

## COMMENTS

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### Comment on “Charging of dust grains in a plasma with negative ions” [Phys. Plasmas 10, 1518 (2003)]

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The shortcoming in the expression of approximated negative ions current to negatively charged dust grains in the case of streaming negative ions distribution by Mamun and Shukla [Phys. Plasmas **10**, 1518 (2003)] is pointed out. Improved estimation in dust grain charging current in the retarding field is presented in the case of streaming dusty plasmas, where the particles streaming velocity is much larger than their thermal velocity. © 2003 American Institute of Physics.  
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In a recent paper, Mamun and Shukla<sup>1</sup> have presented an investigation on the role of negative ions on negatively charged dust grain surface potentials in dusty plasmas. The negative ions are thus in the retarding sheath potential of the dust grain. Two types of negative ions distributions are considered: (1) streaming negative ions and (2) Boltzmannian negative ions. This investigation is important in the trend of current progress of dusty plasma physics. However, in the derivation of negative ions current to the dust grain [Eq. (3) of Ref. 1], when the negative ions streaming velocity ( $v_o$ ) is much larger than the negative ion thermal velocity ( $v_{thn}$ ), they have ignored the lower limit of the integration of particle velocity. This lower limit of integration is  $\sqrt{(2q\phi_d/m)}$  and is the minimum velocity of the particles, which is needed to reach the dust grain by overcoming the retarding sheath potential of the dust grain. The quantities  $q$  and  $m$  are the charge and mass of the particles, respectively, and  $\phi_d$  is the dust grain surface potential. Mamun and Shukla<sup>1</sup> have shown that at the dust grain surface, the ratio of  $\sqrt{(2q\phi_d/m)}$  and  $v_{thn} = (\sqrt{(2k_B T/m)})$  for electrons, i.e.,  $|e\phi_d/k_B T_e| > 2$ . In low temperature laboratory dusty plasmas, negative ions may be multiply charged ( $Z_n e$ ) and their temperature ( $T_n$ ) is much lower than the electron temperature ( $T_e$ ).<sup>2</sup> Here,  $Z_n$  is the number of charge. Therefore, for negative ions the ratio becomes  $|Z_n e \phi_d / k_B T_n| \gg |e \phi_d / k_B T_e|$ . A condition for the velocity of streaming negative ions can be written as  $\sqrt{(2q_n \phi_d / m_n)} > v_o \gg v_{thn}$ . According to this condition, all the streaming negative ions with  $v_o \gg v_{thn}$  cannot graze the dust grain by overcoming the retarding sheath potential of the grain. Therefore, the lower limit of the velocity integration for negative ions is important even when  $v_o \gg v_{thn}$ . Our

objective here is to present the dust grain charging current in a streaming dusty plasma ( $v_o \gg v_{th}$ ) considering the lower limit of velocity integration in the retarding sheath potential of the dust grain.

Dust grain charging currents for the nonstreaming and streaming dusty plasmas are derived considering the proper limit of velocity integration ( $\sqrt{2q\phi_d/m}$  to  $\infty$ ) in the retarding field. In the nonstreaming dusty plasma, the obtained result is exactly consistent with other results.<sup>1,3,4</sup> If the particles streaming velocity is arbitrary, then the dust grain charging current expressions are much more complicated.<sup>5</sup> However, in the limit  $v_o \gg v_{th}$ , the approximate expression of particles current to dust grain can be readily obtained by integrating over function distributions. In this limiting case, we have obtained improved expressions of dust grain charging current compared to Eq. (3) of Ref. 1 in a retarding field. Finally, particles currents are derived in the nonstreaming and streaming dusty plasmas when the limit of velocity integration is extended from 0 to  $\infty$ . Dust charging current integral in the retarding field becomes the current integral for the thick sheath accelerating field when the limit is extended from 0 to  $\infty$ .<sup>6</sup> The particles motion to dust grain is then said to be orbit limited (OL). The obtained current expressions are exactly consistent with other results in the case of OL motion of particles in the accelerating field.<sup>3,4</sup> These suggest that our model for the derivation of dust grain charging currents in the case of nonstreaming and streaming dusty plasmas is correct from both the mathematical and physical points of view.

To derive expressions for charged particle currents ( $I$ ) to dust grain as a function of  $\phi_d$ , we have considered a spherical dust grain immersed in plasma. In this derivation, we have taken advantage of the procedure in the deduction of electric probe formulas proposed by Medicus<sup>7</sup> and, moreover, have followed the book in Ref. 6. The current to the

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dust grain for the particles with uniform initial velocities  $v$  is given by

$$I = \pi q \int p_g^2 v f(v) dv, \quad (1)$$

where  $\pi p_g^2$  is the ‘‘collision’’ cross section of the spherical dust grain for particles with the velocities  $v$ .  $p_g$  is the impact parameter of a particle which grazes the dust grain and  $f(v)$  is the velocity distribution of the particles.

The impact parameter  $p_g$  for which the particles reach the dust grain is given by<sup>6</sup>

$$p_g^2 = r_d^2 (1 - 2q\phi_d/mv^2). \quad (2)$$

In the retarding field, the particles with  $p_g = r_d$  reach the dust grain, provided their velocity is in the following range:

$$v \geq (2q\phi_d/m)^{1/2}. \quad (3)$$

Using the velocity limit [Eq. (3)] and the value of  $p_g$  [Eq. (2)], we get the current to dust grain in the retarding field from Eq. (1),

$$I_r = \pi r_d^2 q \int_{(2q\phi_d/m)^{1/2}}^{\infty} (1 - 2q\phi_d/mv^2) v f(v) dv. \quad (4)$$

We now apply the result [Eq. (4)] to Maxwellian distribution of particles,

$$f(v) = 4\pi n_o (m/2\pi k_B T)^{3/2} v^2 \exp(-mv^2/2k_B T), \quad (5)$$

where  $n_o$  is the density of the particles.

The straightforward integration of Eq. (4) yields the charged particles current to dust grain in the retarding field,

$$I_r = \pi r_d^2 q n_o (8k_B T/\pi m)^{1/2} \exp(-q\phi_d/k_B T). \quad (6)$$

This equation is exactly consistent with other results,<sup>1,3,4</sup> which are the dust grain charging current due to electrons when the dust grains are negatively charged.

Now we consider that the lighter charged particles are streaming with respect to dust grain with a speed  $v_o$ , where  $v_o \gg v_{th}$ . In this limiting case, the particles current to dust grain in retarding field of Eq. (4) becomes

$$I_{rs} = \pi r_d^2 q \left(1 - \frac{2q\phi_d}{mv_o^2}\right) v_o \int_{(2q\phi_d/m)^{1/2}}^{\infty} f(v) dv. \quad (7)$$

The Maxwellian distribution of Eq. (5) can be used for Eq. (7), since in the limit  $v_o \gg v_{th}$ , the distribution function would have a circular contours displace from the origin by  $v_o$ .

The particles current to dust grain in the retarding field for the streaming particles, ( $v_o \gg v_{th}$ ), is then obtained from Eqs. (5) and (7):

$$I_{rs} = \pi r_d^2 q n_o v_o \left(1 - \frac{2q\phi_d}{mv_o^2}\right) \left(\frac{4k_B T}{\pi q\phi_d}\right)^{1/2} \left(1 + \frac{q\phi_d}{k_B T}\right) \times \exp\left(-\frac{q\phi_d}{k_B T}\right). \quad (8a)$$

Now, we consider the two limiting cases  $|q\phi_d/k_B T| > 1$  and  $|q\phi_d/k_B T| < 1$ , then Eq. (8a) becomes

$$I_{rs} \approx \pi r_d^2 q n_o v_o \left(1 - \frac{2q\phi_d}{mv_o^2}\right) \left(\frac{4q\phi_d}{\pi k_B T}\right)^{1/2} \times \exp\left(-\frac{q\phi_d}{k_B T}\right), \quad \text{when } \left|\frac{q\phi_d}{k_B T}\right| > 1, \quad (8b)$$

and

$$I_{rs} \approx \pi r_d^2 q n_o v_o \left(1 - \frac{2q\phi_d}{mv_o^2}\right) \left(\frac{4k_B T}{\pi q\phi_d}\right)^{1/2}, \quad \text{when } \left|\frac{q\phi_d}{k_B T}\right| < 1. \quad (8c)$$

Equation (8) is the improved approximated dust grain charging current by streaming charged particles,  $v_o \gg v_{th}$ , in the retarding sheath potential of dust grain.

Dust charging current integrals in the retarding field [Eqs. (4) and (7)] become the current integrals for the thick sheath accelerating field when the limit of velocity integration is extended from 0 to  $\infty$ .<sup>6</sup> The particles motion to dust grain is then said to be orbit limited (OL). Thus, following the same procedure as before, particles current to dust grain in nonstreaming and streaming dusty plasmas with this limit (0 to  $\infty$ ) can be derived, which are, respectively,

$$I_a = \pi r_d^2 q n_o (8k_B T/\pi m)^{1/2} (1 - q\phi_d/k_B T), \quad (9)$$

$$I_{as} = \pi r_d^2 q n_o v_o (1 - 2q\phi_d/mv_o^2). \quad (10)$$

The obtained current expressions are exactly consistent with other results in the case of OL motion of particles to dust grain in an accelerating field.<sup>3,4</sup>

In summary, we have derived the dust grain charging current expressions in the retarding field for the nonstreaming dusty plasma [Eq. (6)] and for the streaming dusty plasma when  $v_o \gg v_{th}$  [Eq. (8)] considering the proper limit of velocity integration ( $\sqrt{2q\phi_d/m}$  to  $\infty$ ). In the nonstreaming dusty plasma, the obtained result is exactly consistent with other results.<sup>1,3,4</sup> In the streaming dusty plasma, improved approximated dust grain charging current of Eq. (8) is obtained.

Finally, particles currents are derived in the nonstreaming and streaming dusty plasmas when the limit of velocity integration is extended from 0 to  $\infty$ . Dust charging current integrals in the retarding field [Eqs. (4) and (7)] become the current integrals for the thick sheath accelerating field when the limit is extended from 0 to  $\infty$ .<sup>6</sup> Then the particles motion to dust grain is said to be orbit limited (OL). The obtained current expressions [cf. Eqs. (9) and (10)] are exactly consistent with the other results in the case of OL motion of particles to dust grain in an accelerating field.<sup>3,4</sup> These suggest that our model for the derivation of dust grain charging currents in the retarding sheath potential of the grain in the case of nonstreaming and streaming dusty plasmas is correct from both the mathematical and physical points of view. The current expression [Eq. (10)] for streaming dusty plasma in the accelerating field is similar to Eq. (3) of Ref. 1, but four times lower than Eq. (3). Probably, the factor 4 in Eq. (3) of Ref. 1 is a mistake. Equation (3) of Ref. 1 is the negative ions current to negatively charged dust grain. Thus, Mamun

and Shukla<sup>1</sup> have used the dust grain charging current in an accelerating field to the case of the retarding field.

Therefore, the improved approximated dust grain charging current of Eq. (8) due to charged particles, in the case of the retarding sheath potential of dust grain, should be used to understand the role of negative ions—low temperature and highly charged heavy particles compared to electrons—in dust charging related physical phenomenon in streaming dusty plasmas, where  $v_o \gg v_{th}$ .

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