for ccuracy, specifically with big ball.
F2-1-1; Throwing skill for accuracy with big ball, connecting throwing skill with small ball.
F2-1-2; Throwing skill for accuracy with small ball.

Therefore, the model of hierarchical structure was induced as shown in Fig. 3.

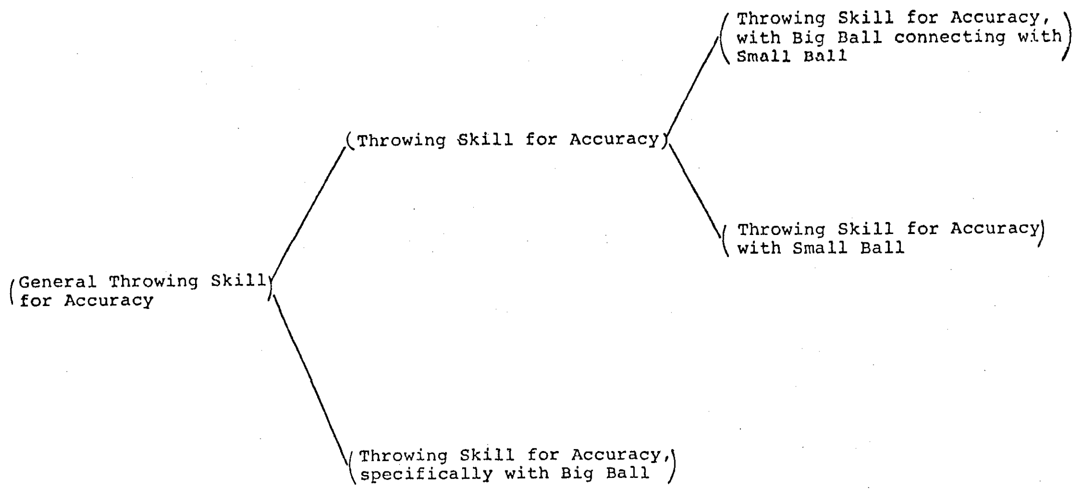


Fig. 3 The hierarchical structure of the general throwing skill for accuracy domain.

Then, Factor 6; Throwing skill for accuracy, connecting with catching skill was factored as shown in Table 15 and 16. The variable 14, 15 and 16 were found to have only one common factor, so factoring stopped. Thus, the following inference was induced;

Table 15. Correlation Matrix

14	1.00000	0.12363	0.13185	0.14666
15	0.12363	1.00000	0.42348	-0.03118
16	0.13185	0.42348	1.00000	0.06878
21	0.14666	-0.03118	0.06878	1.00000

Note: The used variable No. 14, 15, 16, 21

Table 16. Results by Second Step of Factor Analysis on F6

Factor	Amount of Cont.	Degree of Cont.	Item No. Showing Loading Over .4
F6-1	1.50547	37.63675 %	14, 16, 16
F6-2	1.09385	27.34625	21
Total	2.59932	64.98300 %	

F6 ; Throwing skill for accuracy, connecting with catching skill.
F6-1; Throwing skill for accuracy, connecting with catching skill; small ball emphasized.
F6-2; Throwing skill for ccuracy, connecting with catching skill; big ball emphasized.

Therefore, the following model of hierarchical structure was induced as shown in Fig. 4.

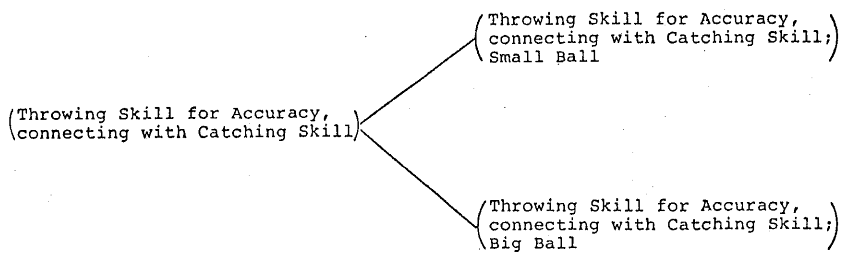


Fig. 4 The hierarchical structure of the throwing skill for accuracy connecting catching skill domain.

2-3. Summary of Application

From the preceding repetition of factor analysis, the following model of hierarchical structure of children's ball handling skill has been resulted as shown in Fig. 5, as far as 24 items used. Fig. 5 suggests

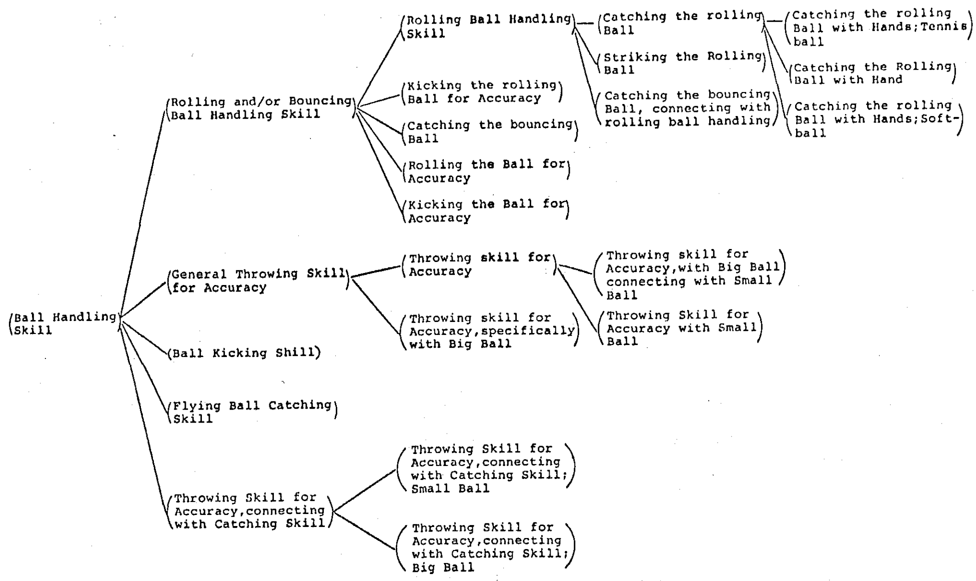


Fig. 5 The hierarchical structure of children's ball handling skill formulated by multi-step factor analysis.

that the children's ball handling skill is structured with these elements hierarchically. As the factoring step goes up, the degree of complexity of factor decreases. The last factor domain, however, has only one common factor, or defined by only one item, so the degrees of factorial complexity are nearly equal to each other. Ball kicking skill is extracted at the first step of factor analysis, but this factor shows significant and usable loadings on only two variables, and "Catching the rolling ball with hands; tennis ball" extracted at 4th step of factor analysis shows significant and usable loadings on only two variables, so both of them are equal on the factorial complexity, although the step to be extracted is different.

From the resulted hierarchical structure of ball handling skill, such a rather easy ball handling skill as "Rolling and/or bouncing ball handling skill" is structured rather complicatedly, but "Ball kincking skill" and "flying ball catching skill" are structured rather simply and independently from other kinds of ball handling skill. And "Throwing skill for accuracy" which is rather difficult for children to perform shows a little complicated structure; that is, "Throwing skill for accuracy with small ball" and "with big ball" are structured as different skill domains, although they are associated to some extend each other. This may suggest that such a difficult skill is influenced by difference of tools with which the motor pattern is performed. And "Throwing skill for accuracy" is divided into the two domains at the first step of factor analysis as shown in Fig. 5: the one is a unique domain for throwing the ball for accuracy and another the one relating to some extent with "catching skill".

Conclusion

A new idea of factoring procedure based on the hierarchical model of factor structure was formulated, and its algorithm was presented in this paper. Then, this was applied to investigate the ability structure of children's ball handling skill based on the assumption of hierarchical model of factor structure. As long as the children's ball handling data of the present study were used, it may be inferred that an elegant solution can be resulted. Therefore, this idea may be applicable to investigate the hierarchical structure of ability and also to cluster the variables into several groups which are classified in temrs of common factor among variables. This method, however, is rather laborious to reach at the final results if the number of variables or the order of initial correlation matrix is very large, but the electronic computer can save such laboriousness greatly. The two kinds of arbitrary values; criterion on amount of variance which is explained by all the extracted factors and criterion of significant and usable loadings for factorial interpretation, must be given arbitrarily. As long as the author's experiences are based upon, as described in this paper too, 80% and 0.4 are recommendable as the criterion values of these two respectively. Anyhow this idea is based upon that the variables or items which belong to one group must have one common factor.

The computer programs developed by the author can be available upon request.

Bibliography

1. Burt, C, "Subdivided Factors", British J. of Psychology (Statistical Section) No. 2, pp. 41–63, 1949
2. Harman, H.H., Modern Factor Analysis, The Univ. of Chicago Press, pp. 471, 1962
3. Holzinger, K.J. and Swine, F.F., "Bi-Factor Method", Psychometrika, No. 2, pp. 41–54, 1937
4. Matsuura, Y., Factorial Structure of Motor Ability, (in Japanese), Fumaido, pp. 358, 1969
5. Matsuura, Y., Factor Analysis for Behavioral Science, Fumaido, (in Japanese), pp. 5–1, 1972
6. Matsuura, Y., "On the Hierarchical Structure of Motor Ability; The Tree Diagram of Motor Ability", R.J. of Physical Education, 17: 5, pp. 297–307, 1973 (in Japanese)
7. Matsuura, Y., "Development of Motor Skill in Childhood; Ball Handling Skill", J.J. of Physical Education, 23: 2, pp. 129–140, 1978 (in Japanese)
8. Schmid, J. and Leiman, J.M., "The Development of Hierarchical Factor Solution", Psychometrika, 22-1, pp. 53–61, 1957.
9. Tucher, L.R., "The Role of Correlated Factors in Factor Analysis", Psychometrika, No. 5, pp. 141–152, 1940
10. Zavala, A., "Determining the Hierarchical Structure of Multidimensional Body of Information", Perceptual and Motor Skills, No. 32, pp. 732–746, 1976

多段因子分析法——階級的因子解又は変量の階級的分類のための新しい方法

運動能力の階級的因子解としては，すでにOut of dateとされているBi-factor, Subdivided factor, 及び厳密な数学的根拠をもつSchmid and Leimanの解，因子構造の質的面を強調し，項目間の関連性を強調するROTHIST解が工夫されて来た。また，一方，変量の分類という事を強調するクラスター分析の諸方法も工夫されてきた。それぞれ，考え方は異るが，また短所，長所をもっているが今日まで体力・運動能力の因子構造の検討や属性の分類等に利用されて来ている。運動能力等の能力の因子解にはSchmid and Leiman及びROTHISTの解が適当であるが，前者は斜交解適用の難しさが含まれ，後者は因子廻転の労力がぼう大である事の難しさがともなっている。ROTHISTの解法の特長を生かし，Schmid and Leimanの考え方も生かしつつ計算時間の短縮，斜交解の困難さの除去等の工夫によって，多段因子解法のアルゴリズムを開発した。この因子解の理論的根拠，アルゴリズムを示し，応用として幼児のボールハンドリング技能の階級的構造を検討し，この方法の適用性を検討した。

この方法の基本的考え方は因子分析によって得られたある因子に高い負荷量を示す変量は一つの変量群を構成すると考えられる（ROTHIST解，及び因子の解釈の考え方）。したがって，この変量群は，さらに単純な因子とその群の変量のみをとりあげれば共通因子として持っている筈である。したがって，前段において，一つの因子を含むと考えられる変量のみについて因子分析を行う事によって，得られた因子の複雑度は前段の因子より低い。このようにして，最後に一つの共通因子しか有しないという変量群にまで達することが出来る（Schmid and Leiman, Tuckerの考え）。この単一因子がもっとも単純な因子という事になり，因子の複雑度が減少してゆくという意味での因子の階級的構造が導かれる。この考えを，24項目のボール・ハンド・リング技能テスト項目から得られた（24×24）相関行列から出発して適用した。それには，主因子解，ノーマル・ベリマックス基準による直交廻転を利用して，幼児のボール・ハンドリング技能の階級的構造を検討した。