PHYSICAL REVIEW D VOLUME 49, NUMBER 7 1 APRIL 1994

Perturbative renormalization factors for bilinear and four-quark operators for Kogut-Susskind fermions on the lattice

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(Received 13 August 1993)

Renormalization factors for bilinear and four-quark operators with the Kogut-Susskind fermion action are perturbatively calculated to one-loop order in the general covariant gauge. Results are presented both for gauge-invariant and -noninvariant operators. For four-quark operators the full renormalization matrix for a complete set of operators with two types of color contraction structures is worked out and detailed numerical tables are given.

PACS number(s): 11.15.Ha, 11.10.Gh

I. INTRODUCTION

Calculation of weak matrix elements of hadrons is one of the central subjects in numerical simulation of lattice QCD. An important ingredient in such studies is the value of renormalization factors that relate operators on the lattice to those in the continuum. In this paper we present a one-loop evaluation of renormalization factors for bilinear and four-quark operators for the Kogut-Susskind (KS) fermion action. The results have been used in our recent work for the pion decay constant [1] and the K^0-$\bar{K}^0$ mixing matrix [2].

Perturbative calculation of renormalization factors for KS fermions has been developed in several previous studies [3–7]. In particular Daniel and Sheard [5,6] calculated the renormalization factors for KS bilinear operators [5] and a subset of four-quark operators [6] in the Feynman gauge. For applications in numerical simulations, however, their results need to be extended in several directions. (1) Weak operators for KS fermions are generally extended in space and time. The calculation of Daniel and Sheard has been carried out for the operators which are made gauge invariant through insertion of gauge link variables between quark and antiquark fields. In recent numerical simulations, however, an alternative method of evaluating matrix elements of gauge-noninvariant operators without link insertions on gauge-fixed configurations has been employed [8]. In fact, whether the two types of operators yield consistent results is the question we have recently addressed [1,2]. Analyzing this problem requires the renormalization factors for gauge noninvariant operators as well as for gauge invariant ones. We have therefore carried out the calculations for both types of operators. (2) Four-quark operators of form $\mathcal{O}_2=(\bar{q}_1^a q_2^a)(\bar{q}_3^b q_4^b)$ with a,b the color indices generally mix with those of form $\mathcal{O}_1=(\bar{q}_1^a q_2^b)(\bar{q}_3^b q_4^a)$ under renormalization. We evaluate the full renormalization factor for the two sets of operators, while the previous work of Sheard [6] listed explicit results only for $\mathcal{O}_2$ mixing with itself and with $\mathcal{O}_1$. (3) Renormalization factors for lattice operators generally take larger values than those for continuum operators due to contribution of gluon tadpoles. Lepage and Mackenzie [9] have argued that the tadpole contributions can be removed by a rescaling of quark and gluon fields. We work out the renormalization factors for the rescaled operators and examine to what extent their values are reduced by rescaling. (4) In addition to the extensions above we have carried out the calculation in the general covariant gauge which allows us to check the gauge parameter independence of the results for gauge invariant operators.

For bilinear quark operators a calculation similar to ours has been reported recently by Patel and Sharpe [10]. Our results are in agreement with theirs and also with those of Ref. [5] for gauge invariant operators. Patel and Sharpe have extended their calculation to four-quark operators [11]. For the gauge noninvariant operators which are relevant for the K meson B parameter their values fully agree with our results. We also find agreement with the results of Sheard [6] for gauge invariant four-quark operators when a comparison is possible.

We should mention that we do not treat penguin operators in this paper. Calculation of their renormalization factors is technically feasible, which should be pursued in future investigations.

This paper is organized as follows. In Sec. II KS operators whose renormalization factors we evaluate are defined and a general strategy for one-loop calculations is summarized following the method of Daniel and Sheard [5,6]. Results for quark bilinear operators are given in Sec. III. Those for four-quark operators are described in Sec. IV where the relation between lattice and continuum operators is illustrated for the case of $\Delta S=2$ operator

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relevant for extraction of the K^0-$\bar{K}^0$ mixing matrix. Analytical expressions for one-loop amplitudes are summarized in Appendixes A, B, and C.

II. FORMALISM

A. Quark operators

The KS fermion action is given by

$$S=a^4\sum_n\left[\frac{1}{2a}\sum_\mu\eta_\mu(n)[\bar{\chi}(n)U_\mu(n)\chi(n+\hat{\mu})-\bar{\chi}(n+\hat{\mu})U_\mu^\dagger(n)\chi(n)]+m\bar{\chi}(n)\chi(n)\right], \quad (1)$$

where a is the lattice spacing, $U_\mu(n)$ denotes the gauge link variable, $\eta_\mu(n)=(-1)^{n_1+\cdots+n_{\mu-1}}$, and χ and $\bar{\chi}$ are the single component KS fermion fields. For the construction of four-component Dirac fields we employ the coordinate-space method of taking a linear combination of the χ fields over a hypercube [12,13]. The Dirac field $Q(2N)$ defined for each hypercube $2N\in(2Z)^4$ is given by

$$Q(2N)_{\alpha i}=\frac{1}{8}\sum_A(\gamma_A)_{\alpha i}\chi(2N+A), \quad (2)$$

where α and i are the Dirac and flavor indices, and

$$\gamma_A=\gamma_1^{A_1}\gamma_2^{A_2}\gamma_3^{A_3}\gamma_4^{A_4} \quad (3)$$

with A running over the vertices of a hypercube (i.e., $A_\mu=0$ or $1, \mu=1,\ldots,4$).

The bilinear quark operator we use is defined by

$$\mathcal{O}_{SF}=\bar{Q}(\gamma_S\otimes\xi_F)Q. \quad (4)$$

Here $\gamma_S=\gamma_1^{S_1}\gamma_2^{S_2}\gamma_3^{S_3}\gamma_4^{S_4}$ and $\xi_F=\gamma_1^{*F_1}\gamma_2^{*F_2}\gamma_3^{*F_3}\gamma_4^{*F_4}$ with the components S_μ and F_μ either 0 or 1. They act on spinor and flavor indices of the Dirac field Q, and represent the spin and flavor SU(4) content of the bilinear operator. In terms of the χ field this operator can be written as

$$\mathcal{O}_{SF}=\frac{1}{16}\sum_{AB}\bar{\chi}_A\overline{(\gamma_S\otimes\xi_F)}_{AB}\chi_B, \quad (5)$$

where we write χ_A instead of $\chi(2N+A)$ for simplicity and

$$\overline{(\gamma_S\otimes\xi_F)}_{AB}=\tfrac{1}{4}\mathrm{tr}(\gamma_A^\dagger\gamma_S\gamma_B\gamma_F^\dagger). \quad (6)$$

The operator (5) is not gauge invariant. In order to make it gauge invariant we insert the average of products of gauge link variables along all possible shortest paths connecting sites $2N+A$ and $2N+B$. Denoting the link factor by U_{AB} we then define the gauge invariant bilinear operator by

$$\mathcal{O}_{SF}=\frac{1}{16}\sum_{AB}\bar{\chi}_A^a\overline{(\gamma_S\otimes\xi_F)}_{AB}U_{AB}^{ab}\chi_B^b, \quad (7)$$

with a and b the color indices.

We consider two types of four-quark operators differing in the color contraction structure defined by

$$\mathcal{O}_2=\bar{Q}^a(\gamma_{S_1}\otimes\xi_{F_1})Q^a\bar{Q}^b(\gamma_{S_2}\otimes\xi_{F_2})Q^b, \quad (8)$$

$$\mathcal{O}_1=\bar{Q}^a(\gamma_{S_1}\otimes\xi_{F_1})Q^b\bar{Q}^b(\gamma_{S_2}\otimes\xi_{F_2})Q^a, \quad (9)$$

with a and b the color indices. In the method of Ref. [14] for calculating weak matrix elements with KS fermions the operator $\mathcal{O}_2$ yields an amplitude with two color contractions after fermion integration and $\mathcal{O}_1$ with a single color contraction. For this reason we shall refer to $\mathcal{O}_2$ and $\mathcal{O}_1$ as color two-loop and one-loop operators. These two operators generally mix under renormalization.

The expression of the operators above in terms of the χ field is given by

$$\mathcal{O}_2=\left[\frac{1}{16}\right]^2\sum_{ABCD}\bar{\chi}_A^a\overline{(\gamma_{S_1}\otimes\xi_{F_1})}_{AB}\chi_B^a\bar{\chi}_C^b\overline{(\gamma_{S_2}\otimes\xi_{F_2})}_{CD}\chi_D^b, \quad (10)$$

$$\mathcal{O}_1=\left[\frac{1}{16}\right]^2\sum_{ABCD}\bar{\chi}_A^a\overline{(\gamma_{S_1}\otimes\xi_{F_1})}_{AB}\chi_B^b\bar{\chi}_C^b\overline{(\gamma_{S_2}\otimes\xi_{F_2})}_{CD}\chi_D^a, \quad (11)$$

where we again ignored the hypercube label $2N$ in $\chi(2N+A)$ for simplicity. To make these operators gauge invariant we insert gauge link factors according to

$$\mathcal{O}_2=\left[\frac{1}{16}\right]^2\sum_{ABCD}\bar{\chi}_A^a\overline{(\gamma_{S_1}\otimes\xi_{F_1})}_{AB}\chi_B^b\bar{\chi}_C^c\overline{(\gamma_{S_2}\otimes\chi_{F_2})}_{CD}\chi_D^d U_{AB}^{ab}U_{CD}^{cd}, \quad (12)$$

$$\mathcal{O}_1=\left[\frac{1}{16}\right]^2\sum_{ABCD}\bar{\chi}_A^a\overline{(\gamma_{S_1}\otimes\xi_{F_1})}_{AB}\chi_B^b\bar{\chi}_C^c\overline{(\gamma_{S_2}\otimes\xi_{F_2})}_{CD}\chi_D^d U_{AD}^{ad}U_{CB}^{cb}. \quad (13)$$

For the color two-loop operators the length of paths for the link factors U_{AB} and U_{CD} is fixed since $\overline{(\gamma_S\otimes\xi_F)}_{AB}$ is nonvanishing only for $A+B=S+F(\mathrm{mod}2)$. On the other hand, the length varies for the factors U_{AD} and U_{CB} that appear in the color one-loop operator.

B. Feynman rules

We adopt the general covariant gauge with a gauge parameter α in our perturbative calculations. The gluon propagator is given by

$$D_{\mu\nu}(k)_{IJ}=\frac{\delta_{IJ}\delta_{\mu\nu}}{\sum_\beta \frac{4}{a^2}\sin^2(ak_\beta/2)}-(1-\alpha)\frac{\delta_{IJ}\frac{4}{a^2}\sin(ak_\mu/2)\sin(ak_\nu/2)}{\left[\sum_\beta \frac{4}{a^2}\sin^2(ak_\beta/2)\right]^2}, \tag{14}$$

and the KS fermion propagator takes the form

$$S(p,-q)_{ab}=\delta_{ab}\frac{\sum_\mu \frac{-i}{a}\sin ap_\mu \bar\delta(p-q+\pi\bar\mu/a)+m\bar\delta(p-q)}{\sum_\mu \frac{1}{a^2}\sin^2 ap_\mu+m^2}, \tag{15}$$

where

$$\bar\mu=\sum_{\nu=1}^{\mu-1}\hat\nu \tag{16}$$

and

$$\bar\delta(p)=(2\pi)^4\sum_n \delta\left[p+\frac{2\pi}{a}n\right]. \tag{17}$$

The one-gluon vertex arising from the action is given by

$$V_\mu(p,-q;k)_{ab}=-ig(T^I)_{ab}\cos a(p+k/2)_\mu\bar\delta(p-q+k+\pi\bar\mu/a), \tag{18}$$

with T^I the SU(3) generators normalized by $\mathrm{Tr}(T^IT^J)=\delta_{IJ}/2$, and the two-gluon vertex by

$$V_{\mu\nu}(p,-q;k_1,k_2)_{ab}=iag^2\tfrac{1}{2}\{T^I,T^J\}_{ab}\sin a\left[p+\frac{k_1+k_2}{2}\right]_\mu\delta_{\mu\nu}\bar\delta(p-q+k_1+k_2+\pi\bar\mu/a), \tag{19}$$

where p,q are the incoming fermion momenta and k,k_1,k_2 the gluon momenta.

Vertices of bilinear operators (7) have the form

$$M_{SF}^{(0)}(p,-q)_{ab}=\delta_{ab}\frac{1}{16}\sum_{AB}e^{iap\cdot A-iaq\cdot B}(\overline{\gamma_S\otimes\xi_F})_{AB}, \tag{20}$$

$$M_{SF}^{(1)}(p,-q;k)_{ab}=-iga(T^I)_{ab}\frac{1}{16}\sum_{AB}e^{iap\cdot A-iaq\cdot B}(\overline{\gamma_S\otimes\xi_F})_{AB}(A-B)_\mu f^\mu_{(AB)}(ak), \tag{21}$$

$$M_{SF}^{(2)}(p,-q;k_1,k_2)_{ab}=\tfrac{1}{2}(iga)^2\tfrac{1}{2}\{T^I,T^J\}_{ab}\frac{1}{16}\sum_{AB}e^{iap\cdot A-iaq\cdot B}(\overline{\gamma_S\otimes\xi_F})_{AB}(A-B)_\mu(A-B)_\nu g^{\mu\nu}_{(AB)}(ak_1,ak_2), \tag{22}$$

where the superscript in parentheses denotes the number of emitted gluons. The function $f^\mu_{(AB)}(ak)$ is defined by

$$f^\mu_{(AB)}(ak)=e^{iaA\cdot k}\frac{1}{12}\sum_{\nu\neq\mu}\sum_{j=1}^{4}e^{i(B-A)\cdot\theta^{(j)}_{\mu\nu}(ak)} \tag{23}$$

with

$$\begin{aligned}
\theta^{(1)}_{\mu\nu}(ak)&=\tfrac{1}{2}ak_\mu\hat\mu,\\
\theta^{(2)}_{\mu\nu}(ak)&=\tfrac{1}{2}ak_\mu\hat\mu+ak_\nu\hat\nu,\\
\theta^{(3)}_{\mu\nu}(ak)&=\sum_{\rho=1}^{4}ak_\rho\hat\rho-\theta^{(1)}_{\mu\nu}(ak),\\
\theta^{(4)}_{\mu\nu}(ak)&=\sum_{\rho=1}^{4}ak_\rho\hat\rho-\theta^{(2)}_{\mu\nu}(ak).
\end{aligned} \tag{24}$$

At the one-loop level the two-gluon vertex appears only through gluon tadpole diagrams. Thus we only need the expression for an equal color index $a=b$ and for the gluon momenta $k_1=-k_2\equiv k$. In this case we find

$$\begin{aligned}
g^{\mu\nu}_{(AB)}(ak,-ak)&=1 \quad \text{for } \mu=\nu,\\
&=e^{iak\cdot(\Delta_\mu+\Delta_\nu)}\left[6+2\sum_{\rho\neq\mu\nu}e^{iak\cdot\Delta_\rho}+2\exp\left[iak\cdot\sum_{\rho\neq\mu\nu}\Delta_\rho\right]\right]+\text{H.c.} \quad \text{for } \mu\neq\nu
\end{aligned} \tag{25}$$

with $\Delta_\mu=(B-A)_\mu\hat{\mu}$.

Vertices for four-quark operators are given by a product of those for the bilinear operators except that the color and site indices have to be interchanged appropriately.

C. Procedure of calculation

Let us consider a one-loop diagram of a bilinear operator with two external fermion lines. The corresponding amplitude generated by the Feynman rules above are written in terms of momenta p taking values in the range $-\pi/a<p\leq\pi/a$. Since the Dirac field $Q(2N)$ is defined on sites with even coordinates the physical momentum $\tilde{p}$ for quarks is related to p through $p=\tilde{p}+C\pi/a$ where the vector C ($C_\mu=0$ or 1) represents the spin-flavor content of quarks. We extract renormalization factors from Feynman amplitudes evaluated at vanishing physical momenta $\tilde{p}=0$ for external fermion lines. We therefore set the external fermion momenta to $p=C\pi/a$ and $D\pi/a$. In this case the spin-flavor part of the tree amplitude $M_{SF}^{(0)}$ takes the form

$$(\overline{\overline{\gamma_S\otimes\xi_F}})_{CD}=\frac{1}{16}\sum_{AB}(-1)^{A\cdot C+B\cdot D}(\overline{\gamma_S\otimes\xi_F})_{AB}\ , \tag{26}$$

which shows that the calculation of renormalization factors requires a conversion of spin-flavor Dirac structure from the "single-bar" basis, in which the Feynman rules are given, to the "double-bar" basis defined in (26).

We employ the technique developed by Daniel and Sheard [5,6] to carry out the conversion. The general form of one-loop amplitudes that results is given by

$$\sum_{MNM'N'}\int_{-\pi/a}^{\pi/a}\frac{d^4k\,a^4}{(2\pi)^4}A_{MNM'N'}(ak)(\overline{\overline{\gamma_{MSN}\otimes\xi_{M'FN'}}})_{CD}\ , \tag{27}$$

where $A_{MNM'N'}$ is a function of loop momenta k. The product of Dirac matrices $(\overline{\overline{\gamma_{MSN}\otimes\xi_{M'FN'}}})$ can be reexpanded in terms of the basis $\{(\gamma_S\otimes\xi_F);S_\mu,F_\mu=0,1\}$:

$$(\overline{\overline{\gamma_{MSN}\otimes\xi_{M'FN'}}})=\sum_{S'F'}C_{MNM'N'}^{SFS'F'}(\overline{\overline{\gamma_{S'}\otimes\xi_{F'}}})\ . \tag{28}$$

The contribution of (27) to the renormalization factor of $\mathcal{O}_{SF}$ is given by

$$\sum_{MNM'N'}C_{MNM'N'}^{SFS'F'}\int_{-\pi/a}^{\pi/a}\frac{d^4k\,a^4}{(2\pi)^4}A_{MNM'N'}(ak)\ . \tag{29}$$

In most cases the decomposition (28) is too tedious to work out analytically. We generate tables of $C_{MNM'N'}^{SFS'F'}$ on a computer and combined them with tables of one-loop integrals $A_{MNM'N'}$, separately evaluated with the Monte Carlo integration routine VEGAS, to calculate the sum (29).

Our calculations are carried out for massless quarks. In this case the amplitudes for the diagrams which have counterparts in the continuum perturbation theory contain infrared divergent terms of the form

$$\int_{-\pi/a}^{\pi/a}\frac{d^4k\,a^4}{(2\pi)^4}\frac{1}{[\sum_\mu 4\sin^2(k_\mu a/2)][\sum_\nu 4\sin^2(k_\nu a/2)]}\ , \tag{30}$$

where the first factor in the denominator arises from the gluon propagator and the second from the massless quark propagator. To regularize the divergence we supply a finite mass κ to the gluon propagator. The integral then takes the value

$$\int_{-\pi/a}^{\pi/a}\frac{d^4k\,a^4}{(2\pi)^4}\frac{1}{[\sum_\mu 4\sin^2(k_\mu a/2)][\sum_\nu 4\sin^2(k_\nu a/2)+(a\kappa)^2]}=\frac{1}{16\pi^2}[-2\ln(a\kappa)+F_{0000}-\gamma_E+1]+O(\kappa a)\ , \tag{31}$$

with $F_{0000}=4.369\,23(1)$ and $\gamma_E=0.577\,216\ldots$.

The infrared regularization above is different from that of Daniel and Sheard who added the mass term $(a\kappa)^2$ to both the quark and gluon propagators, in which case the finite part of the integral is given by $F_{0000}-\gamma_E$. We prefer not to adopt their regulator since it leads to a violation of fermion number conservation in continuum perturbation theory. We also note that the dependence on the gluon mass should cancel out between the renormalization factors in the continuum and on the lattice as long as one employs the same infrared regularization in the two cases.

Evaluation of one-loop amplitudes for four-quark operators is much more cumbersome since they contain a product of two spin-flavor Dirac matrices. The calculational procedure, however, is essentially the same as for bilinear operators.

III. BILINEAR OPERATORS

The one-loop renormalization of the quark bilinear operators $\mathcal{O}_{SF}$ on the lattice can be written as

$$\mathcal{O}_{SF}^{\text{lat}(1)}=\sum_{S'F'}\left[\delta_{SS'}\delta_{FF'}+\frac{g^2}{16\pi^2}z_{SF,S'F'}^{\text{lat}}\right]\mathcal{O}_{S'F'}^{\text{lat}(0)}\ , \tag{32}$$

where the superscript j on $\mathcal{O}_{SF}^{\text{lat}(j)}$ refers to the number of loops. The one-loop diagrams are shown in Fig. 1 and analytic expressions of the amplitudes are collected in Appendix A. Defining the coefficient z^{cont} for the continuum operators in a similar manner the one-loop relation between the lattice and continuum operators is given by

$$\mathcal{O}_{SF}^{\mathrm{cont}(1)} = \sum_{S'F'} \left[\delta_{SS'}\delta_{FF'} + \frac{g^2}{16\pi^2}\left(z_{SF,S'F'}^{\mathrm{cont}} - z_{SF,S'F'}^{\mathrm{lat}}\right) \right] \mathcal{O}_{S'F'}^{\mathrm{lat}(1)} . \tag{33}$$

The continuum renormalization factor for massless quarks can be expressed as

$$z_{SF,S'F'}^{\mathrm{cont}} = \delta_{SS'}\delta_{FF'}\gamma_S \ln(\mu/\kappa) + \delta_{SS'}\delta_{FF'} C_S^{\mathrm{cont}} . \tag{34}$$

We used the same infrared regularization as for the lattice, i.e., a finite gluon mass κ is given to the gluon propagator. The anomalous dimension of the operator γ_S is given by

$$\gamma_S = \tfrac{8}{3}(\sigma_S - 1) , \tag{35}$$

with $\sigma_S = (4,1,0,1,4)$ for the spin structure $\gamma_S = (I, \gamma_\mu, \gamma_{\mu\nu}, \gamma_{\mu 5}, \gamma_5)$. The finite constant C_S^{cont} depends on the continuum regularization and renormalization schemes. For the modified minimal subtraction ($\overline{\mathrm{MS}}$) scheme it takes the values

$$C_S^{\mathrm{cont}} = \begin{cases} (10/3, \;\; 0, \;\; 2/3, \;\; 0, \;\; 10/3) & \text{for NDR,} \\ (14/3, \;\; 0, \;\; -2/3, \;\; 0, \;\; 14/3) & \text{for } \mathrm{DR}\overline{\mathrm{EZ}} \end{cases} \tag{36}$$

for $\gamma_S = (I, \gamma_\mu, \gamma_{\mu\nu}, \gamma_{\mu 5}, \gamma_5)$, where NDR refers to the naive dimensional regularization with an anticommuting γ_5 and $\mathrm{DR}\overline{\mathrm{EZ}}$ to the dimensional reduction with an easy subtraction scheme defined in Ref. [15].

For the renormalization factor on the lattice we find

$$z_{SF,S'F'}^{\mathrm{lat}} = -\delta_{SS'}\delta_{FF'}\gamma_S \ln(a\kappa) + C_{SF,S'F'}^{\mathrm{lat}} . \tag{37}$$

The logarithmically divergent term arises from the diagrams in Figs. 1(a) and 1(d). It takes the same form for gauge invariant and noninvariant operators, and is independent of the gauge parameter α. Comparing (34) and (37) we find that the gluon mass κ cancels out between $z_{SF,S'F'}^{\mathrm{cont}}$ and $z_{SF,S'F'}^{\mathrm{lat}}$ as it should be.

The finite coefficient $C_{SF,S'F'}^{\mathrm{lat}}$ has the following properties. (1) The coefficients have the same value for the two operators with a spin-flavor structure $(\gamma_S \otimes \xi_F)$ and $(\gamma_{S5} \otimes \xi_{F5})$ (see Ref. [10]). (2) Our explicit calculation shows that the Landau gauge part of $C_{SF,S'F'}^{\mathrm{lat}}$ coming from the $(\sin k^\mu/2 \sin k^\nu/2)$ term of the gluon propagator is diagonal in spin and flavor. Their values are the same for the gauge invariant and noninvariant operators. (3) The remaining part of the coefficient generally mixes different spin-flavor structures. The chiral U(1) symmetry of the KS action, however, places a restriction that operators of even distance with $\sum_{\mu=1}^4 (S_\mu + F_\mu)(\mathrm{mod}\,2) = 0,2,4$ do not mix with those having odd distance $(\sum_{\mu=1}^4 (S_\mu + F_\mu)(\mathrm{mod}\,2) = 1,3)$. In fact there are only a few nonvanishing off-diagonal elements [see Table I(b) below]. Furthermore their values are the same for

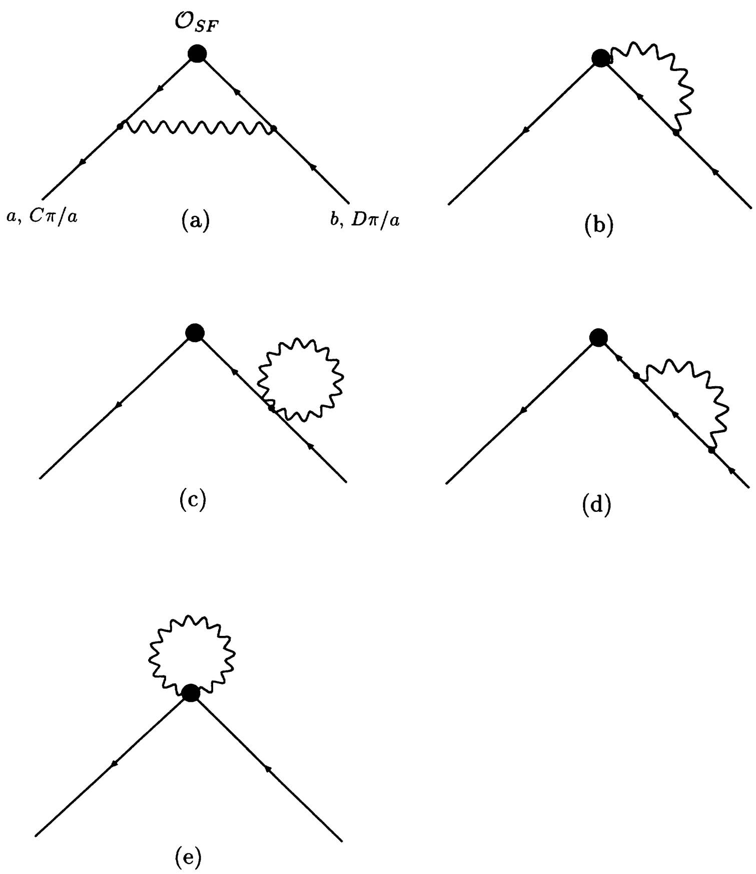

FIG. 1. One-loop diagrams for bilinear operators. Diagrams (b) and (e) are absent for gauge noninvariant operators.

gauge invariant and noninvariant operators.

Numerical values of the diagonal constants $C^{\rm lat}_{SF,SF}$ are tabulated in Table I(a) for both gauge invariant (first column) and noninvariant (second column) operators. The results for the gauge noninvariant operators are given in the Landau gauge. The nonvanishing off-diagonal elements are tabulated in Table I(b). Numerical accuracy is within 0.1%. For the gauge invariant operators we have numerically checked the independence of results on the gauge parameter α. Our results confirm those of Daniel and Sheard (Tables 4 and 5 in Ref. [5]) after correcting our $C^{\rm lat}_{SF,S'F'}$ by $-\delta_{SS'}\delta_{FF'}\gamma_S/2$ to take into account the difference in the regularization of infrared divergence. They are in complete agreement with the recent calculation of the same quantity reported by Patel and Sharpe (Tables 6 and 7 in Ref. [10]). The relationship between our coefficients $C^{\rm lat}_{SF,S'F'}$ and their $C^{\rm PS}_{SF,S'F'}$ is

$$C^{\rm PS}_{SF,S'F'}=\tfrac{3}{4}[\delta_{SS'}\delta_{FF'}(C^{\rm cont}_S+\gamma_S\ln\pi)-C^{\rm lat}_{SF,S'F'}] , \quad (38)$$

where $C^{\rm cont}_S$ are the finite continuum renormalization factor for the $\overline{\rm DREZ}$ scheme given by (36).

We observe in Table I(a) that the coefficients for gauge noninvariant operators have a similar magnitude while those for gauge invariant ones show a substantial variation from operator to operator. This stems from the fact that the gauge invariant operators receive a contribution of gluon tadpoles whose magnitude increases with the distance of the operator $\Delta_{SF}=\sum_\mu(S_\mu+F_\mu)({\rm mod}2)$. Lepage and Mackenzie [9] have argued that the tadpole contributions can be removed by a rescaling of fields which, for KS fermion action, takes the form

$$\chi\to\sqrt{u_0}\chi\ ,\quad \bar\chi\to\sqrt{u_0}\bar\chi\ ,\quad U_\mu\to u_0^{-1}U_\mu\ , \quad (39)$$

where u_0 represents the tadpole renormalization of link variables. A gauge-invariant choice for u_0 is given by

$$u_0=[\tfrac{1}{3}\langle {\rm Tr}U_P\rangle]^{1/4}=1-\tfrac{1}{12}g^2+O(g^4) \quad (40)$$

with $\langle {\rm Tr}U_P\rangle$ the plaquette average. For gauge-invariant bilinear operators the rescaling amounts to a multiplication by a factor $u_0^{1-\Delta_{SF}}$. The renormalization factors for the rescaled operators are obtained by subtracting $4\pi^2(1-\Delta_{SF})/3$ from the second column of Table I(a). The results listed in the third column of Table I(a) show that rescaled gauge invariant operators receive much less renormalization, and that their magnitude becomes less dependent on the flavor of operators. For gauge noninvariant operators without insertion of link variables the

TABLE I. One-loop perturbative corrections for bilinear operators. Operators of form $(\gamma_S\otimes\xi_F)$ and $(\gamma_{S5}\otimes\xi_{F5})$ receive the same one-loop corrections. The components μ, ν, ρ, and σ are all different and not summed. Values of anomalous dimension γ_S are also listed. (a) Diagonal elements of the renormalization factor for gauge invariant (first column) and noninvariant (second column) operators. The third and fourth columns show the values for rescaled operators as discussed in the text. (b) Off-diagonal elements of the correction which take the same values for gauge invariant and noninvariant operators. The first and second columns of operators specify the row and column of the mixing matrix.

(a)

	Operator	γ	Invariant	Noninvariant	Inv (rescaled)	Noninv (rescaled)
1	$(I\otimes I)$	8	55.585	55.585	42.426	42.426
2	$(I\otimes\xi_5)$	8	−47.783	12.813	−8.304	−0.346
3	$(I\otimes\xi_\mu)$	8	14.844	27.077	14.844	13.918
4	$(I\otimes\xi_{\mu5})$	8	−29.948	14.405	−3.629	1.246
5	$(I\otimes\xi_{\mu\nu})$	8	−10.569	17.583	2.589	4.423
6	$(\gamma_\mu\otimes I)$	0	0.000	12.232	0.000	−0.927
7	$(\gamma_\mu\otimes\xi_5)$	0	−30.000	14.353	−3.682	1.193
8	$(\gamma_\mu\otimes\xi_\mu)$	0	19.693	19.693	6.533	6.533
9	$(\gamma_\mu\otimes\xi_\nu)$	0	−13.388	14.764	−0.228	1.605
10	$\gamma_\mu\otimes\xi_{\nu5})$	0	−13.409	14.743	−0.249	1.584
11	$(\gamma_\mu\otimes\xi_{\mu5})$	0	−45.988	14.608	−0.651	1.448
12	$(\gamma_\mu\otimes\xi_{\mu\nu})$	0	4.519	16.752	4.519	3.593
13	$(\gamma_\mu\otimes\xi_{\nu\lambda})$	0	−29.651	14.702	−3.332	1.543
14	$(\gamma_{\mu\nu}\otimes I)$	−8/3	−14.623	13.529	−1.464	0.369
15	$(\gamma_{\mu\nu}\otimes\xi_\mu)$	−8/3	−0.428	11.804	−0.428	−1.355
16	$(\gamma_{\mu\nu}\otimes\xi_\lambda)$	−8/3	−29.668	14.685	−3.349	1.525
17	$(\gamma_{\mu\nu}\otimes\xi_{\mu\nu})$	−8/3	7.728	7.728	−5.430	−5.430
18	$(\gamma_{\mu\nu}\otimes\xi_{\mu\lambda})$	−8/3	−14.200	13.952	−1.041	0.792
19	$(\gamma_{\mu\nu}\otimes\xi_{\lambda\sigma})$	−8/3	−45.390	15.206	−5.911	2.046

(b)

	Operator	Mixed operator	
9	$(\gamma_\mu\otimes\xi_\nu)$	$(\gamma_\mu\otimes\xi_\mu)$	−4.504
11	$(\gamma_\mu\otimes\xi_{\mu5})$	$(\gamma_\mu\otimes\xi_{\nu5})$	0.860
13	$(\gamma_\mu\otimes\xi_{\nu\lambda})$	$(\gamma_\mu\otimes\xi_{\mu\nu}),(\gamma_\mu\otimes\xi_{\lambda\mu})$	1.980
16	$(\gamma_{\mu\nu}\otimes\xi_\lambda)$	$(\gamma_{\mu\nu}\otimes\xi_\mu),(\gamma_{\mu\nu}\otimes\xi_\nu)$	0.902

rescaling factor is universally given by u_0. The rescaling reduces the magnitude of the correction without spoiling the weak flavor dependence already apparent for the original operators.

IV. FOUR-QUARK OPERATORS

A. Lattice result

One-loop diagrams which contribute to the renormalization of the color two-loop four-quark operators $\mathcal{O}_2$ defined in Sec. II are shown in Fig. 2 and those for color one-loop operators $\mathcal{O}_1$ in Fig. 3. In these figures horizontal lines at the four-quark vertices signify contraction of spin-flavor quantum numbers, while dotted lines represent link factors and flow of color indices.

For the diagrams of Figs. 2(a)–2(e) for the color two-loop operator evaluation of momentum and Dirac matrix parts are the same as those of the diagrams in Fig. 1 for the bilinear operator. The color factor is also the same for these diagrams. For the diagrams of Figs. 2(f)–2(h), on the other hand, the color factor takes the form $\sum_I (T^I)_{ab}(T^I)_{a'b'}$ which has to be decomposed into the color one- and two-loop basis. This can be done by the SU(3) identity

$$\sum_I (T^I)_{ab}(T^I)_{a'b'} = -\tfrac{1}{6}\delta_{ab}\delta_{a'b'} + \tfrac{1}{2}\delta_{ab'}\delta_{a'b} \ . \qquad (41)$$

Such a rearrangement is also generally needed for the diagrams of color one-loop operators in Fig. 3 in addition to manipulation of momentum and Dirac parts. Analytic expressions for all the diagrams are summarized in Appendixes B and C.

Because of the mixing of color one- and two-loop operators the renormalization factor for the four-quark operators $\mathcal{O}_i$ takes a 2×2 matrix form

$$\mathcal{O}_i^{\mathrm{lat}(1)} = \left[\delta_{ij} + \frac{g^2}{16\pi^2} z_{ij}^{\mathrm{lat}}\right] \mathcal{O}_j^{\mathrm{lat}(0)} \ , \quad i,j=1,2 \ . \qquad (42)$$

Since the operators $\mathcal{O}_i = \bar{Q}(\gamma_{S_1}\otimes\xi_{F_1})Q\bar{Q}(\gamma_{S_2}\otimes\xi_{F_2})Q$ further depend on the pair of spin-flavor indices $sf \equiv (S_1F_1)(S_2F_2)$, each element z_{ij}^{lat} is a matrix $z_{ij}^{\mathrm{lat}} = \{z_{ij;sf,s'f'}\}$. Treating the infrared divergences as in the case of bilinear operators we find that this matrix can be written as

$$z_{ij;sf,s'f'}^{\mathrm{lat}} = -\delta_{ff'}\gamma_{ij;ss'}^{\mathrm{lat}} \ln(a\kappa) + C_{ij;sf,s'f'}^{\mathrm{lat}} \ . \qquad (43)$$

In Tables II–IX we list the numerical values of the matrices $\gamma_{ij;ss'}^{\mathrm{lat}}$ and $C_{ij;sf,s'f'}^{\mathrm{lat}}$ for the operators with the spin-flavor structure

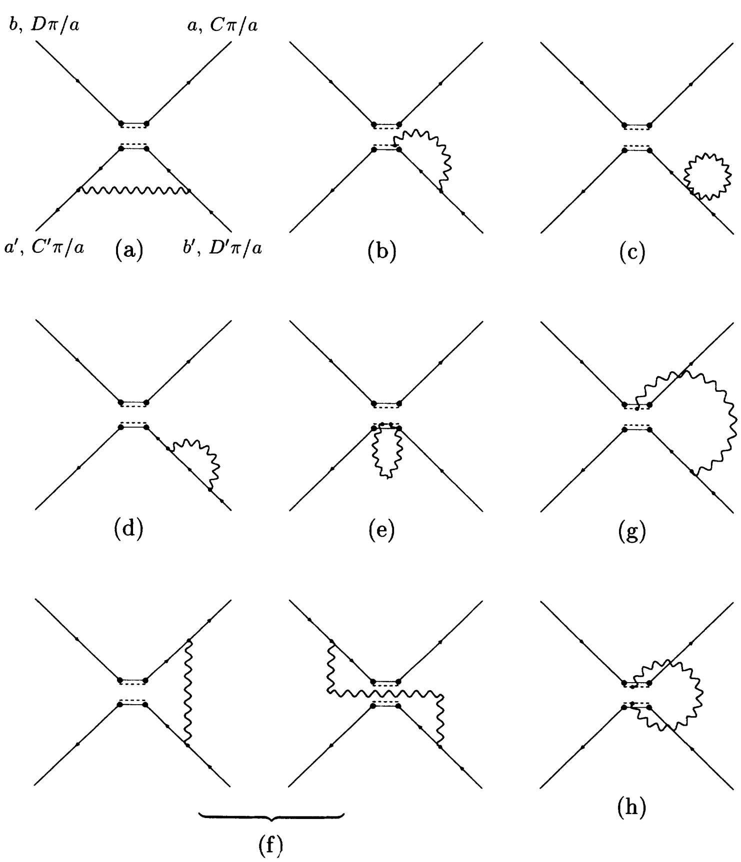

FIG. 2. One-loop diagrams for color two-loop four-quark operators $\bar{Q}^a(\gamma_S\otimes\xi_F)Q^a\bar{Q}^b(\gamma_{S'}\otimes\xi_{F'})Q^b$. Thick horizontal bars at the four-quark vertices signify contraction of spin-flavor quantum numbers, while dotted lines represent link factors and flow of color indices.

$$sf=\begin{cases}(\gamma_\mu\otimes\xi_5)(\gamma_\mu\otimes\xi_5)\ ,\ (\gamma_{\mu5}\otimes\xi_5)(\gamma_{\mu5}\otimes\xi_5)\ \text{(Tables II and VI)}\ ,\\ (\gamma_\mu\otimes\xi_5)(\gamma_{\mu5}\otimes\xi_s)\ \text{(Tables III and VII)}\ ,\\ (\gamma_\mu\otimes I)(\gamma_\mu\otimes\xi_5)\ \text{(Tables IV and VIII)}\ ,\\ (\gamma_\mu\otimes I)(\gamma_{\mu5}\otimes\xi_5),(\gamma_{\mu5}\otimes I)(\gamma_\mu\otimes\xi_5)\ \text{(Tables V and IX)}\ .\end{cases} \tag{44}$$

The results for the gauge invariant operators are in Tables II–V and those for the gauge noninvariant operators are in Tables VI–IX. The anomalous dimension matrix $\gamma^{\text{lat}}_{ij;ss'}$ is gauge independent and takes the same value for gauge invariant and noninvariant operators. The results of $C^{\text{lat}}_{ij;sf,s'f'}$ for the gauge noninvariant operators are for the Landau gauge.

The one-loop renormalization coefficients for the operators having the spin-flavor structure $(\gamma_{S5}\otimes\xi_{F5})(\gamma_{S'5}\otimes\xi_{F'5})$ are the same as those for $(\gamma_S\otimes\xi_F)(\gamma_{S'}\otimes\xi_{F'})$. Hence the tables also cover the renormalization factor for the operators obtained by the interchange $\gamma_\mu\leftrightarrow\gamma_{\mu5}$, $I\leftrightarrow\xi_5$. These operators are the most relevant for calculation of matrix elements of the effective weak Hamiltonian. The gauge noninvariant operators with the spin-flavor structure $(\gamma_{S5}\otimes\xi_{F5})(\gamma_{S'}\otimes\xi_{F'})$ are renormalized in the same way as those for $(\gamma_S\otimes\xi_F)(\gamma_{S'}\otimes\xi_{F'})$ at least to one-loop order, and similarly for the operators $(\gamma_S\otimes\xi_F)(\gamma_{S'5}\otimes\xi_{F'5})$ and $(\gamma_S\otimes\xi_F)(\gamma_{S'}\otimes\xi_{F'})$ [16]. It is not known if this property persists at higher orders. We also note that the operators of even distance do not mix with those of odd distance due to U(1) chiral symmetry, similar to the case of bilinear operators.

The numerical accuracy is within the level of ±0.001 for the majority of elements in the tables, increasing to ±0.01 for large elements whose magnitude is ~10. This accuracy should be sufficient for practical applications. Reducing errors is quite computer time consuming because of a very large number of lattice integrals (~360) which have to be computed.

We have checked the results in two ways. (i) For gauge invariant operators the Landau gauge part proportional to $1-\alpha$ has to vanish. This has been confirmed numerically. (ii) One can rewrite the color one-loop operators as a linear combination of color two-loop operators through the Fierz transformation given by

$$\overline{(\gamma_S\otimes\xi_F)}_{AB}\overline{(\gamma_{S'}\otimes\xi_{F'})}_{A'B'} =\frac{1}{16}\sum_{DE}\overline{(\gamma_S\gamma_D^\dagger\otimes\xi_E^\dagger\xi_{F'})}_{AB'}\overline{(\gamma_{S'}\gamma_D\otimes\xi_E\xi_F)}_{A'B}\ . \tag{45}$$

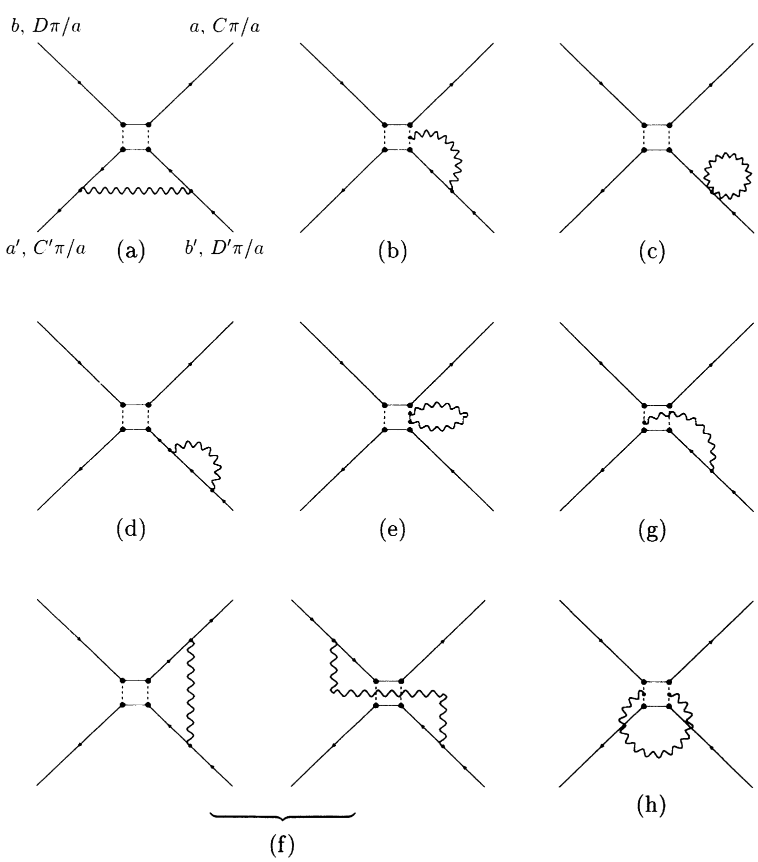

FIG. 3. One-loop diagrams for color one-loop four-quark operators $\overline{Q}^a(\gamma_S\otimes\xi_F)Q^b\overline{Q}^b(\gamma_{S'}\otimes\xi_{F'})Q^a$. The meaning of the symbols is the same as in Fig. 2.

TABLE II. Anomalous dimensions and finite corrections for gauge invariant four-quark operators with the spin-flavor structure $VV=(\gamma_\mu\otimes\xi_5)(\gamma_\mu\otimes\xi_5)$ and $AA=(\gamma_{\mu5}\otimes\xi_5)(\gamma_{\mu5}\otimes\xi_5)$. Subscripts 1 or 2 attached to the operators refer to the number of color loops. Operators listed at the top row mix with those on the first column with the numerical coefficients given (a common factor $g^2/16\pi^2$ is removed). The values after the slash are the finite corrections for the rescaled operators. The two rows of numerical values for each operator in the first column are for the color one-loop operator (first row) and for the color two-loop operator (second column). All indices of operators are summed with different indices not taking equal values.

	Mixed operator	γ	VV_1 finite	γ	VV_2 finite	γ	AA_1 finite	γ	AA_2 finite
1	$(I\otimes\xi_\mu)(I\otimes\xi_\mu)$		4.725				−19.114/−5.954		−5.573
			−1.575				−2.412		1.857
2	$(I\otimes\xi_\mu)(I\otimes\xi_\nu)$		−0.624				−0.485		1.112
			0.208				−0.208		−0.370
3	$(I\otimes\xi_{\sigma5})(I\otimes\xi_{\rho5})$		−0.052		−0.235		−0.067		
			−0.022		0.078		0.022		
4	$(I\otimes\xi_{\sigma5})(I\otimes\xi_{\sigma5})$		−0.876		−1.310		0.565		
			−1.029		0.436		−0.188		
5	$(\gamma_5\otimes\xi_\mu)(\gamma_5\otimes\xi_\mu)$		20.774/7.615				−3.675		−3.150
			−2.569				−1.575		1.050
6	$(\gamma_5\otimes\xi_\mu)(\gamma_5\otimes\xi_\nu)$		0.319				0.485		0.416
			0.706				0.208		−0.138
7	$(\gamma_5\otimes\xi_{\sigma5})(\gamma_5\otimes\xi_{\rho5})$		0.052		0.044		0.130		
			0.022		−0.014		−0.213		
8	$(\gamma_5\otimes\xi_{\sigma5})(\gamma_5\otimes\xi_{\sigma5})$		−0.439		−0.376		1.511		
			−0.188		0.125		−0.872		
9	$(\gamma_\mu\otimes I)(\gamma_\mu\otimes I)$		0.428				−0.086		−1.107
			−0.142				−0.396		0.369
10	$(\gamma_\mu\otimes\xi_5)(\gamma_\mu\otimes\xi_5)$	9	−18.915/7.403			−7	−5.253	−6	−4.502
		−3	−4.772		−60.000/−7.361	−3	−2.251	2	1.501
11	$(\gamma_\mu\otimes\xi_{\mu\nu})(\gamma_\mu\otimes\xi_{\mu\nu})$		−1.289		−1.104		2.683		
			−0.552		0.368		−1.615		
12	$(\gamma_\mu\otimes\xi_{\mu\nu})(\gamma_\mu\otimes\xi_{\mu\rho})$		0.174		0.149		0.390		
			0.074		−0.049		−0.573		
13	$(\gamma_\mu\otimes\xi_{\mu\nu})(\gamma_\mu\otimes\xi_{\nu\rho})$		−0.051		0.402				
			0.153		−0.134				
14	$(\gamma_\mu\otimes\xi_{\nu\rho})(\gamma_\mu\otimes\xi_{\nu\rho})$		−1.139		−2.774		2.342		
			−1.189		0.924		−0.780		
15	$(\gamma_\mu\otimes\xi_{\nu\rho})(\gamma_\mu\otimes\xi_{\nu\sigma})$		0.277		0.149		0.224		
			−0.232		−0.049		−0.074		
16	$(\gamma_{\sigma5}\otimes I)(\gamma_{\sigma5}\otimes I)$		−0.086		−0.478		0.428		
			−0.396		0.159		−0.142		
17	$(\gamma_{\sigma5}\otimes\xi_5)(\gamma_{\sigma5}\otimes\xi_5)$	−7	−5.253	−6	−4.502	9	−19.513/6.805		
		−3	−2.251	2	1.501	−3	−2.977		
18	$(\gamma_{\sigma5}\otimes\xi_{\mu\nu})(\gamma_{\sigma5}\otimes\xi_{\mu\nu})$		2.342				−1.007		−2.774
			−0.780				−1.584		0.924
19	$(\gamma_{\sigma5}\otimes\xi_{\mu\nu})(\gamma_{\sigma5}\otimes\xi_{\mu\rho})$		0.224				0.174		−0.656
			−0.074				0.074		0.218
20	$(\gamma_{\sigma5}\otimes\xi_{\mu\nu})(\gamma_{\sigma5}\otimes\xi_{\mu\sigma})$		0.083						
			−0.249						
21	$(\gamma_{\sigma5}\otimes\xi_{\mu\sigma})(\gamma_{\sigma5}\otimes\xi_{\mu\sigma})$		2.683				−1.289		−1.104
			−1.615				−0.552		0.368
22	$(\gamma_{\sigma5}\otimes\xi_{\mu\sigma})(\gamma_{\sigma5}\otimes\xi_{\nu\sigma})$		0.224				0.174		0.149
			−0.074				0.074		−0.049
23	$(\gamma_{\mu\nu}\otimes\xi_\mu)(\gamma_{\mu\nu}\otimes\xi_\mu)$		−3.224		−2.763		21.486/8.327		
			−1.381		0.921		−3.157		
24	$(\gamma_{\mu\nu}\otimes\xi_\mu)(\gamma_{\mu\nu}\otimes\xi_\nu)$		−0.485		−0.416		−1.465		
			−0.208		0.138		2.733		
25	$(\gamma_{\mu\nu}\otimes\xi_\mu)(\gamma_{\mu\nu}\otimes\xi_\rho)$		−0.222		0.764				
			0.668		−0.254				
26	$(\gamma_{\mu\nu}\otimes\xi_\rho)(\gamma_{\mu\nu}\otimes\xi_\rho)$		−19.434/−6.274		−5.960		4.145		
			−3.000		1.986		−1.381		
27	$(\gamma_{\mu\nu}\otimes\xi_\rho)(\gamma_{\mu\nu}\otimes\xi_\sigma)$		0.485		1.944		0.624		
			0.208		−0.648		−0.208		
28	$(\gamma_{\mu\nu}\otimes\xi_{\nu5})(\gamma_{\mu\nu}\otimes\xi_{\mu5})$		−0.003				−0.052		−0.044

TABLE II. (*Continued*).

		−0.168	−0.022	0.014
29	$(\gamma_{\mu\nu}\otimes\xi_{\nu5})(\gamma_{\mu\nu}\otimes\xi_{\nu5})$	0.918	−0.900	−0.771
		−0.675	−0.385	0.257
30	$(\gamma_{\mu\nu}\otimes\xi_{\sigma5})(\gamma_{\mu\nu}\otimes\xi_{\nu5})$	0.031		
		−0.095		
31	$(\gamma_{\mu\nu}\otimes\xi_{\sigma5})(\gamma_{\mu\nu}\otimes\xi_{\rho5})$	0.067	0.052	−0.528
		−0.022	0.022	0.176
32	$(\gamma_{\mu\nu}\otimes\xi_{\sigma5})(\gamma_{\mu\nu}\otimes\xi_{\sigma5})$	1.157	−0.416	−1.704
		−0.385	−0.832	0.568

TABLE III. Renormalization factors for the gauge invariant operators $VA=(\gamma_\mu\otimes\xi_5)(\gamma_{\mu5}\otimes\xi_5)$. Matrix elements are arranged in the same was as in Table II.

	Mixed operator	γ	VA_1 finite	γ	VA_2 finite
1	$(\gamma_\mu\otimes I)(\gamma_{\mu5}\otimes I)$		0.428		
			−0.142		
2	$(\gamma_\mu\otimes\xi_5)(\gamma_{\mu5}\otimes\xi_5)$	9	−19.319/6.998		
		−3	−3.560		−29.999/−3.680
3	$(\gamma_\mu\otimes\xi_{\mu\nu})(\gamma_{\mu5}\otimes\xi_{\mu\nu})$		−1.289		−1.104
			−0.552		0.368
4	$(\gamma_\mu\otimes\xi_{\mu\nu})(\gamma_{\mu5}\otimes\xi_{\mu\rho})$		0.174		0.149
			0.074		−0.049
5	$(\gamma_\mu\otimes\xi_{\mu\nu})(\gamma_{\mu5}\otimes\xi_{\nu\rho})$		−0.051		
			0.153		
6	$(\gamma_\mu\otimes\xi_{\nu\rho})(\gamma_{\mu5}\otimes\xi_{\mu\nu})$				0.402
					−0.134
7	$(\gamma_\mu\otimes\xi_{\nu\rho})(\gamma_{\mu5}\otimes\xi_{\nu\rho})$		−1.344		−1.152
			−0.576		0.384
8	$(\gamma_\mu\otimes\xi_{\nu\rho})(\gamma_{\mu5}\otimes\xi_{\nu\sigma})$		0.225		−0.253
			−0.078		0.084
9	$(\gamma_{\sigma5}\otimes I)(\gamma_\sigma\otimes I)$		−0.191		−0.164
			−0.082		0.054
10	$(\gamma_{\sigma5}\otimes\xi_5)(\gamma_\sigma\otimes\xi_5)$	−7	−5.253	−6	−4.502
		−3	−2.251	2	1.501
11	$(\gamma_{\sigma5}\otimes\xi_{\mu\nu})(\gamma_\sigma\otimes\xi_{\mu\nu})$		2.342		
			−0.780		
12	$(\gamma_{\sigma5}\otimes\xi_{\mu\nu})(\gamma_\sigma\otimes\xi_{\mu\rho})$		0.224		
			−0.074		
13	$(\gamma_{\sigma5}\otimes\xi_{\mu\nu})(\gamma_\sigma\otimes\xi_{\mu\sigma})$		0.083		
			−0.249		
14	$(\gamma_{\sigma5}\otimes\xi_{\mu\sigma})(\gamma_\sigma\otimes\xi_{\mu\sigma})$		2.412		
			−0.804		
15	$(\gamma_{\sigma5}\otimes\xi_{\mu\sigma})(\gamma_\sigma\otimes\xi_{\nu\sigma})$		0.307		
			−0.324		
16	$(\gamma_{\mu\nu}\otimes\xi_\mu)(\gamma_{\rho\sigma}\otimes\xi_{\sigma5})$		0.174		0.149
			0.074		−0.049
17	$(\gamma_{\mu\nu}\otimes\xi_\rho)(\gamma_{\rho\sigma}\otimes\xi_{\nu5})$		−0.091		0.253
			−0.324		−0.084
18	$(\gamma_{\mu\nu}\otimes\xi_{\nu5})(\gamma_{\rho\sigma}\otimes\xi_\rho)$		0.173		
			0.078		
19	$(\gamma_{\mu\nu}\otimes\xi_{\sigma5})(\gamma_{\rho\sigma}\otimes\xi_\mu)$		−0.224		
			0.074		

The results for the color two-loop operators can then be used to evaluated the renormalization factors for the color one-loop operators. The results obtained in this way agree with those of a direct calculation of the color one-loop operators.

The matrix elements for the mixing of gauge invariant color two-loop operators with color one- and two-loop operators have been computed previously by Sheard [6]. He took a basis quite different from ours, and we found it difficult to make a full comparison. For those cases where we can compare, however, our results are in agreement with his results. Also the results in Table VI for gauge noninvariant operators agree with those of Patel and Sharpe [11].

Let us finally consider improvement of four-quark operators by factoring out tadpole renormalizations through rescaling of fields as suggested by Lepage and Mackenzie [9]. For the color two-loop operators the rescaling (39) yields a simple form for the improved operators:

$$\mathcal{O}^{\mathrm{imp}}_{2;sf}=u_0^{2-\Delta_{sf}}\,\mathcal{O}_{2;sf}\ , \tag{46}$$

where $\Delta_{sf}=\Delta_{S_1F_1}+\Delta_{S_2F_2}$ for gauge invariant operators having the spin-flavor structure $sf=(\gamma_{S_1}\otimes\xi_{F_1})(\gamma_{S_2}\otimes\xi_{F_2})$ with $\Delta_{SF}=\sum_\mu(S_\mu+F_\mu)(\mathrm{mod}2)$ the distance of bilinear operators, while $\Delta_{sf}=0$ for the gauge noninvariant operators. For the color one-loop operators, on the other hand, rescaling is not straightforward since the link insertion factors have lengths ranging from 0 to 4. To handle this case we recall the Fierz formula (45) and rewrite the color one-loop operators in terms of color two-loop operators as

TABLE IV. Renormalization factors for the gauge invariant operator $VV=(\gamma_\mu\otimes I)(\gamma_\mu\otimes\xi_5)$. Matrix elements are arranged in the same way as in Table II.

	Mixed operator	γ	VV_1 finite	γ	VV_2 finite
1	$(\gamma_\mu\otimes I)(\gamma_\mu\otimes\xi_5)$	9	−19.319/6.999		
		−3	−3.560		−29.999/−3.680
2	$(\gamma_\mu\otimes\xi_5)(\gamma_\mu\otimes I)$		0.428		
			−0.142		
3	$(\gamma_\mu\otimes\xi_{\mu\nu})(\gamma_\mu\otimes\xi_{\nu\sigma5})$		0.051		
			−0.153		
4	$(\gamma_\mu\otimes\xi_{\mu\nu})(\gamma_\mu\otimes\xi_{\mu\sigma5})$		−0.225		0.253
			0.078		−0.084
5	$(\gamma_\mu\otimes\xi_{\mu\nu})(\gamma_\mu\otimes\xi_{\mu\nu5})$		−1.344		−1.152
			−0.576		0.384
6	$(\gamma_\mu\otimes\xi_{\nu\rho})(\gamma_\mu\otimes\xi_{\rho\sigma5})$		0.174		0.149
			0.074		−0.049
7	$(\gamma_\mu\otimes\xi_{\nu\rho})(\gamma_\mu\otimes\xi_{\nu\rho5})$		−1.289		−1.104
			−0.552		0.368
8	$(\gamma_\mu\otimes\xi_{\nu\rho})(\gamma_\mu\otimes\xi_{\mu\rho5})$				0.402
					−0.134
9	$(\gamma_{\sigma5}\otimes I)(\gamma_{\sigma5}\otimes\xi_5)$	−7	−5.253	−6	−4.502
		−3	−2.251	2	1.501
10	$(\gamma_{\sigma5}\otimes\xi_5)(\gamma_{\sigma5}\otimes I)$		−0.191		−0.164
			−0.082		0.054
11	$(\gamma_{\sigma5}\otimes\xi_{\mu\nu})(\gamma_{\sigma5}\otimes\xi_{\nu\sigma5})$		−0.083		
			0.249		
12	$(\gamma_{\sigma5}\otimes\xi_{\mu\nu})(\gamma_{\sigma5}\otimes\xi_{\nu\rho5})$		0.307		
			−0.324		
13	$(\gamma_{\sigma5}\otimes\xi_{\mu\nu})(\gamma_{\sigma5}\otimes\xi_{\mu\nu5})$		2.412		
			−0.804		
14	$(\gamma_{\sigma5}\otimes\xi_{\mu\sigma})(\gamma_{\sigma5}\otimes\xi_{\rho\sigma5})$		−0.224		
			0.074		
15	$(\gamma_{\sigma5}\otimes\xi_{\mu\sigma})(\gamma_{\sigma5}\otimes\xi_{\mu\sigma5})$		2.342		
			−0.780		
16	$(\gamma_{\mu\nu}\otimes\xi_\mu)(\gamma_{\rho\sigma5}\otimes\xi_\rho)$		0.091		−0.253
			0.324		0.084
17	$(\gamma_{\mu\nu}\otimes\xi_\rho)(\gamma_{\rho\sigma5}\otimes\xi_\mu)$		−0.174		−0.149
			−0.074		0.049
18	$(\gamma_{\mu\nu}\otimes\xi_{\nu5})(\gamma_{\rho\sigma5}\otimes\xi_{\sigma5})$		0.224		
			−0.074		
19	$(\gamma_{\mu\nu}\otimes\xi_{\sigma5})(\gamma_{\rho\sigma5}\otimes\xi_{\nu5})$		−0.173		
			−0.078		

TABLE V. Renormalization factors for the gauge invariant operator $VA=(\gamma_\mu\otimes I)(\gamma_{\mu 5}\otimes\xi_5)$ and $AV=(\gamma_{\mu 5}\otimes I)(\gamma_\mu\otimes\xi_5)$. Matrix elements are arranged in the same way as in Table II.

	Mixed operator	γ	VA_1 finite	γ	VA_2 finite	γ	AV_1 finite	γ	AV_2 finite
1	$(I\otimes\xi_\mu)(\gamma_5\otimes\xi_{\mu 5})$		1.511				−0.439		−0.376
			−0.872				−0.188		0.125
2	$(I\otimes\xi_\mu)(\gamma_5\otimes\xi_{\sigma 5})$		0.130				0.052		0.044
			−0.213				0.022		−0.014
3	$(I\otimes\xi_{\sigma 5})(\gamma_5\otimes\xi_\mu)$		0.485		0.416		0.319		
			0.208		−0.138		0.706		
4	$(I\otimes\xi_{\sigma 5})(\gamma_5\otimes\xi_\sigma)$		−3.675		−3.150		20.774/7.615		
			−1.575		1.050		−2.569		
5	$(\gamma_5\otimes\xi_\mu)(I\otimes\xi_{\mu 5})$		0.565				−0.876		−1.310
			−0.188				−1.029		0.436
6	$(\gamma_5\otimes\xi_\mu)(I\otimes\xi_{\sigma 5})$		−0.067				−0.052		−0.235
			0.022				−0.022		0.078
7	$(\gamma_5\otimes\xi_{\sigma 5})(I\otimes\xi_\mu)$		−0.485		1.112		−0.624		
			−0.208		−0.370		0.208		
8	$(\gamma_5\otimes\xi_{\sigma 5})(I\otimes\xi_\sigma)$		−19.114/−5.954		−5.573		4.725		
			−2.412		1.857		−1.575		
9	$(\gamma_\mu\otimes I)(\gamma_{\mu 5}\otimes\xi_5)$	9	−19.513/6.805			−7	−5.253	−6	−4.502
		−3	−2.977			−3	−2.251	2	1.501
10	$(\gamma_\mu\otimes\xi_5)(\gamma_{\mu 5}\otimes I)$		0.428				−0.086		−0.478
			−0.142				−0.396		0.159
11	$(\gamma_\mu\otimes\xi_{\mu\nu})(\gamma_{\mu 5}\otimes\xi_{\nu\sigma 5})$						0.083		
							−0.249		
12	$(\gamma_\mu\otimes\xi_{\mu\nu})(\gamma_{\mu 5}\otimes\xi_{\mu\sigma 5})$		−0.174		0.656		−0.224		
			−0.074		−0.218		0.074		
13	$(\gamma_\mu\otimes\xi_{\mu\nu})(\gamma_{\mu 5}\otimes\xi_{\mu\nu 5})$		−1.007		−2.774		2.342		
			−1.584		0.924		−0.780		
14	$(\gamma_\mu\otimes\xi_{\nu\rho})(\gamma_{\mu 5}\otimes\xi_{\rho\sigma 5})$		0.174		0.149		0.224		
			0.074		−0.049		−0.074		
15	$(\gamma_\mu\otimes\xi_{\nu\rho})(\gamma_{\mu 5}\otimes\xi_{\nu\rho 5})$		−1.289		−1.104		2.683		
			−0.552		0.368		−1.615		
16	$(\gamma_\mu\otimes\xi_{\nu\rho})(\gamma_{\mu 5}\otimes\xi_{\mu\rho 5})$						−0.083		
							0.249		
17	$(\gamma_{\sigma 5}\otimes I)(\gamma_\sigma\otimes\xi_5)$	−7	−5.253	−6	−4.502	9	−18.915/7.403		
		−3	−2.251	2	1.501	−3	−4.772		−60.000/−7.361
18	$(\gamma_{\sigma 5}\otimes\xi_5)(\gamma_\sigma\otimes I)$		−0.086		−1.107		0.428		
			−0.396		0.369		−0.142		
19	$(\gamma_{\sigma 5}\otimes\xi_{\mu\nu})(\gamma_\sigma\otimes\xi_{\nu\sigma 5})$						0.051		−0.402
							−0.153		0.134
20	$(\gamma_{\sigma 5}\otimes\xi_{\mu\nu})(\gamma_\sigma\otimes\xi_{\nu\rho 5})$		0.390				0.174		0.149
			−0.573				0.074		−0.049
21	$(\gamma_{\sigma 5}\otimes\xi_{\mu\nu})(\gamma_\sigma\otimes\xi_{\mu\nu 5})$		2.683				−1.289		−1.104
			−1.615				−0.552		0.368
22	$(\gamma_{\sigma 5}\otimes\xi_{\mu\sigma})(\gamma_\sigma\otimes\xi_{\rho\sigma 5})$		−0.224				−0.277		−0.149
			0.074				0.232		0.049
23	$(\gamma_{\sigma 5}\otimes\xi_{\mu\sigma})(\gamma_\sigma\otimes\xi_{\mu\sigma 5})$		2.342				−1.139		−2.774
			−0.780				−1.189		0.924
24	$(\gamma_{\sigma 5}\otimes\xi_{\mu\sigma})(\gamma_\sigma\otimes\xi_{\mu\rho 5})$						−0.051		0.402
							0.153		−0.134
25	$(\gamma_{\mu\nu}\otimes\xi_\mu)(\gamma_{\mu\nu 5}\otimes\xi_{\mu 5})$		−0.416		−1.704		1.157		
			−0.832		0.568		−0.385		
26	$(\gamma_{\mu\nu}\otimes\xi_\mu)(\gamma_{\mu\nu 5}\otimes\xi_{\nu 5})$		0.052		−0.528		0.067		
			0.022		0.176		−0.022		
27	$(\gamma_{\mu\nu}\otimes\xi_\mu)(\gamma_{\mu\nu 5}\otimes\xi_{\sigma 5})$						0.031		
							−0.095		
28	$(\gamma_{\mu\nu}\otimes\xi_\rho)(\gamma_{\mu\nu 5}\otimes\xi_{\nu 5})$						0.031		
							−0.095		
29	$(\gamma_{\mu\nu}\otimes\xi_\rho)(\gamma_{\mu\nu 5}\otimes\xi_{\rho 5})$		−0.900		−0.771		0.918		
			−0.385		0.257		−0.675		

TABLE V. (*Continued*).

	Mixed operator	γ	VA_1 finite	γ	VA_2 finite	γ	AV_1 finite	γ	AV_2 finite
30	$(\gamma_{\mu\nu}\otimes\xi_{\rho})(\gamma_{\mu\nu 5}\otimes\xi_{\sigma 5})$		-0.052		-0.044		-0.003		
			-0.022		0.014		-0.168		
31	$(\gamma_{\mu\nu}\otimes\xi_{\nu 5})(\gamma_{\mu\nu 5}\otimes\xi_{\mu})$		0.624				0.485		1.944
			-0.208				0.208		-0.648
32	$(\gamma_{\mu\nu}\otimes\xi_{\nu 5})(\gamma_{\mu\nu 5}\otimes\xi_{\nu})$		4.145				$-19.434/-6.274$		-5.960
			-1.381				-3.000		1.986
33	$(\gamma_{\mu\nu}\otimes\xi_{\nu 5})(\gamma_{\mu\nu 5}\otimes\xi_{\rho})$						-0.222		0.764
							0.668		-0.254
34	$(\gamma_{\mu\nu}\otimes\xi_{\sigma 5})(\gamma_{\mu\nu 5}\otimes\xi_{\mu})$						-0.222		0.764
							0.668		-0.254
35	$(\gamma_{\mu\nu}\otimes\xi_{\sigma 5})(\gamma_{\mu\nu 5}\otimes\xi_{\rho})$		-1.465				-0.485		-0.416
			2.733				-0.208		0.138
36	$(\gamma_{\mu\nu}\otimes\xi_{\sigma 5})(\gamma_{\mu\nu 5}\otimes\xi_{\sigma})$		$21.486/8.327$				-3.224		-2.763
			-3.157				-1.381		0.921

$$\mathcal{O}_{1;sf}=\sum_{s'f'}F_{sf,s'f'}\mathcal{O}_{2;s'f'}\ , \tag{47}$$

with $F_{sf,s'f'}$ numerical constants. The rescaled color one-loop operators can then be defined as [16]

$$\mathcal{O}_{1;sf}^{\mathrm{imp}}=\sum_{s'f'}F_{sf,s'f'}u_0^{2-\Delta_{s'f'}}\mathcal{O}_{2;s'f'}\ . \tag{48}$$

For the gauge invariant choice (40) for the tadpole factor u_0 the finite renormalization factors for rescaled four-quark operators are listed in Tables II–IX where those elements changed by rescaling are given after a slash symbol. As one can see in the tables the rescaling indeed reduces the magnitude of the renormalization correction.

B. Relation with continuum operators

In order to obtain the physical values of weak matrix elements the renormalization factor on the lattice obtained in the previous section has to be combined with those in the continuum. In this section we illustrate the procedure for the K meson B parameter relevant for the K^0-$\bar{K}^0$ mixing matrix.

The K meson B parameter B_K in the continuum theory is defined by

$$B_K=\frac{\langle\bar{K}^0|\bar{s}\gamma_\mu(1-\gamma_5)d\bar{s}\gamma_\mu(1-\gamma_5)d|K^0\rangle}{\frac{8}{3}f_K^2m_K^2}\ . \tag{49}$$

In the method of Ref. [14] for calculating weak matrix elements with KS fermions, the operator in the numerator is replaced by the sum of the four operators

$$\begin{aligned}
\mathcal{V}_1&=\bar{S}^a(\gamma_\mu\otimes\xi_5)D^b\bar{S}^b(\gamma_\mu\otimes\xi_5)D^a\ ,\\
\mathcal{V}_2&=\bar{S}^a(\gamma_\mu\otimes\xi_5)D^a\bar{S}^b(\gamma_\mu\otimes\xi_5)D^b\ ,\\
\mathcal{A}_1&=\bar{S}^a(\gamma_{\mu 5}\otimes\xi_5)D^b\bar{S}^b(\gamma_{\mu 5}\otimes\xi_5)D^a\ ,\\
\mathcal{A}_2&=\bar{S}^a(\gamma_{\mu 5}\otimes\xi_5)D^a\bar{S}^b(\gamma_{\mu 5}\otimes\xi_5)D^b\ ,
\end{aligned} \tag{50}$$

where S and D are the KS quark fields introduced for s and d quarks separately, a and b the color indices, and the quark fields in the first current are to be contracted with $\bar{K}^0$ and those in the second current with K^0. The choice of flavor ξ_5 in these operators corresponds to the use of $\bar{D}(\gamma_5\otimes\xi_5)S$ for creating the external K^0 and $\bar{K}^0$ in the Nambu-Goldstone channel associated with U(1) chiral symmetry of the KS fermion action.

The renormalization factor for these operators can be read off from Table II for gauge invariant operators and from Table VI for gauge noninvariant operators. As can be seen, the four operators not only mix among themselves but also with a large number of others having different spin-flavor structures. We note that the extra operators all have the flavor matrix $\xi_F\neq\xi_5$. Since the K^0 and $\bar{K}^0$ mesons are created with the flavor ξ_5, the matrix element of the extra operators should vanish in the continuum limit where a restoration of SU(4) flavor symmetry is expected. The renormalization factor for some of the extra operators is numerically not small, however. Whether they yield negligibly small contributions at the current range of inverse lattice spacing $1/a\sim 2$–3 GeV has to be checked through actual simulations. For simplicity we disregard the mixing with the extra operators in the following. Extensions to the general case are straightforward.

Denoting the four operators (50) as $\{\mathcal{O}_\alpha^{\mathrm{lat}};\alpha=1,\ldots,4\}=\{\mathcal{V}_1,\ldots,\mathcal{A}_2\}$, we find that the 4×4 anomalous dimension matrix $\gamma_{\alpha\beta}^{\mathrm{lat}}$ is given by

$$\gamma_{\alpha\beta}^{\mathrm{lat}}=\begin{bmatrix} 9 & -3 & -7 & -3\\ 0 & 0 & -6 & 2\\ -7 & -3 & 9 & -3\\ -6 & 2 & 0 & 0\end{bmatrix}\ , \tag{51}$$

which takes the same form for the gauge invariant and noninvariant cases. The finite part $C_{\alpha\beta}^{\mathrm{lat}}$ takes the values

TABLE VI. Renormalization factors for the gauge noninvariant operators $VV=(\gamma_\mu\otimes\xi_5)(\gamma_\mu\otimes\xi_5)$ and $AA=(\gamma_{\mu5}\otimes\xi_5)(\gamma_{\mu5}\otimes\xi_5)$ in the Landau gauge. Matrix elements are arranged in the same way as in Table II.

	Mixed operator	γ	VV_1 finite	γ	VV_2 finite	γ	AA_1 finite	γ	AA_2 finite
1	$(I\otimes\xi_\mu)(I\otimes\xi_\mu)$		4.725				−3.774		−3.235
			−1.575				−1.617		1.078
2	$(I\otimes\xi_\mu)(I\otimes\xi_\nu)$		−0.624				−0.485		−0.416
			0.208				−0.208		0.138
3	$(I\otimes\xi_{\sigma5})(I\otimes\xi_{\rho5})$		−0.052		−0.044		−0.067		
			−0.022		0.014		0.022		
4	$(I\otimes\xi_{\sigma5})(I\otimes\xi_{\sigma5})$		−1.466		−1.257		0.565		
			−0.628		0.419		−0.188		
5	$(\gamma_5\otimes\xi_\mu)(\gamma_5\otimes\xi_\mu)$		4.853				−3.675		−3.150
			−1.617				−1.575		1.050
6	$(\gamma_5\otimes\xi_\mu)(\gamma_5\otimes\xi_\nu)$		0.624				0.485		0.416
			−0.208				0.208		−0.138
7	$(\gamma_5\otimes\xi_{\sigma5})(\gamma_5\otimes\xi_{\rho5})$		0.052		0.044		0.067		
			0.022		−0.014		−0.022		
8	$(\gamma_5\otimes\xi_{\sigma5})(\gamma_5\otimes\xi_{\sigma5})$		−0.439		−0.376		1.885		
			−0.188		0.125		−0.628		
9	$(\gamma_\mu\otimes I)(\gamma_\mu\otimes I)$		0.428				−0.538		−0.461
			−0.142				−0.230		0.153
10	$(\gamma_\mu\otimes\xi_5)(\gamma_\mu\otimes\xi_5)$	9	37.446/11.127			−7	−5.253	−6	−4.502
		−3	−2.913		28.706/2.387	−3	−2.251	2	1.501
11	$(\gamma_\mu\otimes\xi_{\mu\nu})(\gamma_\mu\otimes\xi_{\mu\nu})$		−1.289		−1.104		3.016		
			−0.552		0.368		−1.005		
12	$(\gamma_\mu\otimes\xi_{\mu\nu})(\gamma_\mu\otimes\xi_{\mu\rho})$		0.174		0.149		0.224		
			0.074		−0.049		−0.074		
13	$(\gamma_\mu\otimes\xi_{\nu\rho})(\gamma_\mu\otimes\xi_{\nu\rho})$		−1.813		−1.554		2.342		
			−0.777		0.518		−0.780		
14	$(\gamma_\mu\otimes\xi_{\nu\rho})(\gamma_\mu\otimes\xi_{\nu\sigma})$		0.174		0.149		0.224		
			0.074		−0.049		−0.074		
15	$(\gamma_{\sigma5}\otimes I)(\gamma_{\sigma5}\otimes I)$		−0.538		−0.461		0.428		
			−0.230		0.153		−0.142		
16	$(\gamma_{\sigma5}\otimes\xi_5)(\gamma_{\sigma5}\otimes\xi_5)$	−7	−5.253	−6	−4.502	9	37.976/11.657		
		−3	−2.251	2	1.501	−3	−4.504		24.464/−1.854
17	$(\gamma_{\sigma5}\otimes\xi_{\mu\nu})(\gamma_{\sigma5}\otimes\xi_{\mu\nu})$		2.342				−1.813		−1.554
			−0.780				−0.777		0.518
18	$(\gamma_{\sigma5}\otimes\xi_{\mu\nu})(\gamma_{\sigma5}\otimes\xi_{\mu\rho})$		0.224				0.174		0.149
			−0.074				0.074		−0.049
19	$(\gamma_{\sigma5}\otimes\xi_{\mu\sigma})(\gamma_{\sigma5}\otimes\xi_{\mu\sigma})$		3.016				−1.289		−1.104
			−1.005				−0.552		0.368
20	$(\gamma_{\sigma5}\otimes\xi_{\mu\sigma})(\gamma_{\sigma5}\otimes\xi_{\nu\sigma})$		0.224				0.174		0.149
			−0.074				0.074		−0.049
21	$(\gamma_{\mu\nu}\otimes\xi_\mu)(\gamma_{\mu\nu}\otimes\xi_\mu)$		−3.224		−2.763		5.433		
			−1.381		0.921		−1.811		
22	$(\gamma_{\mu\nu}\otimes\xi_\mu)(\gamma_{\mu\nu}\otimes\xi_\nu)$		−0.485		−0.416		−0.624		
			−0.208		0.138		0.208		
23	$(\gamma_{\mu\nu}\otimes\xi_\rho)(\gamma_{\mu\nu}\otimes\xi_\rho)$		−4.226		−3.622		4.145		
			−1.811		1.207		−1.381		
24	$(\gamma_{\mu\nu}\otimes\xi_\rho)(\gamma_{\mu\nu}\otimes\xi_\sigma)$		0.485		0.416		0.624		
			0.208		−0.138		−0.208		
25	$(\gamma_{\mu\nu}\otimes\xi_{\nu5})(\gamma_{\mu\nu}\otimes\xi_{\mu5})$		−0.067				−0.052		−0.044
			0.022				−0.022		0.014
26	$(\gamma_{\mu\nu}\otimes\xi_{\nu5})(\gamma_{\mu\nu}\otimes\xi_{\nu5})$		1.293				−0.900		−0.771
			−0.431				−0.385		0.257
27	$(\gamma_{\mu\nu}\otimes\xi_{\sigma5})(\gamma_{\mu\nu}\otimes\xi_{\rho5})$		0.067				0.052		0.044
			−0.022				0.022		−0.014
28	$(\gamma_{\mu\nu}\otimes\xi_{\sigma5})(\gamma_{\mu\nu}\otimes\xi_{\sigma5})$		1.157				−1.005		−0.862
			−0.385				−0.431		0.287

$$C^{\rm lat}_{\alpha\beta}=\begin{bmatrix} -18.915 & -4.772 & -5.253 & -2.251 \\ 0 & -60.000 & -4.502 & 1.501 \\ -5.253 & -2.251 & -19.513 & -2.977 \\ -4.502 & 1.501 & 0 & 0 \end{bmatrix} \tag{52}$$

for the gauge invariant operator, and

$$C^{\rm lat}_{\alpha\beta}=\begin{bmatrix} 37.446 & -2.913 & -5.253 & -2.251 \\ 0 & 28.706 & -4.502 & 1.501 \\ -5.253 & -2.251 & 37.976 & -4.504 \\ -4.502 & 1.501 & 0 & 24.464 \end{bmatrix} \tag{53}$$

for the gauge noninvariant operator in the Landau gauge. The result for the gauge noninvariant operator (53) is in agreement with those of Ref. [11]. The second and the fourth rows of (52) have previously been calculated by Sheard, with which our results agree. For the rescaled operators discussed in Sec. IV A the diagonal elements of $C^{\rm lat}_{\alpha\beta}$ changes to $(7.403, -7.361, 6.805, 0)$ for the gauge invariant operators and to $(11.127, 2.387, 11.657, -1.854)$ for the gauge noninvariant operators (off-diagonal elements are not affected).

The $\Delta S=2$ continuum operators are given by

$$\mathcal{L}_1=\bar{s}^a\gamma_\mu(1-\gamma_5)d^b\bar{s}^b\gamma_\mu(1-\gamma_5)d^a\ ,$$
$$\mathcal{L}_2=\bar{s}^a\gamma_\mu(1-\gamma_5)d^a\bar{s}^b\gamma_\mu(1-\gamma_5)d^b\ , \tag{54}$$

with the one-loop renormalization taking the form

$$\mathcal{L}_i^{(1)}=\left[\delta_{ij}+\frac{g^2}{16\pi^2}(\gamma^{\rm cont}_{ij}\ln(\mu/\kappa)+C^{\rm cont}_{ij}\right]\mathcal{L}_j^{(0)}\ , \tag{55}$$

where the anomalous dimension matrix $\gamma^{\rm cont}_{ij}$ is given by

$$\gamma^{\rm cont}_{ij}=\begin{bmatrix} 2 & -6 \\ -6 & 2 \end{bmatrix}\ . \tag{56}$$

For the finite part $C^{\rm cont}_{ij}$ we find that

$$C^{\rm cont}_{ij}=c\gamma^{\rm cont}_{ij} \tag{57}$$

with

TABLE VII. Renormalization factors for the gauge noninvariant operators $VA=(\gamma_\mu\otimes\xi_5)(\gamma_{\mu5}\otimes\xi_5)$ in the Landau gauge. Matrix elements are arragned in the same way as in Table II.

	Mixed operator	γ	VA_1 finite	γ	VA_2 finite
1	$(\gamma_\mu\otimes I)(\gamma_{\mu5}\otimes I)$		0.428		
			-0.142		
2	$(\gamma_\mu\otimes\xi_5)(\gamma_{\mu5}\otimes\xi_5)$	9	37.711/11.392		
		-3	-3.708		26.585/0.266
3	$(\gamma_\mu\otimes\xi_{\mu\nu})(\gamma_{\mu5}\otimes\xi_{\mu\nu})$		-1.289		-1.104
			-0.552		0.368
4	$(\gamma_\mu\otimes\xi_{\mu\nu})(\gamma_{\mu5}\otimes\xi_{\mu\rho})$		0.174		0.149
			0.074		-0.049
5	$(\gamma_\mu\otimes\xi_{\nu\rho})(\gamma_{\mu5}\otimes\xi_{\nu\rho})$		-1.813		-1.554
			-0.777		0.518
6	$(\gamma_\mu\otimes\xi_{\nu\rho})(\gamma_{\mu5}\otimes\xi_{\nu\sigma})$		0.174		0.149
			0.074		-0.049
7	$(\gamma_{\sigma5}\otimes I)(\gamma_\sigma\otimes I)$		-0.538		-0.461
			-0.230		0.153
8	$(\gamma_{\sigma5}\otimes\xi_5)(\gamma_\sigma\otimes\xi_5)$	-7	-5.253	-6	-4.502
		-3	-2.251	2	1.501
9	$(\gamma_{\sigma5}\otimes\xi_{\mu\nu})(\gamma_\sigma\otimes\xi_{\mu\nu})$		2.342		
			-0.780		
10	$(\gamma_{\sigma5}\otimes\xi_{\mu\nu})(\gamma_\sigma\otimes\xi_{\mu\rho})$		0.224		
			-0.074		
11	$(\gamma_{\sigma5}\otimes\xi_{\mu\sigma})(\gamma_\sigma\otimes\xi_{\mu\sigma})$		3.016		
			-1.005		
12	$(\gamma_{\sigma5}\otimes\xi_{\mu\sigma})(\gamma_\sigma\otimes\xi_{\nu\sigma})$		0.224		
			-0.074		
13	$(\gamma_{\mu\nu}\otimes\xi_\mu)(\gamma_{\rho\sigma}\otimes\xi_{\sigma5})$		0.174		0.149
			0.074		-0.049
14	$(\gamma_{\mu\nu}\otimes\xi_\rho)(\gamma_{\rho\sigma}\otimes\xi_{\nu5})$		-0.174		-0.149
			-0.074		0.049
15	$(\gamma_{\mu\nu}\otimes\xi_{\nu5})(\gamma_{\rho\sigma}\otimes\xi_\rho)$		0.224		
			-0.074		
16	$(\gamma_{\mu\nu}\otimes\xi_{\sigma5})(\gamma_{\rho\sigma}\otimes\xi_\mu)$		-0.224		
			0.074		

$$c=\begin{cases}11/12 & \text{for NDR [17]} ,\\ 7/12 & \text{for } \mathrm{DR\overline{EZ}} .\end{cases} \tag{58}$$

In order to relate the lattice operators to those in the continuum let us define a 2×4 matrix $M_{i\alpha}$ by

$$M=\begin{bmatrix}1 & 0 & 1 & 0\\ 0 & 1 & 0 & 1\end{bmatrix} \tag{59}$$

with which one can write $\mathcal{L}_i^{(0)}=M_{i\alpha}\mathcal{O}_\alpha^{\mathrm{lat}(0)}$. Using the intertwining property $M_{i\alpha}\gamma_{\alpha\beta}^{\mathrm{lat}}=\gamma_{ij}^{\mathrm{cont}}M_{j\beta}$ it is easy to see that the one-loop renormalization relation between the continuum and lattice operators is given by

$$\mathcal{L}_i^{(1)}=M_{i\alpha}\left\{\delta_{\alpha\beta}+\frac{g^2}{16\pi^2}[\gamma^{\mathrm{lat}}\ln(\mu a)+c\gamma^{\mathrm{lat}} - C^{\mathrm{lat}}]_{\alpha\beta}\right\}\mathcal{O}_\beta^{\mathrm{lat}(1)} . \tag{60}$$

The numerator of the B_K parameter with one-loop renormalization correction equals the sum $\langle \overline{K}^0|\mathcal{L}_1^{(1)}|K^0\rangle+\langle \overline{K}^0|\mathcal{L}_2^{(1)}|K^0\rangle$.

ACKNOWLEDGMENTS

Valuable discussions and correspondence with Steve Sharpe and Aproova Patel are gratefully acknowledged. We thank A. Ukawa and M. Okawa for useful discussions, and M. Fukugita and H. Mino for comments on the manuscript.

APPENDIX A: ONE-LOOP AMPLITUDES FOR BILINEAR OPERATORS

The one-loop amplitudes corresponding to the diagrams of Fig. 1 for the external quark momenta $p=C\pi/a, D\pi/a$ and color indices a,b are as follows [the common factor $4/3\delta_{ab}g^2/(16\pi^2)$ is not included]:

TABLE VIII. Renormalization factors for the gauge noninvariant operators $VV=(\gamma_\mu\otimes I)(\gamma_\mu\otimes\xi_5)$ in the Landau gauge. Matrix elements are arranged in the same way as in Table II.

	Mixed operator	γ	VV_1 finite	γ	VV_2 finite
1	$(\gamma_\mu\otimes I)(\gamma_\mu\otimes\xi_5)$	9	37.711/11.392		
		-3	-3.708		26.585/0.266
2	$(\gamma_\mu\otimes\xi_5)(\gamma_\mu\otimes I)$		0.428		
			-0.142		
3	$(\gamma_\mu\otimes\xi_{\mu\nu})(\gamma_\mu\otimes\xi_{\mu\sigma 5})$		-0.174		-0.149
			-0.074		0.049
4	$(\gamma_\mu\otimes\xi_{\mu\nu})(\gamma_\mu\otimes\xi_{\mu\nu 5})$		-1.813		-1.554
			-0.777		0.518
5	$(\gamma_\mu\otimes\xi_{\nu\rho})(\gamma_\mu\otimes\xi_{\rho\sigma 5})$		0.174		0.149
			0.074		-0.049
6	$(\gamma_\mu\otimes\xi_{\nu\rho})(\gamma_\mu\otimes\xi_{\nu\rho 5})$		-1.289		-1.104
			-0.552		0.368
7	$(\gamma_{\sigma 5}\otimes I)(\gamma_{\sigma 5}\otimes\xi_5)$	-7	-5.253	-6	-4.502
		-3	-2.251	2	1.501
8	$(\gamma_{\sigma 5}\otimes\xi_5)(\gamma_{\sigma 5}\otimes I)$		-0.538		-0.461
			-0.230		0.153
9	$(\gamma_{\sigma 5}\otimes\xi_{\mu\nu})(\gamma_{\sigma 5}\otimes\xi_{\nu\rho 5})$		0.224		
			-0.074		
10	$(\gamma_{\sigma 5}\otimes\xi_{\mu\nu})(\gamma_{\sigma 5}\otimes\xi_{\mu\nu 5})$		3.016		
			-1.005		
11	$(\gamma_{\sigma 5}\otimes\xi_{\mu\sigma})(\gamma_{\sigma 5}\otimes\xi_{\rho\sigma 5})$		-0.224		
			0.074		
12	$(\gamma_{\sigma 5}\otimes\xi_{\mu\sigma})(\gamma_{\sigma 5}\otimes\xi_{\mu\sigma 5})$		2.342		
			-0.780		
13	$(\gamma_{\mu\nu}\otimes\xi_\mu)(\gamma_{\rho\sigma 5}\otimes\xi_\rho)$		0.174		0.149
			0.074		-0.049
14	$(\gamma_{\mu\nu}\otimes\xi_\rho)(\gamma_{\rho\sigma 5}\otimes\xi_\mu)$		-0.174		-0.149
			-0.074		0.049
15	$(\gamma_{\mu\nu}\otimes\xi_{\nu 5})(\gamma_{\rho\sigma 5}\otimes\xi_{\sigma 5})$		0.224		
			-0.074		
16	$(\gamma_{\mu\nu}\otimes\xi_{\sigma 5})(\gamma_{\rho\sigma 5}\otimes\xi_{\nu 5})$		-0.224		
			0.074		

TABLE IX. Renormalization factors for the gauge noninvariant operators $VA=(\gamma_\mu\otimes I)(\gamma_{\mu5}\otimes\xi_5)$ and $AV=(\gamma_{\mu5}\otimes I)(\gamma_\mu\otimes\xi_5)$ in the Landau gauge. Matrix elements are arranged in the same way as in Table II.

	Mixed operator	γ	VA_1 finite	γ	VA_2 finite	γ	AV_1 finite	γ	AV_2 finite
1	$(I\otimes\xi_\mu)(\gamma_5\otimes\xi_{\mu5})$		1.885				−0.439		−0.376
			−0.628				−0.188		0.125
2	$(I\otimes\xi_\mu)(\gamma_5\otimes\xi_{\sigma5})$		0.067				0.052		0.044
			−0.022				0.022		−0.014
3	$(I\otimes\xi_{\sigma5})(\gamma_5\otimes\xi_\mu)$		0.485		0.416		0.624		
			0.208		−0.138		−0.208		
4	$(I\otimes\xi_{\sigma5})(\gamma_5\otimes\xi_\sigma)$		−3.675		−3.150		4.853		
			−1.575		1.050		−1.617		
5	$(\gamma_5\otimes\xi_\mu)(I\otimes\xi_{\mu5})$		0.565				−1.466		−1.257
			−0.188				−0.628		0.419
6	$(\gamma_5\otimes\xi_\mu)(I\otimes\xi_{\sigma5})$		−0.067				−0.052		−0.044
			0.022				−0.022		0.014
7	$(\gamma_5\otimes\xi_{\sigma5})(I\otimes\xi_\mu)$		−0.485		−0.416		−0.624		
			−0.208		0.138		−0.208		
8	$(\gamma_5\otimes\xi_{\sigma5})(I\otimes\xi_\sigma)$		−3.774		−3.235		4.725		
			−1.617		1.078		−1.575		
9	$(\gamma_\mu\otimes I)(\gamma_{\mu5}\otimes\xi_5)$	9	37.976/11.657			−7	−5.253	−6	−4.502
		−3	−4.503		24.464/−1.854	−3	−2.251	2	1.501
10	$(\gamma_\mu\otimes\xi_5)(\gamma_{\mu5}\otimes I)$		0.428				−0.538		−0.461
			−0.142				−0.230		0.153
11	$(\gamma_\mu\otimes\xi_{\mu\nu})(\gamma_{\mu5}\otimes\xi_{\mu\sigma5})$		−0.174		−0.149		−0.224		
			−0.074		0.049		0.074		
12	$(\gamma_\mu\otimes\xi_{\mu\nu})(\gamma_{\mu5}\otimes\xi_{\mu\nu5})$		−1.813		−1.554		2.342		
			−0.777		0.518		−0.780		
13	$(\gamma_\mu\otimes\xi_{\nu\rho})(\gamma_{\mu5}\otimes\xi_{\rho\sigma5})$		0.174		0.149		0.224		
			0.074		−0.049		−0.074		
14	$(\gamma_\mu\otimes\xi_{\nu\rho})(\gamma_{\mu5}\otimes\xi_{\nu\rho5})$		−1.289		−1.104		3.016		
			−0.552		0.368		−1.005		
15	$(\gamma_{\sigma5}\otimes I)(\gamma_\sigma\otimes\xi_5)$	−7	−5.253	−6	−4.502	9	37.446/11.127		
		−3	−2.251	2	1.501	−3	−2.913		28.706/2.387
16	$(\gamma_{\sigma5}\otimes\xi_5)(\gamma_\sigma\otimes I)$		−0.538		−0.461		0.428		
			−0.230		0.153		−0.142		
17	$(\gamma_{\sigma5}\otimes\xi_{\mu\nu})(\gamma_\sigma\otimes\xi_{\nu\rho5})$		0.224				0.174		0.149
			−0.074				0.074		−0.049
18	$(\gamma_{\sigma5}\otimes\xi_{\mu\nu})(\gamma_\sigma\otimes\xi_{\mu\nu5})$		3.016				−1.289		−1.104
			−1.005				−0.552		0.368
19	$(\gamma_{\sigma5}\otimes\xi_{\mu\sigma})(\gamma_\sigma\otimes\xi_{\rho\sigma5})$		−0.224				−0.174		−0.149
			0.074				−0.074		0.049
20	$(\gamma_{\sigma5}\otimes\xi_{\mu\sigma})(\gamma_\sigma\otimes\xi_{\mu\sigma5})$		2.342				−1.813		−1.554
			−0.780				−0.777		0.518
21	$(\gamma_{\mu\nu}\otimes\xi_\mu)(\gamma_{\mu\nu5}\otimes\xi_{\mu5})$		−1.005		−0.862		1.157		
			−0.431		0.287		−0.385		
22	$(\gamma_{\mu\nu}\otimes\xi_\mu)(\gamma_{\mu\nu5}\otimes\xi_{\nu5})$		0.052		0.044		0.067		
			0.022		−0.014		−0.022		
23	$(\gamma_{\mu\nu}\otimes\xi_\rho)(\gamma_{\mu\nu5}\otimes\xi_{\rho5})$		−0.900		−0.771		1.293		
			−0.385		0.257		−0.431		
24	$(\gamma_{\mu\nu}\otimes\xi_\rho)(\gamma_{\mu\nu5}\otimes\xi_{\sigma5})$		−0.052		−0.044		−0.067		
			−0.022		0.014		0.022		
25	$(\gamma_{\mu\nu}\otimes\xi_{\nu5})(\gamma_{\mu\nu5}\otimes\xi_\mu)$		0.624				0.485		0.416
			−0.208				0.208		−0.138
26	$(\gamma_{\mu\nu}\otimes\xi_{\nu5})(\gamma_{\mu\nu5}\otimes\xi_\nu)$		4.145				−4.226		−3.622
			−1.381				−1.811		1.207
27	$(\gamma_{\mu\nu}\otimes\xi_{\sigma5})(\gamma_{\mu\nu5}\otimes\xi_\rho)$		−0.624				−0.485		−0.416
			0.208				−0.208		0.138
28	$(\gamma_{\mu\nu}\otimes\xi_{\sigma5})(\gamma_{\mu\nu5}\otimes\xi_\sigma)$		5.433				−3.224		−2.763
			−1.811				−1.381		0.921

$$G^{1(a)}=[\sigma_S-(1-\alpha)]x(\overline{\overline{\gamma_S\otimes\xi_F}})_{CD}+\sum_{\mu\rho\sigma MN}X_{MN}^{\mu,\rho\sigma}(\overline{\overline{\gamma_{\mu\rho MSN\sigma\mu}\otimes\xi_{MFN}}})_{CD}-(1-\alpha)\sum_M X_M(\overline{\overline{\gamma_{MSM}\otimes\xi_{MFM}}})_{CD}\ , \tag{A1}$$

$$\begin{aligned}G^{1(b)}=&\sum_{MN\mu\rho}Y_{MN}^{\mu,\rho}|S+F|_\mu(\overline{\overline{\gamma_{\mu\rho\mu5MSN}\otimes\xi_{\mu5MFN}}}+\overline{\overline{\gamma_{\mu5MSN\rho\mu}\otimes\xi_{\mu5MFN}}})_{CD}\\&-4(1-\alpha)\sum_{MN\mu}Y_{MN}^{\mu}|S+F|_\mu(\overline{\overline{\gamma_{\mu5MSN}\otimes\xi_{\mu5MFN}}})_{CD}\ ,\end{aligned} \tag{A2}$$

$$G^{1(c)}=\tfrac{1}{8}(3+\alpha)Z_{0000}(\overline{\overline{\gamma_S\otimes\xi_F}})_{CD}\ , \tag{A3}$$

$$G^{1(d)}=[-x\alpha-\tfrac{1}{8}(1+\alpha)Z_{0000}-R\,](\overline{\overline{\gamma_S\otimes\xi_F}})_{CD}\ , \tag{A4}$$

$$G^{1(e)}=[-\tfrac{1}{8}(3+\alpha)\Delta_{SF}Z_{0000}-2(1-\alpha)T_{\Delta_{SF}}](\overline{\overline{\gamma_S\otimes\xi_F}})_{CD}\ , \tag{A5}$$

where $\sigma_S=(4,1,0,1,4)$ for $\gamma_S=(I,\gamma_\mu,\gamma_{\mu\nu},\gamma_{\mu5},\gamma_5)$, $x=-2\ln a\kappa+F_{0000}-\gamma_E+1$, $|S+F|_\mu=(S_\mu+F_\mu)(\mathrm{mod}2)$, and $\Delta_{SF}=\sum_\mu|S+F|_\mu$.

We use the following notation to simplify the expressions for the loop integrals which appear in the amplitudes above:

$$\begin{aligned}&s_\mu=\sin\phi_\mu\ ,\quad \bar{s}_\mu=\sin\phi_\mu/2\ ,\quad B=\left[4\sum_\mu\bar{s}_\mu^2\right]^{-1}\ ,\quad c_\mu=\cos\phi_\mu\ ,\quad \bar{c}_\mu=\cos\phi_\mu/2\ ,\\&F=\left[\sum_\mu s_\mu^2\right]^{-1}\ ,\quad \int_\phi=16\pi^2\int_{-\pi}^{\pi}\frac{d^4\phi}{(2\pi)^4}\ ,\end{aligned} \tag{A6}$$

where ϕ is the loop momentum and B and F originate from gluon and fermion propagators. In terms of these symbols the loop integrals are defined as

$$X_{MN}^{\mu,\rho\sigma}=\int_\phi[\bar{c}_\mu^2 s_\rho s_\sigma E_M(\phi)E_N(-\phi)BF^2-\tfrac{1}{4}\delta_{\rho\sigma}\delta_{M0}\delta_{N0}B^2]\ , \tag{A7}$$

$$X_M=\int_\phi[E_M(\phi)E_M(-\phi)-\delta_{M0}]B^2\ , \tag{A8}$$

$$Y_{MN}^{\mu,\rho}=\int_\phi i\bar{c}_\mu s_\rho BF\frac{1}{12}\sum_{j=1}^{4}\sum_{\sigma\neq\mu}E_M(\theta_{\mu\sigma}^{(j)})E_N(-\theta_{\mu\sigma}^{(j)})\ , \tag{A9}$$

$$Y_{MN}^{\mu}=\int_\phi i\bar{s}_\mu B^2\frac{1}{12}\sum_{j=1}^{4}\sum_{\sigma\neq\mu}E_M(\theta_{\mu\sigma}^{(j)})E_N(-\theta_{\mu\sigma}^{(j)})\ , \tag{A10}$$

$$Z_{0000}=\int_\phi B\ , \tag{A11}$$

$$T_\Delta=\int_\phi 2\bar{s}_1^2\bar{s}_2^2B^2(0,0,1,2+c_3,3+2c_3+c_3c_4)\quad\text{for }\Delta=(0,1,2,3,4)\ , \tag{A12}$$

$$R=\int_\phi\left[c_1\left[2s_1^2-\frac{1}{F}\right]\left[-2-2\bar{s}_1^2+\frac{1}{4B}\right]BF^2-B^2\right]\ , \tag{A13}$$

where $E_M(\phi)$ and $\theta_{\mu\nu}$ are given by

$$E_M(\phi)=\prod_\mu\frac{1}{2}[e^{-i\phi_\mu/2}+(-1)^{\tilde{M}_\mu}e^{i\phi_\mu/2}]\ , \tag{A14}$$

$$\tilde{M}_\mu=\sum_{\nu\neq\mu}M_\nu\ , \tag{A15}$$

$$\begin{aligned}&\theta_{\mu\nu}^{(1)}=\tfrac{1}{2}\phi_\mu\hat{\nu}\ ,\quad \theta_{\mu\nu}^{(2)}=\tfrac{1}{2}\phi_\mu\hat{\mu}+\phi_\nu\hat{\nu}\\&\theta_{\mu\nu}^{(3)}=\sum_{\rho=1}^{4}\phi_\rho\hat{\rho}-\theta_{\mu\nu}^{(1)}\ ,\quad \theta_{\mu\nu}^{(4)}=\sum_{\rho=1}^{4}\phi_\rho\hat{\rho}-\theta_{\mu\nu}^{(2)}\ .\end{aligned} \tag{A16}$$

The integrals $X_{MN}^{\mu,\rho\sigma}$ and $Y_{MN}^{\mu,\rho}$ were evaluated numerically by Daniel and Sheard [5]. We have confirmed their results except for some sign reversals for $Y_{MN}^{1,1}$ in their Table 3. For the calculation of the integral we employed the Monte Carlo integration routine VEGAS.

APPENDIX B: ONE-LOOP AMPLITUDES FOR COLOR TWO-LOOP FOUR-QUARK OPERATORS

Analytic expressions for one-loop diagrams in Fig. 2 are listed below for color two-loop four-quark operators of a general spin-flavor structure $(\gamma_S\otimes\xi_F)(\gamma_{S'}\otimes\xi_{F'})$. The amplitudes for the diagrams Figs. 2(a)–2(e) are products of tree and one-loop bilinear amplitudes. The diagrams in Figs. 2(f)–2(h) cannot be factorized in this way. They take the following form, where we drop the common factor $\sum_I(T^I)_{ab}(T^I)_{a'b'}$ and $g^2/16\pi^2$. External fermion lines have momenta $p=C\pi/a$, $D\pi/a$, $C'\pi/a$, $D'\pi/a$ and color indices a,b,a',b' as specified in Fig. 2:

$$
\begin{aligned}
G^{2(f)}= & -\frac{1}{4}x\sum_{\mu\rho}(\overline{\overline{\gamma_{\mu\rho S}\otimes\xi_F}}-\overline{\overline{\gamma_{S\rho\mu}\otimes\xi_F}})_{CD}(\overline{\overline{\gamma_{\mu\rho S'}\otimes\xi_{F'}}}-\overline{\overline{\gamma_{S'\rho\mu}\otimes\xi_{F'}}})_{C'D'}\\
& -\sum_{\mu\rho\sigma MN}X^{\mu,\rho\sigma}_{MN}(\overline{\overline{\gamma_{\mu\rho MS}\otimes\xi_{MF}}}-\overline{\overline{\gamma_{SM\rho\mu}\otimes\xi_{FM}}})_{CD}(\overline{\overline{\gamma_{\mu\sigma NS'}\otimes\xi_{NF'}}}-\overline{\overline{\gamma_{S'N\sigma\mu}\otimes\xi_{F'N}}})_{C'D'}\\
& +(1-\alpha)\sum_{M}X_M(\overline{\overline{\gamma_{MS}\otimes\xi_{MF}}}-\overline{\overline{\gamma_{SM}\otimes\xi_{FM}}})_{CD}(\overline{\overline{\gamma_{MS'}\otimes\xi_{MF'}}}-\overline{\overline{\gamma_{S'M}\otimes\xi_{F'M}}})_{C'D'}\ ,
\end{aligned}
\tag{B1}
$$

$$
\begin{aligned}
G^{2(g)}= & \sum_{\mu\rho LMN}U^{\mu,\rho}_{LMN}\\
& \times\{|S+F|_\mu(\overline{\overline{\gamma_{\mu 5MSL}\otimes\xi_{\mu 5MFL}}})_{CD}(\overline{\overline{\gamma_{S'N\rho\mu}\otimes\xi_{F'N}}}-\overline{\overline{\gamma_{\mu\rho NS'}\otimes\xi_{NF'}}})_{C'D'}\\
& +|S'+F'|_\mu(\overline{\overline{\gamma_{SN\rho\mu}\otimes\xi_{FN}}}-\overline{\overline{\gamma_{\mu\rho NS}\otimes\xi_{NF}}})_{CD}(\overline{\overline{\gamma_{\mu 5MS'L}\otimes\xi_{\mu 5MF'L}}})_{C'D'}\}-(1-\alpha)\sum_{\mu LMN}U^{\mu}_{LMN}\\
& \times\{|S+F|_\mu(\overline{\overline{\gamma_{\mu 5MSL}\otimes\xi_{\mu 5MFL}}})_{CD}(\overline{\overline{\gamma_{S'N}\otimes\xi_{F'N}}}-\overline{\overline{\gamma_{NS'}\otimes\xi_{NF'}}})_{C'D'}\\
& +|S'+F'|_\mu(\overline{\overline{\gamma_{SN}\otimes\xi_{FN}}}-\overline{\overline{\gamma_{NS}\otimes\xi_{NF}}})_{CD}(\overline{\overline{\gamma_{\mu 5MS'L}\otimes\xi_{\mu 5MF'L}}})_{C'D'}\}\ ,
\end{aligned}
\tag{B2}
$$

$$
\begin{aligned}
G^{2(h)}= & -\sum_{\mu KLMN}V^{\mu}_{KLMN}|S+F|_\mu|S'+F'|_\mu(\overline{\overline{\gamma_{\mu 5KSL}\otimes\xi_{\mu 5KFL}}})_{CD}(\overline{\overline{\gamma_{\mu 5MS'N}\otimes\xi_{\mu 5MF'N}}})_{C'D'}\\
& +(1-\alpha)\sum_{\mu\nu KLMN}V^{\mu\nu}_{KLMN}|S+F|_\mu(|S'+F'|)_\nu(\overline{\overline{\gamma_{\mu 5KSL}\otimes\xi_{\mu 5KFL}}})_{CD}(\overline{\overline{\gamma_{\nu 5MS'N}\otimes\xi_{\nu 5MF'N}}})_{C'D'}\ ,
\end{aligned}
\tag{B3}
$$

where $|S+F|_\mu=(S_\mu+F_\mu)(\mathrm{mod}2)$. The four types of loop integrals are defined by

$$
U^{\mu,\rho}_{LMN}=\int_\phi i\bar{c}_\mu s_\rho BF\frac{1}{12}\sum_{\nu\neq\mu}\sum_{j=1}^{4}E_L(\theta^{(j)}_{\mu\nu})E_M(\phi-\theta^{(j)}_{\mu\nu})E_N(-\phi)\ , \tag{B4}
$$

$$
U^{\mu}_{LMN}=\int_\phi i2\bar{s}_\mu B^2\frac{1}{12}\sum_{\nu\neq\mu}\sum_{j=1}^{4}E_L(\theta^{(j)}_{\mu\nu})E_M(\phi-\theta^{(j)}_{\mu\nu})E_N(-\phi)\ , \tag{B5}
$$

$$
V^{\mu}_{KLMN}=\int_\phi B\frac{1}{12}\sum_{\nu\neq\mu}\sum_{i=1}^{4}E_K(\theta^{(i)}_{\mu\nu})E_L(\phi-\theta^{(i)}_{\mu\nu})\frac{1}{12}\sum_{\lambda\neq\mu}\sum_{j=1}^{4}E_M(-\theta^{(j)}_{\mu\lambda})E_N(-\phi+\theta^{(j)}_{\mu\lambda})\ , \tag{B6}
$$

$$
V^{\mu\nu}_{KLMN}=\int_\phi 4\bar{s}_\mu\bar{s}_\nu B^2\frac{1}{12}\sum_{\lambda\neq\mu}\sum_{i=1}^{4}E_K(\theta^{(i)}_{\mu\lambda})E_L(\phi-\theta^{(i)}_{\mu\lambda})\frac{1}{12}\sum_{\sigma\neq\nu}\sum_{j=1}^{4}E_M(-\theta^{(j)}_{\nu\sigma})E_N(-\phi+\theta^{(j)}_{\nu\sigma})\ . \tag{B7}
$$

The numerical values of the last three integrals reported in Tables 4 and 5 in Ref. [6] have some minus signs missing. We have corrected the sign and evaluated the values of $U^{\mu,\rho}_{LMN}$ which are not listed in Ref. [6].

APPENDIX C: ONE-LOOP AMPLITUDES FOR COLOR ONE-LOOP FOUR-QUARK OPERATORS

Analytic expressions for one-loop diagrams in Fig. 3 are listed below for color one-loop four-quark operators of a general spin-flavor structure $(\gamma_S\otimes\xi_F)(\gamma_{S'}\otimes\xi_{F'})$. We set the momenta of the external fermion lines to $p=C\pi/a$, $D\pi/a$, $C'\pi/a$, $D'\pi/a$ and assign the color indices a,b,a',b' as shown in Fig. 3. The amplitudes of the diagrams Figs. 3(a), 3(c), and 3(d) are the same as those for the color two-loop operators except for the color factor which takes the form $\sum_I(T^I)_{ab'}(T^I)_{a'b}$ for diagram (a) and $\delta_{ab'}\delta_{a'b}$ for diagrams (c) and (d). Other amplitudes take the following form, where we drop the common factor $g^2/16\pi^2$:

$$
\begin{aligned}
G^{3(b)}= & \frac{4}{3}\delta_{ab'}\delta_{a'b}\\
& \times\Bigg[-\sum_{\mu\rho MN}\frac{1}{2}(Y^{\mu,\rho}_{M[\mu 5N]}+Y^{\mu,\rho}_{N[\mu 5M]})\\
& \quad\times\{(\overline{\overline{\gamma_{\mu\rho MS}\otimes\xi_{MF}}})_{CD}(\overline{\overline{\gamma_{S'N}\otimes\xi_{F'N}}})_{C'D'}+(\overline{\overline{\gamma_{SM\rho\mu}\otimes\xi_{FM}}})_{CD}(\overline{\overline{\gamma_{NS'}\otimes\xi_{NF'}}})_{C'D'}\\
& \quad+(\overline{\overline{\gamma_{SN}\otimes\xi_{FN}}})_{CD}(\overline{\overline{\gamma_{\mu\rho MS'}\otimes\xi_{MF'}}})_{C'D'}+(\overline{\overline{\gamma_{NS}\otimes\xi_{NF}}})_{CD}(\overline{\overline{\gamma_{S'M\rho\mu}\otimes\xi_{F'M}}})_{C'D'}\}\\
& \quad-4(1-\alpha)\sum_{\mu M}Y^{\mu}_{[\mu 5M]M}[(-1)^{\tilde{M}\cdot(S+F)}+(-1)^{\tilde{M}\cdot(S'+F')}](\overline{\overline{\gamma_{SM}\otimes\xi_{FM}}})_{CD}(\overline{\overline{\gamma_{S'M}\otimes\xi_{F'M}}})_{C'D'}\Bigg]\ ,
\end{aligned}
\tag{C1}
$$

$$
\begin{aligned}
G^{3(e)}=\frac{4}{3}\delta_{ab'}\delta_{a'b}\Bigg[&-2Z_{0000}(\overline{\overline{\gamma_S\otimes\xi_F}})_{CD}(\overline{\overline{\gamma_{S'}\otimes\xi_{F'}}})_{C'D'}\\
&+\sum_{\mu}\frac{1}{4}Z_{0000}[(-1)^{(S+F)_\mu}+(-1)^{(S'+F')_\mu}](\overline{\overline{\gamma_{\mu5S}\otimes\xi_{\mu5F}}})_{CD}(\overline{\overline{\gamma_{\mu5S'}\otimes\xi_{\mu5F'}}})_{C'D'}\\
&+(1-\alpha)\tfrac{1}{2}Z_{0000}(\overline{\overline{\gamma_S\otimes\xi_F}})_{CD}(\overline{\overline{\gamma_{S'}\otimes\xi_{F'}}})_{C'D'}\\
&-(1-\alpha)\sum_{\mu}Z_{0000}\tfrac{1}{2}[(-1)^{(S+F)_\mu}+(-1)^{(S'+F')_\mu}](\overline{\overline{\gamma_{\mu5S}\otimes\xi_{\mu5F}}})_{CD}(\overline{\overline{\gamma_{\mu5S'}\otimes\xi_{\mu5F'}}})_{C'D'}\\
&+(1-\alpha)\frac{1}{2}\sum_{\mu\neq\nu,M}T_M^{\mu\nu}\{-[(-1)^{\tilde{M}\cdot(S+F)}+(-1)^{\tilde{M}\cdot(S'+F')}](\overline{\overline{\gamma_{MS}\otimes\xi_{MF}}})_{CD}(\overline{\overline{\gamma_{MS'}\otimes\xi_{MF'}}})_{C'D'}\\
&+2[(-1)^{(S+F)_\mu+\tilde{M}\cdot(S+F)}+(-1)^{(S'+F')_\mu+\tilde{M}\cdot(S'+F')}](\overline{\overline{\gamma_{\mu5MS}\otimes\xi_{\mu5MF}}})_{CD}(\overline{\overline{\gamma_{\mu5MS'}\otimes\xi_{\mu5MF'}}})_{C'D'}\\
&-[(-1)^{(S+F)_\mu+(S+F)_\nu+\tilde{M}\cdot(S+F)}+(-1)^{(S'+F')_\mu+(S'+F')_\nu+\tilde{M}\cdot(S'+F')}]\\
&\times(\overline{\overline{\gamma_{\mu\nu MS}\otimes\xi_{\mu\nu MF}}})_{CD}(\overline{\overline{\gamma_{\mu\nu MS'}\otimes\xi_{\mu\nu MF'}}})_{C'D'}\}\Bigg]\ ,
\end{aligned}
\tag{C2}
$$

$$
\begin{aligned}
G^{3(f)}=\sum_{\mu\rho\sigma MN}\left(X^{\mu\rho\sigma}_{MN}+\frac{x}{4}\delta_{\rho\sigma}\delta_{M0}\delta_{N0}\right)\Bigg[&\frac{4}{3}\delta_{ab'}\delta_{a'b}\{(\overline{\overline{\gamma_{\mu\rho MS}\otimes\xi_{MF}}})_{CD}(\overline{\overline{\gamma_{S'N\sigma\mu}\otimes\xi_{F'N}}})_{C'D'}\\
&+(\overline{\overline{\gamma_{SM\rho\mu}\otimes\xi_{FM}}})_{CD}(\overline{\overline{\gamma_{\mu\sigma NS'}\otimes\xi_{NF'}}})_{C'D'}\}\\
&-\sum_{I}(T^I)_{ab'}(T^I)_{a'b}\{(\overline{\overline{\gamma_{\mu\rho MS}\otimes\xi_{MF}}})_{CD}(\overline{\overline{\gamma_{\mu\sigma NS'}\otimes{}_{NF'}}})_{C'D'}\\
&+(\overline{\overline{\gamma_{SM\rho\mu}\otimes\xi_{FM}}})_{CD}(\overline{\overline{\gamma_{S'N\sigma\mu}\otimes\xi_{F'N}}})_{C'D'}\}\Bigg]\\
+(1-\alpha)\sum_{M}(X_M+x\delta_{M0})(\overline{\overline{\gamma_{SM}\otimes\xi_{FM}}})_{CD}(\overline{\overline{\gamma_{S'M}\otimes\xi_{F'M}}})_{C'D'}\Bigg[&-\frac{4}{3}\delta_{ab'}\delta_{a'b}[(-1)^{\tilde{M}\cdot(S+F)}+(-1)^{\tilde{M}\cdot(S'+F')}]\\
&+\sum_{I}(T^I)_{ab'}(T^I)_{a'b}[1+(-1)^{\tilde{M}\cdot(S+F+S'+F')}]\Bigg]\ ,
\end{aligned}
\tag{C3}
$$

$$
\begin{aligned}
G^{3(g)}=\sum_{I}(T^I)_{ab'}(T^I)_{a'b}\Bigg[\sum_{\mu\rho LMN}\frac{1}{2}(U^{\mu,\rho}_{[\mu5L]MN}-U^{\mu,\rho}_{[\mu5M]LN}\Bigg\{&(\overline{\overline{\gamma_{LSN\rho\mu}\otimes\xi_{LFN}}})_{CD}(\overline{\overline{\gamma_{S'M}\otimes\xi_{F'M}}})_{C'D'}\\
&+(\overline{\overline{\gamma_{SM}\otimes\xi_{FM}}})_{CD}(\overline{\overline{\gamma_{LS'N\rho\mu}\otimes\xi_{LF'N}}})_{C'D'}\\
&+(\overline{\overline{\gamma_{\mu\rho NSL}\otimes\xi_{NFL}}})_{CD}(\overline{\overline{\gamma_{MS'}\otimes\xi_{MF'}}})_{C'D'}\\
&+(\overline{\overline{\gamma_{MS}\otimes\xi_{MF}}})_{CD}(\overline{\overline{\gamma_{\mu\rho NS'L}\otimes{}_{NF'L}}})_{C'D'}\Bigg\}\\
-(1-\alpha)\sum_{\mu LMN}\frac{1}{2}(U^{\mu}_{[\mu5L]MN}-U^{\mu}_{[\mu5M]LN})\{&(\overline{\overline{\gamma_{SM}\otimes\xi_{FM}}})_{CD}(\overline{\overline{\gamma_{LS'N}\otimes\xi_{LF'N}}})_{C'D'}\\
&+(\overline{\overline{\gamma_{MS}\otimes\xi_{MF}}})_{CD}(\overline{\overline{\gamma_{NS'L}\otimes\xi_{NF'L}}})_{C'D'}\\
&+(\overline{\overline{\gamma_{LSN}\otimes\xi_{LFN}}})_{CD}(\overline{\overline{\gamma_{S'M}\otimes\xi_{F'M}}})_{C'D'}\\
&+(\overline{\overline{\gamma_{NSL}\otimes\xi_{NFL}}})_{CD}(\overline{\overline{\gamma_{MS'}\otimes\xi_{MF'}}})_{C'D'}\}\Bigg]\ ,
\end{aligned}
\tag{C4}
$$

$$G^{3(h)}=\sum_I (T^I)_{ab'}(T^I)_{a'b}\left[\sum_{\mu KLMN}\frac{1}{4}(\overline{\overline{\gamma_{KSN}\otimes\xi_{KFN}}})_{CD}(\overline{\overline{\gamma_{MS'L}\otimes\xi_{MF'L}}})_{C'D'}\{[(-1)^{(S+F)_\mu}+(-1)^{(S'+F')_\mu}]V^\mu_{KLMN} - [1+(-1)^{(S+F+S'+F')_\mu}]V^\mu_{[\mu 5K]L[\mu 5M]N}\}\right.$$
$$\left.+(1-\alpha)\sum_{\mu\nu KLMN}\frac{1}{4}(\overline{\overline{\gamma_{LSM}\otimes\xi_{LFM}}})_{CD}(\overline{\overline{\gamma_{NS'K}\otimes\xi_{NF'K}}})_{C'D'}\{V^{\mu\nu}_{K[\mu 5L]M[\nu 5N]}+V^{\mu\nu}_{L[\mu 5K]N[\nu 5M]} - V^{\mu\nu}_{K[\mu 5L]N[\nu 5M]}-V^{\mu\nu}_{L[\mu 5K]M[\nu 5N]}\}\right] . \tag{C5}$$

The new integral $T^{\mu\nu}_M$ is defined by

$$T^{\mu\nu}_M=\int_\phi \bar{s}^2_\mu \bar{s}^2_\nu B^2\left[\frac{1}{6}\sum_{j=1}^{3}E_M(\psi^{(j)}_{(\mu\nu)})E_M(-\psi^{(j)}_{(\mu\nu)})+\frac{1}{2}\delta_{M0}\right] , \tag{C6}$$

where

$$\psi^{(1)}_{(\mu\nu)}=\phi_\rho\hat{\rho} , \quad \psi^{(2)}_{(\mu\nu)}=\phi_\sigma\hat{\sigma} , \quad \psi^{(3)}_{(\mu\nu)}=\phi_\rho\hat{\rho}+\phi_\sigma\hat{\sigma} , \tag{C7}$$

the components μ, ν, ρ, and σ are all different.

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[16] We thank Steve Sharpe and Aproova Patel for valuable correspondence on this point.
[17] To obtain this result we followed the procedure of Buras and Weisz [A. Buras and P. Weisz, Nucl. Phys. **B333**, 69 (1990)] and decomposed the one-loop amplitudes for four-quark operators as a sum of two terms, or proportional to $\Gamma\otimes\Gamma=\gamma_\mu(1-\gamma_5)\otimes\gamma_\mu(1-\gamma_5)$ and the other proportional to $E=\mathcal{R}_{\alpha\beta}\Gamma\otimes\Gamma\mathcal{R}_{\beta\alpha}$ with $\mathcal{R}_{\alpha\beta}=\gamma_\alpha\gamma_\beta\otimes I-I\otimes\gamma_\alpha\gamma_\beta$. The operator E is independent of the left-left operator $\Gamma\otimes\Gamma$ in general dimensions, vanishing in four dimensions. It can be ignored at one loop, but has to be retained at two-loop order and beyond.