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INTERACTIVE INFINITE MARKOV PARTICLE SYSTEMS WITH JUMPS

By
Seiji Hiraba

Abstract. In [2] we investigated independent infinite Markov particle systems as measure-valued Markov processes with jumps, and we gave sample path properties and martingale characterizations. In particular, we investigated the exponent of Hölder-right continuity in case that the motion process is absorbing $\alpha$-stable motion on $(0, \infty)$ with $0 < \alpha < 2$, that is, time-changed absorbing Brownian motions on $(0, \infty)$ by the increasing $\alpha/2$-stable Lévy processes.

In the present paper we shall extend the results to the case of simple interactive infinite Markov particle systems. We also consider the absorbing stable motion on a half space $H = \mathbb{R}^{d-1} \times (0, \infty)$ as a motion process.

1. Settings and Previous Results

In this section we give the general settings and the known results which are given in [2] in order to describe the main results in §3 and §4.

Let $S$ be a domain of $\mathbb{R}^d$. Let $(\nu(t), P_x)_{t \geq 0, x \in S}$ be a $S$-valued Markov process having life time $\xi(w) \in (0, \infty)$ such that $w \in \mathcal{D}([0, \xi(w)) \to S)$, i.e., $w : [0, \xi(w)) \to S$ is right continuous and has left-hand limits. For convenience, we fix an extra point $\Delta \notin S$ and set $\nu(t) = \Delta$ if $t \geq \xi(w)$. Moreover we shall extend functions $f$ on $S$ to on $\{\Delta\}$ by $f(\Delta) = 0$, if necessary.

We use the following notations: Let $S \subset \mathbb{R}^d$ be a domain.

- If $x = (x_1, \ldots, x_d) \in \mathbb{R}^d$, then $\delta^{k}_{i \rightarrow \Delta} = \partial^k/(\partial x_{i_1} \cdots \partial x_{i_k})$, $\delta^k_i = \partial^k/(\partial x_i)$ and $\delta_i = \partial/\partial t$ for each $k = 0, 1, \ldots$, $i = 1, \ldots, d$. Moreover $\delta_i = \partial/\partial t$ for time $t \geq 0$. 

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$f \in C_c \equiv C_c(S) \quad \text{if} \quad f$ is a continuous function on $S$ with compact support in $S$, and $C^\infty_c = C^\infty_c(S) := C_c(S) \cap C^\infty(S)$.

- For each integer $k \geq 0$, $C^k_b := C^k_b(\mathbb{R}^d)|_S$, that is, $f \in C^k_b$ is a restriction to $S$ of $k$-times continuously differentiable function on $\mathbb{R}^d$ with bounded derivatives of order between 0 and $k$. Moreover $f \in C_0 \quad \text{if} \quad f$ is continuous on $S$ and $f(x) \to 0$ whenever $x \to \partial S$ or $|x| \to \infty$. Furthermore $C_b := C^0_b$, $C^\infty_b := \bigcap_k C^k_b$, $C^k := C^k \cap C^k_b$ and $C^\infty := \bigcap_k C^\infty_k$.

- For a function space $D$ on $S$, $f \in D^+ \quad \text{if} \quad f \in D$, $f \geq 0$.

- $\langle \mu, f \rangle := \int_S f(x)\mu(dx)$ for a function $f$ on $S$ and a measure $\mu$ on $S$.

The following two assumptions are the same as in [2].

**ASSUMPTION 1.** Let $(P_t)_{t \geq 0}$ be the transition semigroup of $(w(t), P_x)$, i.e., $P_t f(x) = E_x[f(w(t)) : t < \zeta]$.

(i) $(P_t)$ is a strongly continuous nonnegative contraction semigroup on $(C_0, \| \cdot \|_\infty)$ with generator $(A, \mathcal{D}(A))$, where $\| f \|_\infty = \sup_{x \in S} |f(x)|$.

(ii) $C^\infty_c \subset \mathcal{D}(A)$ and there is a strictly positive function $g_0 \in C^\infty_c$ such that $g_0 \in \mathcal{D}(A)$ and that $g_0^{-1} A f \in C_b$ with $g_0^{-1} = 1/g_0$ for every $f \in C^\infty_c$ and $f = g_0$.

(iii) $\sup_{t \leq T} \| g_0^{-1} P_t g_0 \|_\infty < \infty$ for every $T > 0$.

Under this assumption we introduce a function space $D_{g_0} \subset \mathcal{D}(A)$ as follows:

$f \in D_{g_0} \quad \text{if} \quad f \in \mathcal{D}(A) \quad \text{such that} \quad \| g_0^{-1} f \|_\infty < \infty$ and $\| g_0^{-1} A f \|_\infty < \infty$.

Clearly $g_0 \in D_{g_0}$ and $C^\infty_c \subset D_{g_0}$. Moreover since $C^\infty_c$ is dense in $C_0$ and $P, C^\infty_c \subset D_{g_0}$ for every $t \geq 0$, $D_{g_0}$ is a core for $A$. However, $D_{g_0}$ may be too large, so we further need the following assumption:

**ASSUMPTION 2.** There exist a bounded function $g_1 \in C^\infty_c; \ g_1 \geq g_0(> 0)$ and a core $D \subset D_{g_0}$ (we denote $D = D_g$ with $g = (g_0, g_1)$) satisfying the following:

(i) If $f \in D_g$, then $\lim_{t \downarrow 0} \frac{1}{t} (P_t f^2(x) - f(x)^2)$ exists for each $x \in S$ (we also denote the limit as $A f^2(x)$), $\partial_t P_t f^2(x) = A P_t f^2(x) = P_t A f^2(x)$ ($\to A f^2(x)$ as $t \downarrow 0$ for each $x \in S$), $A f^2 \in C_b$ and $\| g_1^{-1} A f^2 \|_\infty < \infty$.

(ii) For each $T > 0$, $\sup_{t \leq T} \| g_1^{-1} P_t g_1 \|_\infty < \infty$.

(iii) For each $0 < s < T$, $\sup_{t \leq s} \| g_0^{-1} P_t g_1 \|_\infty < \infty$.

(iv) There exist constants $0 \leq \gamma < 1, \delta > 0$ such that $\sup_{0 \leq t \leq \delta} t^{\gamma} \| g_0^{-1} P_t g_1 \|_\infty < \infty$.

(v) $g_0 \in D_g$. 
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All through the present paper we suppose that Assumption 1 and 2 are fulfilled. We shall sometime use the notation \( \| \cdot \|_{g_0} = \| \cdot / g_0 \|_{\infty} \).

Let \( \mathscr{M}_{g_0} = \mathscr{M}_{g_0}(S) \) be a space of counting measures on \( S \) defined as

\[ \mu \in \mathscr{M}_{g_0} \iff \mu = \sum_n \delta_{x_n} \text{ such that } \langle \mu, g_0 \rangle < \infty \text{ and} \]

\[ \mu_n \to \mu \quad \text{in } \mathscr{M}_{g_0} \iff \sup \langle \mu_n, g_0 \rangle < \infty, \]

\[ \langle \mu_n, f \rangle \to \langle \mu, f \rangle \quad \text{for all } f \in C_C \text{ and } f = g_0, \]

where \( C_C = C_c(S) \) denotes the space of all continuous functions with compact supports on \( S \). Then \( \mathscr{M}_{g_0} \) is metrizable and separable.

We mainly consider the case that the generator has the form

\[ A = A^c + A^d, \]

with

\[ A^c f(x) = \frac{1}{2} \sum_{i,j=1}^d a^{ij}(x) \partial^2_{ij} f(x) + \sum_{i=1}^d b^i(x) \partial_i f(x), \]

\[ A^d f(x) = \int_{S \setminus \{x\}} \left[ f(y) - f(x) - \nabla f(x) \cdot (y - x) I(|y - x| < 1) \right] \nu(x, dy) \]

\[ - k(x) f(x) + \sum_{i=1}^d c^i(x) \partial_i f(x) \]

for \( f \in D_B \), where \( a^{ij}, b^i \in C_b(S) \), \( (a^{ij}) \) is positive definite, \( k(x) \geq 0 \) denotes the killing rate by jumps, \( (c^i(x)) \) depends on jumps, and \( \nu(x, dy) \) is the Lévy kernel on \( S \times (S \setminus \{x\} \) satisfying that

\[ \sup_{x \in S} \int_{S \setminus \{x\}} (1 \wedge |y - x|^2) \nu(x, dy) < \infty. \]

Independent IMPS; Results in [2]

Let \( (X_t, P_\mu) \) be an (indistinguishable) independent Markov particle system (IMP) associated with the motion process \( (w(t), P_\mu) \), i.e., for many independent motions \( (w_n(t), P_{\mu_n}) \overset{(d)}{=} (w(t), P_\mu), \)

\[ X_t = \sum_n \delta_{w_n(t)} I_S \quad \text{if } \mu = \sum_n \delta_{x_n} \text{ on } S, \text{ and } P_\mu = \prod_n P_{\mu_n}. \]
The generator $\mathcal{L}_0$ of this particle system is given by the following: for $f \in C_c^\infty$,

$$\mathcal{L}_0 e^{-\langle \xi, f \rangle}(\mu) = -\langle \mu, e^f A(1 - e^{-f}) \rangle e^{-\langle \mu, f \rangle}$$

$$= -\langle \mu, A f - \Gamma f \rangle e^{-\langle \mu, f \rangle},$$

where $\Gamma f := Af - e^f A(1 - e^{-f})$. In fact, let $\{\mathcal{F}_t\}$ be the filtration generated by $\{X_t\}$ and set

$$V_tf(x) = -\log P_x[\exp -f(w(t))] = -\log \{1 - P_t(1 - e^{-f})(x)\}.$$ 

We have that if $0 \leq s < t$, then

$$E_\mu[e^{-\langle X_t, V_t \rangle} | \mathcal{F}_s] = \exp[-\langle X_s, V_{t-s}f \rangle].$$

It is easy to see that $(V_t)_{t \geq 0}$ is a nonnegative contraction semigroup on $C_0$ and that by (ii) of Assumption 1 if $f \in C_c^\infty$, then $1 - e^{-f} \in C_c^\infty \subset D_g$, hence we have

$$\partial_t V_t f = \frac{P_t A(1 - e^{-f})}{1 - P_t(1 - e^{-f})} = \frac{AP_t(1 - e^{-f})}{1 - P_t(1 - e^{-f})} = e^{V_t} A(1 - e^{-V_t f})$$

$$\rightarrow e^f A(1 - e^{-f}) = Af - \Gamma f \quad (t \downarrow 0).$$

Note that since $V_t f \leq P_t f$ (by Jensen's inequality), $\Gamma$ is nonnegative;

$$\Gamma f = Af - \partial_t V_t f |_{t=0+} = \lim_{t \downarrow 0} \frac{1}{t} [(P_t f - f) - (V_t f - f)] \geq 0$$

and that for each $f \in C_c^\infty$, $v_t = V_t f$ is the unique solution to the following equation:

$$\partial_t v_t = e^h A(1 - e^{-v_t}), \quad v_0 = f$$

(because $u_t := 1 - e^{-v_t}$ satisfies $\partial_t u_t = Au_t$, $u_0 = 1 - e^{-f}$ and the unique solution is given as $u_t = P_t(1 - e^{-f})$). Moreover if $Av_t(x)$ is well-defined for $t > 0$, $x \in S$, then

$$\partial_t v_t = Au_t - \Gamma v_t, \quad v_0 = f.$$ 

By using the Markov property and by induction we have

**Proposition 1 (Prop. 1 in [1]).** For every $0 \leq t_1 \leq \cdots \leq t_n$ and $f_i \in D_g^+$, $i = 1, 2, \ldots, n,$
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\[ E_\mu[\langle X_{t_1}, f_1 \rangle \cdots \langle X_{t_n}, f_n \rangle] \]

\[ \leq \prod_{i=1}^{n} \langle \mu, P_i f_i \rangle + C_1^{(n)} \sum_{i=1}^{n} \prod_{j \neq i} \langle \mu, P_j f_j \rangle \]

\[ + C_2^{(n)} \sum_{i, \neq i} \prod_{j=i, i} \langle \mu, P_j f_j \rangle + \cdots + C_{n-1}^{(n)} \sum_{j=1}^{n} \langle \mu, P_j f_j \rangle + C_n^{(n)}, \]

where \( C_k^{(n)}, k = 1, \ldots, n \) are positive constants, depending on \( (n, \| f \|_\infty, t\leq n). \)

We introduce a non-negative operator \( Q \) as \( Qf = Af^2 - 2Af \) for \( f \in D_q \), which is well-defined by (i) of Assumption 2 and plays an important role to investigate the exponents of Hölder (right) continuity. The non-negativity follows from \( (P_i f^2 - f^2)^2 - 2f(P_i f - f)^2 + f^2 \) for \( f \in D_q \).

**Theorem 1** (Th. 2.3 and Cor. 2.1 in [2]). Let \( (w(t), P_t) \) be a discontinuous Markov process in \( (\mathbb{D}(\{0, \xi(w)\}) \to \mathbb{S}) \) with transition semigroup \( (P_t) \) satisfying Assumption 1 and 2. Let \( \mu \in \mathbb{M}_{\mathbb{S}} \). The following holds with \( P_\mu \)-probability one.

(i) \( \{\langle X_t, g_0 \rangle \} \) is \( (1-\gamma)/2 - \varepsilon \)-Hölder right continuous at \( t = 0 \) for sufficiently small \( \varepsilon > 0 \), where the constant \( 0 \leq \gamma < 1 \) is in (iv) of Assumption 2.

(ii) If \( \langle \mu, g_1 \rangle < \infty \), in particular, if \( g_1(x) = g_0(x) \) then \( \{\langle X_t, g_0 \rangle \} \) is \( (1/2 - \varepsilon) \)-Hölder right continuous at \( t = 0 \) for sufficiently small \( \varepsilon > 0 \).

(iii) For each fixed \( t_0 > 0 \), \( \{\langle X_t, g_0 \rangle \} \) is \( (1/2 - \varepsilon) \)-Hölder right continuous at \( t = t_0 \) for sufficiently small \( \varepsilon > 0 \).

2. Sampling Replacement Markov Particle Systems

Let \( \mu = \sum_n \delta_{x_n} \in \mathbb{M}_{\mathbb{S}} \). Let \( (Y_t, P_t^\mu) \) be a sampling replacement Markov particle system associated with the motion process \( (w(t), P_t) \), sampling replacement rate \( \lambda > 0 \) and sampling replacement probability \( q(d(m,n)) = \sum_{k,l} p_{k,l} \delta_{(k,l)}(d(m,n)) \) on \( \mathbb{N}^2 \), where \( p_{k,l} \geq 0, p_{k,k} = 0 \) and \( \sum p_{k,l} = 1 \). Each particle first moves independently each other. After a \( \lambda \)-exponential random time, two particles are selected randomly, for example, \( m \)-th and \( n \)-th particles are selected with probability \( p_{m,n} \), and at that time the \( m \)-th particle jumps to the place of the \( n \)-th particle. Then the \( m \)-th particle moves independently. And these operations are continued. We denote each particle by \( Y_n^*(t) \) such that \( Y_n^*(0) = x_n \), and hence \( Y_t = \sum_n \delta_{Y_n^*(t)} \). Note that if \( (P_t) \) is non-conservative, then it is possible that the dead particles come life again.
Recall \((X_t, \mathbf{P}_\mu)\) is the independent Markov particle system with the motion process \((\omega(t), \mathbf{P}_\mu)\). For \(f \in D_g\), set \(L^Y_t(\mu) = E^Y_\mu[\exp - \langle Y_t, f \rangle]\) and \(L_t(\mu) = E^Y_\mu[\exp - \langle X_t, f \rangle]\). Then
\[
L_t(\mu) = e^{-\langle \mu, \omega(t) \rangle} \quad \text{with} \quad V_t f(x) = -\log E_x[\exp - f(\omega(t))] = -\log(1 - P_t(1 - e^{-f})).
\]
It is easy to see that \(L^Y_t(\mu)\) satisfies the following equation:
\[
L^Y_t(\mu) = e^{-\lambda t} L_t(\mu) + \lambda \int_0^t ds e^{-\lambda s} \int_{\mathbb{N}^2} q(d(m, n)) \mathbf{P}_s(\Theta_{m,n} L_{t-s})(\mu),
\]
where \((\mathbf{P}_t)\) is the transition semigroup of \((X_t, \mathbf{P}_\mu)\) and \(\Theta_{m,n}\) is an operator such that it makes the \(m\)-th particle jump to the place of \(n\)-th particle of \(\mu = \sum \delta_k \in \mathcal{M}_{\geq 0}\) on a class of all functions \(F(\mu)\) and it is defined by \(\Theta_{m,n} F(\mu) = F(\mu^{m,n})\) with \(\mu^{m,n} = \mu - \delta_{x_m} + \delta_{x_n}\). Note that \(P_t \Theta_{m,n} = \Theta_{m,n} P_t\) holds. The solution is given as
\[
(2.1) \quad L^Y_t(\mu) = T_t e^{-\langle \cdot, \omega(t) \rangle}(\mu) \quad \text{with} \quad T_t = \sum_k e^{-\lambda t} \frac{(\lambda t)^k}{k!} \left( \int_{\mathbb{N}^2} q(d(m, n)) \Theta_{m,n} \right)^k,
\]
where \(T_t\) is an operator on a class of functions \(F(\mu)\) with polynomial growth of \(\langle \mu, f_1 \rangle, \langle \mu, f_2 \rangle, \ldots, \langle \mu, f_n \rangle\) \((f_i \in D_g)\) and
\[
\left( \int_{\mathbb{N}^2} q(d(m, n)) \Theta_{m,n} \right)^k F(\mu) = \left( \int_{\mathbb{N}^2} q(d(m, n)) \Theta_{m,n} \right)^{k-1} \sum_{m,n \in \mathbb{N}} p_{m,n} F(\mu^{m,n}).
\]

The generator \(\mathcal{L}^Y\) of this particle system is given by the following: for \(f \in C_c^{\infty}\),
\[
\mathcal{L}^Y e^{-\langle \cdot, f \rangle}(\mu) = \mathcal{L}^Y_0 e^{-\langle \cdot, f \rangle}(\mu) + \lambda \int (e^{-\langle \mu^{m,n} f \rangle} - e^{-\langle \mu, f \rangle}) q(d(m, n))
\]
\[
= -\left\{ \langle \mu, Af - \Gamma f \rangle + \lambda \langle \delta_{x_m} - \delta_{x_m}, f \rangle q(d(m, n)) \right\}
\]
\[
= -\lambda \left\{ (e^{-\langle \delta_{x_m} - \delta_{x_m}, f \rangle} - 1 + \langle \delta_{x_m} - \delta_{x_m}, f \rangle) q(d(m, n)) \right\} e^{-\langle \mu, f \rangle}
\]
(more general formula of \(\mathcal{L}^Y F(\mu)\) is given in §5). We have the following result.
Recall that we denote the particles of \(Y_t\) by \(w_n^a(i)\), i.e., \(Y_t = \sum \delta_{w_n^a(i)}\). Note that \(w_n^a(i)\) moves like as \(w_n(i)\) during the jump times.
**Theorem 2** (Semi-martingale Representation of $Y_t$). Under Assumptions 1 and 2 for $(P_t)$, if $\mu \in \mathcal{M}_{0\alpha}$, then $(Y_t, P^Y_t)$ is an $\mathcal{M}_{\alpha 0}$-valued Markov process with sample paths in $\mathbb{D}(\mathbb{R})$ satisfying the following:

(i) $\langle Y_t, \gamma_0 \rangle$ has the same exponent of Hölder right continuity as in Theorem 1.

(ii) If the motion process $(w(t), P_t)$ has generator $A$ of the form as in (1.1), then for $f \in D_0$,

$$
\langle Y_t, f \rangle = \langle Y_0, f \rangle + \int_0^t \left\{ \langle Y_s, Af \rangle + \lambda \left[ \langle \delta_w(s) - \delta_{w_0}(s), f \rangle q(d(m,n)) \right] \right\} ds
+ M^c_t(f) + M^d_t(f),
$$

where

$$M^c_t(f) \text{ is a continuous } L^2\text{-martingale}
$$

with quadratic variation $\langle \langle M^c(f) \rangle \rangle_t = \int_0^t \langle Y_s, Q^c f \rangle ds = 2 \int_0^t \langle Y_s, \Gamma^c f \rangle ds$ and

$$M^d_t(f) = \int_0^t \int_{\mathbb{R}^2} \langle \mu, f \rangle \tilde{N}(ds, d\mu) \text{ is a purely discontinuous martingale}
$$

where $\tilde{N} = N - \hat{N}$ is the martingale measure with

$$N(ds, d\mu) = \sum_{\omega: \Delta Y_{\omega} \neq 0} \delta_{(\omega, \Delta Y_{\omega})}(ds, d\mu): \text{ the jump measure of } \{Y_t\},
$$

$$\hat{N}(ds, d\mu) = ds \left\{ \int Y_s(dx) \left( \int v(x, dy) \delta_{(\theta_i - \theta_s)} + k(x) \delta_{-\theta_s} \right)
+ \lambda \left\{ q(d(m,n)) \delta_{(\theta^+ - \theta_{m,n}(\theta))}(d\mu) : \text{ the compensator of } N. \right\}
$$

**Proof.** The proof is the same as the independent case (Proof of Theorem 2.4 in [2]). However, we need some computations. First the Markov property can be shown by mathematical induction. For $t_1 < t_2$, $f_1, f_2 \in C^\infty$, let $L^{f_1, f_2}_{t_1, t_2}(\mu) = \mathbb{E}^\mu_{t_1}[\exp(-\langle Y_{t_1}, f_1 \rangle - \langle Y_{t_2}, f_2 \rangle)]$. Recall that $L^{f_1}_t(\mu) = L^{f_1}_1(\mu) = \mathbb{E}^\mu_{t_1}[\exp(-\langle Y_t, f \rangle)]$ satisfies (2.1) and the solution is given as $L^{f_1}_1(\mu) = T_t \mathbb{P}_t[\exp(-\langle \cdot, f \rangle)(\mu)]$. Hence it is easy to see that $L^{f_1, f_2}_{t_1, t_2}(\mu)$ satisfies the following equation:

$$L^{f_1, f_2}_{t_1, t_2}(\mu) = e^{-i\lambda} \mathbb{P}_{t_1}[e^{-i\lambda} L^{f_2}_{t_2 - t_1}(\mu)]
+ \lambda \int_0^{t_1} ds e^{-i\lambda} \int q(d(m,n)) \mathbb{P}_s(\Theta^{m,n}_s L^{f_1, f_2}_{t_1, t_2 - s})(\mu).$$
The solution is given as

\[ L_{\mu}^{f_1, f_2} = \mathbb{P}_\mu \left( e^{-\langle \xi, f_1 \rangle} L_{\mu}^{f_2} \left( \int_{\mathbb{R}^d} e^{-\langle \xi, f_2 \rangle} \mathbb{P}_\mu \left( e^{-\langle \xi, f_1 \rangle} L_{\mu}^{f_2} \right) \mu \right) \right). \]

Therefore by induction, for every \( n \in \mathbb{N} \), if \( t_1 < t_2 < \cdots < t_n, f_1, \ldots, f_n \in C_c^\infty \), then it holds that

\[ \mathbb{E}_\mu^Y \left[ \exp(-\langle Y_{t_1}, f_1 \rangle - \cdots - \langle Y_{t_n}, f_n \rangle) \right] \]

\[ = \mathbb{E}_\mu^Y \left[ \exp(-\langle Y_{t_2}, f_2 \rangle \cdots \mathbb{E}_{Y_{t_1}}^Y \left[ \exp(-\langle Y_{t_1}, f_1 \rangle \right) \cdots \mathbb{E}_{Y_{t_{n-1}}}^Y \left[ \exp(-\langle Y_{t_{n-1}}, f_{n-1} \rangle \right) \cdots \right]. \]

Next we shall show that \((Y_t, \mathbb{P}_\mu^Y)\) satisfies a moment inequality of the same type as in Proposition 1.

**Proposition 2.** Let \( T > 0 \) and \( n \in \mathbb{N} \). For every \( 0 \leq t_1 \leq \cdots \leq t_n \leq T \) and \( f_i \in D_g^+, i = 1, 2, \ldots, n \),

\[ \mathbb{E}_\mu^Y \left[ \langle Y_{t_1}, f_1 \rangle \cdots \langle Y_{t_n}, f_n \rangle \right] \]

\[ \leq \prod_{i=1}^n \langle \mu, f_i \rangle + C_{n, T}^{(n)} \sum_{i=1}^n \prod_{j \neq i} \langle \mu, f_j \rangle \]

\[ + C_{2, T}^{(n)} \sum_{i \neq j_1, j_2} \prod_{i \neq j_1, j_2} \langle \mu, f_j \rangle + \cdots + C_{n-1, T}^{(n)} \sum_{j=1}^n \langle \mu, f_j \rangle + C_n^{(n)}, \]

where \( C_{k, T}, k = 1, \ldots, n \) are positive constants depending on \((n, T, \{\|f_i\| \}_{i \leq n})\).

**Proof.** For simplicity, we use notations \( f_m = f(s_m) \) and \( \| \cdot \| = \| \cdot \|_\infty \). Since \( \Theta_{m, \ell} \langle \mu, f \rangle = \langle \mu, f \rangle + f_m - f_m \leq \langle \mu, f \rangle + \|f\| \), we have for every \( k \in \mathbb{N} \),

\[ \left( \int_{\mathbb{N}^2} q(d(m, \ell)) \Theta_{m, \ell} \right)^k \langle \mu, f_1 \rangle \cdots \langle \mu, f_n \rangle \leq \langle \mu, f_1 \rangle + k \|f_1\| \cdots \langle \mu, f_n \rangle + k \|f_n\|. \]

Moreover if we denote by \( M(j; \lambda t) \) the \( j \)-th moment of \( \lambda t\)-Poisson distribution, then

\[ T_j \langle \cdot, f_1 \rangle \cdots \langle \cdot, f_n \rangle(\mu) \]

\[ = \sum_{k=0}^n \mathbb{E}_\mu \left[ e^{-\lambda t} \mathbb{P}_\mu \left( \int_{\mathbb{N}^2} q(d(m, \ell)) \Theta_{m, \ell} \right)^k \langle \cdot, f_1 \rangle \cdots \langle \cdot, f_n \rangle(\mu) \right] \]

\[ \leq \sum_{j=0}^n M(n-j; \lambda t) \sum_{\{i_1, \ldots, i_n\} \in \{1, \ldots, n\}} \langle \mu, f_{i_1} \rangle \cdots \langle \mu, f_{i_j} \rangle \|f_{i_{j+1}}\| \cdots \|f_n\|. \]
Therefore by applying Proposition 1 and the above inequality to
\[ E_\mu^Y \langle Y_{n+1}, f_{n+1} \rangle \cdots \langle Y_{n}, f_n \rangle = T_n (E_\mu^X \langle X_{n+1}, f_{n+1} \rangle \cdots \langle X_{n}, f_n \rangle) (\mu), \]
we can get the desired inequality. \hfill \square

Thus the proof can be proceeded by the same way as in the independent case (see §4 in [2]). In that way we can get the following result. For \( \mu = \sum_n \delta_{x_n} \in \mathcal{M}_{\mathfrak{g}0} \), \( f \in C_c^\infty \), let
\[
\Psi(\mu; f) = \langle \mu, Af - \Gamma f \rangle - \lambda \left[ e^{-\langle \partial_{x_n} - \beta_n, f \rangle} - 1 \right] q(d(m, n)).
\]

**Theorem 3.** For \( f \in C_c^\infty \),
\[
e^{-\langle Y_{n+1}, f_{n+1} \rangle} - e^{-\langle Y_n, f \rangle} - \int_0^t \mathbb{P}_\mu^Y e^{-\langle \cdot, f \rangle} (Y_s) \, ds
\]
is a \( \mathbb{P}_\mu^Y \)-martingale. Moreover
\[
H_t(f) = \exp \left[ -\langle Y_t, f \rangle + \int_0^t \Psi(Y_s; f) \, ds \right]
\]
is also a \( \mathbb{P}_\mu^Y \)-martingale.

**Proof.** By the same way as in the proof of Theorem 4.1 in [2] we have if \( s < t \), then
\[
\partial_s E_\mu^Y (e^{-\langle Y_s, f \rangle} \, | \mathcal{F}_s) = \partial_s T_{t-s} (e^{-\langle \cdot, V_{t-s} f \rangle})(Y_s)
\]
\[
= \partial_{u=0} T_{t-s+u} (e^{-\langle \cdot, V_{t-s+u} f \rangle})(Y_s)
\]
\[
= \partial_{u=0} E_\mu^Y [ T_{u} e^{-\langle \cdot, V_{t-s+u} f \rangle}(Y_t) \, | \mathcal{F}_s]
\]
\[
= E_\mu^Y [ \partial_{u=0} \mathbb{P}_\mu^Y e^{-\langle \cdot, f \rangle}(Y_t) \, | \mathcal{F}_s]
\]
\[
= E_\mu^Y [ \mathbb{P}_\mu^Y e^{-\langle \cdot, f \rangle}(Y_t) \, | \mathcal{F}_s]. \hfill \square
\]

By using the above results it is not difficult to prove the semi-martingale representation of \( Y_t \), as of \( X_t \), in [2]. In fact, for \( \mu \in C_c^\infty \), \( \langle Y_t, f \rangle \) is a special semi-martingale, thus,
\[
\langle Y_t, f \rangle = \langle Y_0, f \rangle + C_t(f) + M_t^c(f) + \tilde{N}_t(f) + N_t(f),
\]
where $C_t(f)$ is a continuous process of locally bounded variation, $M_t^c(f)$ is a continuous $L^2$-martingale with quadratic variation $\langle M_t^c(f) \rangle_t$, and

$$
\hat{N}_t(f) = \int_0^t \int_{\mathbb{R}^d} \langle \mu, f \rangle I(||\mu|| < 1) \hat{N}(ds, d\mu),
$$

$$
N_t(f) = \int_0^t \int_{\mathbb{R}^d} \langle \mu, f \rangle I(||\mu|| \geq 1) N(ds, d\mu)
$$

with the jump measure $N$ of $\{Y_t\}$, its compensator $\hat{N}$ and $\hat{N} = N - \hat{N}$. If we set

$$
B_t(f) = C_t(f) + \int_0^t \int_{\mathbb{R}^d} \langle \mu, f \rangle \hat{N}(ds, d\mu)
$$

$$
+ \lambda \int_0^t ds \int \langle \delta_{y_n(t)} - \delta_{y_n(t)}, f \rangle q(d(m, n)),
$$

then by applying Itô's formula for $Z_t(f)$ we can get

$$
-dB_t(f) + \frac{1}{2} \frac{d}{dt} \langle M_t^c(f) \rangle_t + \int |e^{-\langle \mu, f \rangle} - 1 + \langle \mu, f \rangle| \hat{N}(dt, d\mu)
$$

$$
= -\Psi(Y_t; f) dt
$$

$$
= \left\{-\langle Y_t, Af \rangle + \langle Y_t, Gf \rangle + \lambda \left( e^{-\langle \delta_{y_n(t)} - \delta_{y_n(t)}, f \rangle} - 1 \right) q(d(m, n)) \right\} dt
$$

$$
= \left\{-\langle Y_t, Af \rangle + \lambda \int \langle \delta_{y_n(t)} - \delta_{y_n(t)}, f \rangle q(d(m, n)) \right\} + \langle Y_t, G^c f \rangle
$$

$$
+ \langle Y_t, G^c f \rangle + \lambda \left( e^{-\langle \delta_{y_n(t)} - \delta_{y_n(t)}, f \rangle} - 1 + \langle \delta_{y_n(t)} - \delta_{y_n(t)}, f \rangle \right) q(d(m, n)) \right\} dt
$$

Thus we have

$$
B_t(f) = \int_0^t \langle Y_s, Af \rangle ds + \lambda \int \langle \delta_{y_n(t)} - \delta_{y_n(t)}, f \rangle q(d(m, n)),
$$

$$
\langle M_t^c(f) \rangle_t = 2 \int_0^t \langle Y_s, G^c f \rangle ds = \int_0^t \langle Y_s, Q^c f \rangle ds
$$

and
Therefore the proof is completed. 

3. Martingale Problems for $\mathcal{L}^\gamma$

The following assumption is needed to prove the well-posedness of martingale problems.

**Assumption 3.** For each $f \in (C^\infty_c)^+$, $AV_i f = -A \log(1 - P_i(1 - e^{-f}))$ is well-defined and $AV_i f$ is continuous in $t$ under the norm $\| g \|_\infty$, i.e.,

$$\| (AV_i f - AV_{i,0} f) / g \|_\infty \to 0 \quad (t \to t_0).$$

In the following we suppose that the generator $A$ of the motion process has the form of (1.1).

For $\eta \in \mathcal{M}_{g_0}$, let $F(\eta) = \Phi(\langle \eta, f_1 \rangle, \ldots, \langle \eta, f_n \rangle) \in \mathcal{D}_0 \overset{\text{def}}{=} \Phi(x) \in C^\infty(\mathbb{R}^n)$ is a polynomial growth function with polynomial growth derivatives of all orders and $f_i \in D_{g_0}, i = 1, \ldots, n$. For this $F(\eta)$, the generator $\mathcal{L}_0$ of $X$, will be extended to the following form:

$$\mathcal{L}_0 F(\eta) = \sum_{i=1}^n \partial_i \Phi(\langle \eta, f_1 \rangle, \ldots, \langle \eta, f_n \rangle) \langle \eta, A f_i \rangle$$

$$+ \frac{1}{2} \sum_{i,j=1}^n \partial^2_{ij} \Phi(\langle \eta, f_1 \rangle, \ldots, \langle \eta, f_n \rangle) \langle \eta, Q^\circ(f_i, f_j) \rangle$$

$$+ \int_S \left( \int_{S \setminus \{x\}} \nu(x, dy) \left[ \Phi(\langle \eta, f_1 \rangle + f_i(y) - f_i(x), \ldots, \langle \eta, f_n \rangle + f_n(y) - f_n(x)) - \Phi(\langle \eta, f_1 \rangle, \ldots, \langle \eta, f_n \rangle) \right. 

\left. + \sum_{i=1}^n \partial_i \Phi(\langle \eta, f_1 \rangle, \ldots, \langle \eta, f_n \rangle)(f_i(y) - f_i(x)) \right] \right)$$
+ k(x) \left[ \Phi(\langle \eta, f_1 \rangle - f_1(x), \ldots, \langle \eta, f_n \rangle - f_n(x)) \\
- \Phi(\langle \eta, f_1 \rangle, \ldots, \langle \eta, f_n \rangle) \\
+ \sum_{i=1}^{n} \delta_i \Phi(\langle \eta, f_1 \rangle, \ldots, \langle \eta, f_n \rangle) f_i(x) \right] \right \} \eta(dx),

where

\[ Q^\nu(f, g)(x) = \sum_{i,j} a^\nu(x) \delta_i f(x) \delta_j g(x). \]

For \( F(\eta) \in \mathfrak{B}_0 \), the generator \( \mathfrak{L}^Y \) of \( Y \), will be extended to

\[ \mathfrak{L}^Y F(\eta) = \mathfrak{L}_0 F(\eta) + \lambda \int (Q_{m,n} F(\eta) - F(\eta)) q(m,n). \]

**Theorem 4** (Martingale Problem for \((\mathfrak{L}^Y, \mathfrak{B}_0, \mu))\). Under Assumption 1, 2 and 3, suppose that the generator \( A \) is given as in (1.1). Let \( \mu \in \mathfrak{M}_0 \).

(i) \( P_\mu^Y(Y_0 = \mu) = 1 \) holds and for each \( F(\mu) = \Phi(\langle \mu, f_1 \rangle, \ldots, \langle \mu, f_n \rangle) \in \mathfrak{B}_0 \),

\[ M_t^F = F(Y_t) - F(Y_0) - \int_0^t \mathfrak{L}^Y F(Y_s) \, ds \quad \text{is} \quad P_\mu^Y \text{-martingale.} \]

(ii) If there is a probability measure \( Q_\mu \) on \( \mathfrak{D} = \mathfrak{D}(\{0, \infty\} \to \mathfrak{M}_0) \) such that the canonical process \( \tilde{Y}_t(\omega) = \omega(t) \) (\( \omega \in \mathfrak{D} \)) satisfies the same conditions as \( (Y_t, P_\mu^Y) \) in (i) and

\[ \int_0^t \langle \tilde{Y}_s, g_1 \rangle \, ds < \infty \quad Q_\mu \text{-a.s. for all } t \geq 0, \]

then \( Q_\mu = P_\mu^Y \circ Y^{-1} \) on \( \mathfrak{D} \), that is, martingale problem for \((\mathfrak{L}^Y, \mathfrak{B}_0, \mu)\) on \( \mathfrak{D} \) is well-posed.

**Proof.** The proof is essentially the same as the independent case (see §5 in [2]). However, the computations are more complicated, so we give the outline of the proof. (i) is easily obtained. We show (ii). We always fix \( f \in C_c^{\infty} \), \( T > 0 \), and set \( \Psi_t^T(\eta) = T_{T-t}(\exp - \langle \cdot, \mathcal{V}_{T-t} f \rangle)(\eta) \) (\( 0 \leq t \leq T, \eta \in \mathfrak{M}_0 \)). It is no difficult to show that \( \{\Psi_t^T(\tilde{Y}_t)\}_{t \leq T} \) is a \( Q_\mu \)-martingale. In fact, by using Ito's formula
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\[ d(\exp -\langle \tilde{Y}_t, f \rangle) = -\langle \tilde{Y}_t, Af - \Gamma f \rangle e^{-\langle \tilde{Y}_t, f \rangle} \, dt \]
\[ + \lambda \int q(d(m,n))(\Theta_{m,n} - I)e^{-\langle \cdot, \nu \rangle}(\tilde{Y}_t) \, dt + d(Q_\mu\text{-martingale}). \]

Since \( T_t \) is a bounded operator, we have (set \( v_t = v^{T_t} = V_{T_t} f \) again)

\[ d(\Psi^T_t(\tilde{Y}_t)) = T_{T_t-1} \left( -\lambda \int q(d(m,n))(\Theta_{m,n} - I)e^{-\langle \cdot, \nu \rangle} - \langle \cdot, A v_t \rangle e^{-\langle \cdot, \nu \rangle} \right) (\tilde{Y}_t) \, dt \]
\[ + d(Q_\mu\text{-martingale}) \]
\[ = T_{T_t-1}(-\langle \cdot, \partial_t v_t + A v_t - \Gamma v_t \rangle e^{-\langle \cdot, \nu \rangle})(\tilde{Y}_t) + d(Q_\mu\text{-martingale}) \]
\[ = d(Q_\mu\text{-martingale}) \]

Hence for \( 0 \leq s < t \leq T \), we have

\[ Q_\mu[\Psi^T_t(\tilde{Y}_t)] |_{\mathcal{F}_s} = \Psi^T_s(\tilde{Y}_s) \]

and set \( T = t \), then

\[ Q_\mu[e^{-\langle \tilde{Y}_t, f \rangle}] |_{\mathcal{F}_s} = T_{T-\lambda} e^{-\langle \cdot, \nu \rangle}(\tilde{Y}_s). \]

Therefore \( P_\mu = Q_\mu \) on \( D \).

4. Multi-Dimensional Absorbing Stable Motions on a Half Space

In §3 of [2] as a motion process we considered absorbing Brownian motion and absorbing stable motion on \((0, \infty)\) and discussed the Hölder (right) continuities of \( \{X_t\} \). It is possible to consider absorbing motions on \( H = \mathbb{R}^{d-1} \times (0, \infty) \) and we can get the same results as in Theorem 3.1 and in Corollary 3.1 of [2]. For the absorbing Brownian motion, it is not so difficult and essentially done in [1]. So in this section we only discuss the absorbing stable motion on \( H \).

For a function \( f \) on \( H \), let \( \tilde{f} \) be an extension of \( f \) to on \( \mathbb{R}^d \) defined as

\[ \tilde{f}(x) = \begin{cases} f(x) & (x_d > 0), \\ f(\bar{x}, 0^+) = 0 & (x_d = 0), \\ -f(\bar{x}, -x_d) & (x_d < 0), \end{cases} \]
where $x = (\tilde{x}, x_d) \in H$. Note that if $x \in H$, then $\tilde{f}(x) = f(x)$. The generator $A^{-\alpha}$ of absorbing $\alpha$-stable motion $(w^{-\alpha}(t), P_{x}^{-\alpha}) = (w^{-\alpha}(t), P_{x}^{-\alpha})$ on $H$ is given as $A^{-\alpha}f(x) = A^{\alpha}\tilde{f}(x)$; ($A^{-\alpha}$ is the same as $L^{-\alpha}$ in §4 of [1], however, in which we have some miss-prints)

\begin{equation}
A^{-\alpha}f(x) = c \int_{R^{d+1}(0)} \left[ \tilde{f}(x + y) - \tilde{f}(x) - \nabla \tilde{f}(x) \cdot y I(|y| < 1) \right] \frac{dy}{|y|^{d+\alpha}}
\end{equation}

\begin{align*}
&= c \int_{R^{d+1}} d\tilde{y} \int_{-\infty}^{\infty} \left[ f(x + \tilde{y}) - f(x) - \nabla f(x) \cdot y I(|y| < 1) \right] \frac{dy_d}{|y|^{d+\alpha}} \\
&\quad + c \int_{R^{d+1}} d\tilde{y} \int_{\infty}^{\infty} \left[ f(x + \tilde{y}) - f(\tilde{x} + \tilde{y}, y_d - x_d) - 2f(x) \right] \frac{dy_d}{|y|^{d+\alpha}}
\end{align*}

with some positive constant $c$, where in the last term the integral corresponding to $\nabla f(x) \cdot y$ is equal to zero by the symmetric property (of course, it is integrable). We can also write that if $0 < \alpha < 1$, then

\begin{align*}
A^{-\alpha}f(x) &= c \int_{R^{d+1}(x)} \left[ f(y) - f(x) \right] \frac{dy}{|y - x|^{d+\alpha}} \\
&= c \int_{R^{d+1}} d\tilde{y} \left\{ \int_{0}^{\infty} [f(y) - f(x)] K(x, y) dy_d \\
&\quad - 2f(x) \int_{0}^{\infty} \frac{dy_d}{|y - \tilde{x}, y_d - x_d|^{d+\alpha}} \right\},
\end{align*}

and that if $1 \leq \alpha < 2$, then

\begin{align*}
A^{-\alpha}f(x) &= c \int_{R^{d+1}(x)} \left[ f(y) - f(x) - \nabla f(x) \cdot (y - x) I(|y - x| < 1) \right] \frac{dy}{|y - x|^{d+\alpha}} \\
&= c \int_{R^{d+1}} d\tilde{y} \left\{ \int_{0}^{\infty} [f(y) - f(x) - \nabla f(x) \cdot (y - x) I(|y - x| < 1)] K(x, y) dy_d \\
&\quad + \int_{0}^{\infty} [-2f(x) - \nabla f(x) \cdot (y - x) I(|y - x| < 1) \\
&\quad - \nabla f(x) \cdot (y - \tilde{x}, y_d - x_d) I(|y - \tilde{x}, y_d - x_d| < 1)] \frac{dy_d}{|y - \tilde{x}, y_d - x_d|^{d+\alpha}} \right\}
\end{align*}
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\[ = c \int_{\mathbb{R}^{d+1}} d\tilde{y} \int_0^\infty [f(y) - f(x) - \nabla f(x) \cdot (y - x) I(|y - x| < 1)]K(x, y) \, dy_d, \]

\[ - f(x)k(x) + \nabla f(x) \cdot c(x), \]

where

\[ K(x, y) = \frac{I(y \neq x)}{|y - x|^{d+\alpha}} \frac{1}{|I(\tilde{y} - \tilde{x}, y_d + x_d)|^{d+\alpha}}, \]

\[ k(x) = k(x_d) = 2c \int_{\mathbb{R}^{d+1}} d\tilde{y} \int_0^\infty \frac{dy_d}{|y|^{d+\alpha}} \]

and

\[ c(x) = c \int_{\mathbb{R}^{d+1}} d\tilde{y} \int_0^\infty \left[ -(\tilde{y} - \tilde{x}, \tilde{y}_d - x_d) I(|(\tilde{y} - \tilde{x}, y_d + x_d)| < 1) \right] \]

\[ - (y - x) I(|y - x| < 1) \frac{dy_d}{|I(\tilde{y} - \tilde{x}, y_d + x_d)|^{d+\alpha}} \]

\[ = c \int_{\mathbb{R}^{d+1}} d\tilde{y} \int_0^\infty \left[ ((\tilde{y}, y_d + x_d) I(|(\tilde{y}, y_d + x_d)| < 1) \right] \]

\[ - (\tilde{y}, y_d - x_d) I(|(\tilde{y}, y_d + x_d)| < 1) \frac{dy_d}{|I(\tilde{y}, y_d + x_d)|^{d+\alpha}}. \]

Let \( h_0(v) \) be a \( C^\infty \)-function on \((0, \infty)\) such that \( 0 < h_0 \leq 1 \) on \((0, \infty)\),

\( h_0(v) = v \) for \( v \in (0, 1/2) \) and \( h_0(v) = 1 \) for \( v \geq 1 \). Let \( d < p < d + \alpha \). Set \( g_p(x) = (1 + |x|^2)^{-p/2} \) and \( g_{p,0}(x) := g_p(x) h_0(x_d) \) for \( x \in H \). Let \( f \in C_p \iff f \in C(\mathbb{R}^d)|_H; \|f/g_p\|_\infty < \infty, f \in C_{p,0} \iff f \in C(\mathbb{R}^d)|_H; \|f/g_{p,0}\| < \infty \). Moreover set

\[ f \in C_p \iff f \in C_p^\infty \]

for \( i, j \neq d, f, \partial_\tilde{x}^2 f, \partial_i f, \partial_\tilde{x}^2 f \in C_{p,0} \) and \( \partial_{d,i} f, \partial_{d,i}^2 f \in C_p \).

Then we can take \( D_0 = C_{p,0}^\infty \).

Moreover for each \( 0 < \alpha < 2 \), \( Q^{-\alpha} f \equiv Q^{-\alpha} f = Af^2 - fAf \) is given by the following formula:

\[ Q^{-\alpha} f(x) = c \int_{\mathbb{R}^{d+1}} d\tilde{y} \int_{-x_d}^{x_d} [f(x + y) - f(x)]^2 \frac{dy_d}{|y|^{d+\alpha}} \]

\[ + c \int_{\mathbb{R}^{d+1}} d\tilde{y} \int_{x_d}^\infty [f(x + y) - f(x + \tilde{x}, y_d - x_d)]. \]
\[ \{ f(x + y) + f(\tilde{y} + \tilde{x}, y_d - x_d) - 2f(x) \} + 2f(x)^2 \frac{dy_d}{|y|^{d+2}} \]

\[ \int_{\mathbb{R}^{d+1}} d\tilde{y} \int_0^\infty |f(y) - f(x)| K(x, y) \, dy_d + f(x)^2 k(x). \]

**Theorem 5.** Let \( d \geq 1 \), \( d < p < d + \alpha \), \( \mu \in \mathcal{M}_{p, 0} \) and let the motion process be absorbing \( \alpha \)-stable motion on \( H \) with \( 0 < \alpha < 2 \). Let \( \epsilon > 0 \) denote an arbitrary small number.

(i) For \((X_t, P_{\mu})\) the following holds.

(a) Under \( P_{\mu} \), \( \{X_t, g_{p, 0}\} \) is \((1/2(\alpha + 1)) - \epsilon\)-Hölder right continuous at \( t = 0 \). Moreover in case of \( 1 < \alpha < 2 \), if \( \langle \mu, g_1 \rangle < \infty \) with \( g_1(x) = g_p(x) h_0(x_d)^{2-\alpha} \), then \( \{X_t, g_{p, 0}\} \) is \((1/2 - \epsilon)\)-Hölder right continuous at \( t = 0 \).

(b) If \( \theta > 0 \), then under \( P_{\mu} \), \( \{X_t, g_{p, 0}\} \) is \((1/2 - \epsilon)\)-Hölder right continuous at \( t = \theta \) for every \( 0 < \alpha < 2 \).

(ii) For \((Y_t, P_{\mu}^r)\) the same results hold as above.

**Proof.** Let \( d \geq 2 \). The proof is proceeded in the same way as the case of \( d = 1 \). It suffices to check that the conditions in Assumption 1 and 2 are fulfilled with \( g_0 = g_{p, 0} \) and with suitable \( g_1 \in C^\infty \), \( 0 \leq \gamma < 1 \) as follows. Let \( h_1 \in C^\infty \), \( 0 < h_1 \leq 1 \), \( h_1(v) = v \log(1/v) \) for \( v \in (0, 1/e] \) and \( h_1(v) = 1 \) for \( v \geq 1 \).

(i) If \( 0 < \alpha < 1 \), then \( g_1(x) = g_{p, 0}(x), \gamma = 0 \).

(ii) If \( \alpha = 1 \), then \( g_1(x) = g_p(x) h_1(x_d), \gamma = \delta \) for any small \( 0 < \delta < 1 \).

(iii) If \( 1 < \alpha < 2 \), then \( g_1(x) = g_p(x) h_0(x_d)^{2-\alpha}, \gamma = 1 - 1/\alpha \).

Note that as \( x_d \downarrow 0 \),

\[ g_1(x) \sim x_d (0 < \alpha < 1), \quad \sim x_d \log(1/x_d) (\alpha = 1), \quad \sim x_d^{2-\alpha} (1 < \alpha < 2). \]

For simplicity of the notations we omit the superscript "\( \alpha \)" as \( P^{-\alpha}_t = P_t^{-\alpha}, A^{-\alpha}_t = A^{-\alpha} \). We shall show the following. Since they imply \( \|g_{p, 0}P^{-\alpha}_t g_1\|_\infty \leq C r^{-\gamma} \), we can get the \( ((1 - \gamma)/2 - \epsilon)\)-Hölder right continuity.

(C1) \( C_{p, 0}^3 \subset \mathcal{Z}(A^{-\alpha}), P^{-\alpha}_t C^\infty \subset C_{p, 0}^3 \) for every \( t \geq 0 \), \( \sup_{t \geq 0, 0 < x_d < 1} |x_d^{-1} P^{-\alpha}_t g_{p, 0}(x)| < \infty \) and \( A^{-\alpha} C_{p, 0}^3 \subset C_{p, 0} \) (these imply Assumption 1 and that \( C_{p, 0}^3 \) is a core).
(C2) For every \( f \in C^3_{\beta,0} \), \( \partial_1 P_{\gamma} f^2(x) = A^{-1} P_{\gamma} f^2(x) = P_{\gamma} A^{-1} f^2(x) \) \( (x \in H) \), \( A^{-1} f^2 \in C_0 \) and \( \| \gamma^{-1} Q^{-1} f \|_{\infty} < \infty \) (these imply (i) of Assumption 2).

(C3) For each \( 0 < \beta \leq 1 \), \( \sup_{x \geq 0} P_{\gamma} (y_{\beta}^\infty)(x) \leq 2(1 + \beta)x_d^\beta \) for all \( x \in H \) (this implies (ii) of Assumption 2).

(C4) For each \( 0 < \beta \leq 1 \), \( \sup_{0 < x_d \leq 1} x_d^{-1} P_{\gamma} (y_{\beta}^\infty)(x) \leq C_{\beta} x_d^{-1-\beta/\alpha} \) with a constant \( C_{\beta} > 0 \) depending only on \( \beta \) (this implies (iii), (iv) of Assumption 2).

Note that we take \( \gamma = (1 - \beta)/\alpha \) in Assumption 2. More exactly, if \( 0 < \alpha < 1 \), then \( \beta = 1 \), i.e., \( \gamma = 0 \). If \( \alpha = 1 \), then \( \beta = 1 - \epsilon \) for any small \( 0 < \epsilon < 1 \), i.e., \( \gamma = \epsilon \). If \( 1 < \alpha < 2 \), then \( \beta = 2 - \alpha \), i.e., \( \gamma = 1 - 1/\alpha \). (C3) and (C4) can be shown in a way similar to the case of \( d = 1 \); (B3) and (B4) in [2] by using the following. For the density \( p^\alpha(x) \) of the rotation invariant \( \alpha \)-stable motion on \( \mathbb{R}^d \) starting from 0, \( p^\alpha(x) = \gamma^{-d/2} p^{1/\alpha}(t^{-1/\alpha} x) \) and \( p_\gamma(x) \leq C(1 + |x|^{-d-\alpha}) \). The transition density \( p_{\gamma}(x, y) \equiv p^{-\infty}(x, y) \) of absorbing \( \alpha \)-stable motion on \( H \) is given as

\[
p_{\gamma}(x, y) = p_{\gamma}(y - x) - p_{\gamma}(y - \bar{x}, y_d + x_d) = - \int_{-x_d}^{y_d} \partial_v p_{\gamma}(y - \bar{x}, y_d + v) dv.
\]

We also use the following result.

\[
\int_{\mathbb{R}} \int_{H} z_{d+1}^{\infty} p^\infty(x, z_d + u) dz \text{ is bounded in } u \in \mathbb{R}.
\]

From these results we can get (C3), (C4).

In each (C1), (C2), the claims except the last one can be shown by the same way as in \( d = 1 \). In order to show the last claims of (C1), (C2), it is enough to prove that for each \( f \in C^3_{\alpha,0} \), there is a constant \( C > 0 \) such that

\[
|A^{-1} f(x)| \leq C x_d \quad \text{for } 0 < x_d \leq 1/2 \quad \text{and} \quad Q^{-1} f(x) \leq C g_1(x) \quad \text{for all } x \in H.
\]

Let \( 0 < x_d \leq 1/2 \). For \( A^{-1} \) we decompose as \( A^{-1} = (J_{1,1} + J_{1,2}) + (J_2 + J_3) \) and we shall show each term has order of \( x_d^\gamma, x_d, x_d \), respectively. The main calculus is of \( J_{1,2} (1 < \alpha < 2) \) and \( J_3 \). In the first term of (4.1) we divide the integral area to \( \{ |y| \geq 1 \} \cup \{ |y| < 1 \} \) and denote the corresponding terms by \( J_{1,1}(x), J_{1,2}(x) \) respectively. In the following we use the same symbols \( C', C'' \) as any positive finite constants which are independent of \( x \). First note that if \( |y| \geq 1 \) and \( |y_d| \leq x_d \leq 1/2 \), then \( |y|^2 \geq 1 - x_d^2 \geq 3/4 =: b \). By \( |f(y)| \leq C x_d \),
Next note that if $|y| < 1$, then $|\tilde{y}| < 1$ and that for some $\theta \in (0, 1)$,

$$
|f(x + y) - f(x) - \nabla f(x) \cdot y| = \frac{1}{2} \|f^{(2)}(x + \theta y)\cdot y\|^2 \leq \frac{1}{2} \|f^{(2)}\|_{\infty} |y|^2.
$$

If $0 < \alpha < 1$, then $|y|^{d-2+\alpha} \geq |y|^{d-2+\alpha}$ by $d - 2 + \alpha > 0$, and

$$
|J_{1,2}(x)| = \left| e \int_{\mathbb{R}^{d-1}} d\tilde{y} \int_{-\tilde{y}^d}^{\tilde{y}^d} [f(x + y) - f(x) - \nabla f(x) \cdot y] I(|y| < 1) \frac{dy_d}{|y|^{d+\alpha}} \right|
$$

$$
\leq e \int_{|\tilde{y}| < 1} d\tilde{y} \int_{-\tilde{y}^d}^{\tilde{y}^d} |f(x + y) - f(x) - \nabla f(x) \cdot y| \frac{dy_d}{|y|^{d+\alpha}}
$$

$$
\leq e \int_{|\tilde{y}| < 1} \tilde{y} \int_{0}^{\tilde{y}^d} \|f^{(2)}\|_{\infty} \frac{|y|^2}{|y|^{d+\alpha}} dy_d
$$

$$
\leq e \int_{|\tilde{y}| < 1} \tilde{y} \int_{0}^{\tilde{y}^d} \|f^{(2)}\|_{\infty} x_d dy_d
$$

$$
\leq e \int_{|\tilde{y}| < 1} \tilde{y} \int_{0}^{\tilde{y}^d} \|f^{(2)}\|_{\infty} x_d dy_d
$$

$$
= e \frac{\|f^{(2)}\|_{\infty} x_d}{1 - \alpha} = C' x_d.
$$

On the other hand if $1 \leq \alpha < 2$, then by using

$$
(f(x + y) - f(x) - \nabla f(x) \cdot y) = \frac{1}{2} f^{(2)}(x) \cdot y^2 + \frac{1}{6} f^{(3)}(x + \theta y) \cdot y^3
$$
with some \( \theta \in (0,1) \), and the corresponding integral to \( \sum_{i=1}^{d-1} \partial_{i}^{2} f(x) \cdot y_{i} \cdot y_{d} \) is equal to zero by symmetric property in \( y_{d} \), we have

\[
|J_{1,2}(x)| \leq c \int_{|\tilde{y}|<1} d\tilde{y} \int_{0}^{x_{d}} \left\{ \sum_{i,j=1}^{d-1} |\partial_{i} \partial_{j} f(x)| \cdot |y_{i}| \cdot |y_{j}| + |\partial_{i}^{2} f(x)| \cdot y_{d}^{2} \right. \\
+ \left. \frac{1}{3} \|f^{(3)}\| \cdot |y|^{3} \right\} \frac{dy_{d}}{|y|^{d+\alpha}}.
\]

Let \( 0 < \epsilon < 2 - \alpha \) and set \( \alpha_{\epsilon} := \alpha + \epsilon \in (1,2) \), then \( |y|^{d+\alpha-2} \geq |\tilde{y}|^{d-1-\epsilon} \cdot |y_{d}|^{-1+\epsilon} \).

By \( |\partial_{i} \partial_{j} f(x)|, |\partial_{i}^{2} f(x)| \leq Cx_{d} \) for \( i, j \neq d \), corresponding terms to \( f^{(2)} \) are less than or equal to

\[
Cx_{d} \int_{|\tilde{y}|<1} \frac{d\tilde{y}}{|\tilde{y}|^{d-1-\epsilon}} \int_{0}^{x_{d}} \frac{dy_{d}}{y_{d}^{2-1+\epsilon}} = Cx_{d} \int_{0}^{1} r^{\alpha-1} dr \cdot \frac{2\epsilon r^{2}}{2 - \alpha_{\epsilon}} = \frac{C}{2 - \alpha_{\epsilon}} x_{d}^{3-\alpha_{\epsilon}}.
\]

For the last term, by \( d \geq 2, \alpha \geq 1 \), i.e., \( d + \alpha - 3 \geq 0 \), we have \( |y|^{d+\alpha-3} \geq |\tilde{y}|^{d+\alpha-3} \). Hence the last term is less than or equal to

\[
\int_{|y|<1} \frac{d\tilde{y}}{|\tilde{y}|^{d+\alpha-3}} \cdot \|f^{(3)}\|_{\infty} \cdot x_{d} = \int_{0}^{1} r^{1-\alpha} dr \cdot \|f^{(3)}\|_{\infty} \cdot x_{d} = \frac{\|f^{(3)}\|_{\infty}}{2 - \alpha} x_{d}.
\]

These estimates imply \( |J_{1,2}(x)| \leq C'x_{d} \). In the second term of (4.1) we also divide the integral area to \( \{|y| \geq 1\} \), \( \{|y| < 1\} \) and denote the corresponding terms by \( J_{2}(x), J_{3}(x) \) respectively. For \( J_{2}(x) \), by

\[
|f(x + y) - f(\tilde{x} + \tilde{y}, y_{d} - x_{d})| \leq 2x_{d} \|\partial_{d} f\|_{\infty}
\]

and \( |f(x)| \leq Cx_{d} \), we have

\[
|J_{2}(x)| = \left| \int_{R^{d-1}} d\tilde{y} \int_{x_{d}}^{\infty} \left[ f(x + y) - f(\tilde{x} + \tilde{y}, y_{d} - x_{d}) - 2f(x) \right] I(|y| \geq 1) \frac{dy_{d}}{|y|^{d+\alpha}} \right| \\
\leq c \int_{H} 2\|\partial_{d} f\|_{\infty} + Cx_{d} I(|y| \geq 1) \frac{dy}{|y|^{d+\alpha}} \\
\leq C'x_{d} \int_{|y|<1} \frac{dy}{|y|^{d+\alpha}} = C''x_{d}.
\]

For \( J_{3}(x) = c \int_{R^{d-1}} d\tilde{y} \int_{x_{d}}^{\infty} \left[ f(x + y) - f(\tilde{x} + \tilde{y}, y_{d} - x_{d}) - 2f(x) \right] I(|y| < 1) \frac{dy_{d}}{|y|^{d+\alpha}}, \)

\[
f(x + y) - f(\tilde{x} + \tilde{y}, y_{d} - x_{d}) - 2f(x) \\
= \left[ f(x + y) - f(\tilde{x} + \tilde{y}, y_{d} - x_{d}) - 2f(\tilde{x} + \tilde{y}, x_{d}) \right] - 2\left[ f(\tilde{x} + \tilde{y}, x_{d}) - f(x) \right].
\]
For the first term, by the same way as in case of $d = 1$ (in $J_3(x)$) corresponding to the variable $y_d$, we have

$$|f(x + y) - f(\tilde{x} + \tilde{y}, y_d - x_d) - 2f(x)|$$

$$\leq 2\|\partial_d^3 f\|_{\infty} x_d(y_d^2 + x_d^2) + C(y_d + x_d)x_d y_d + 2C x_d^3.$$  

For the second term, note that

$$f(\tilde{x} + \tilde{y}, x_d) - f(x) = \sum_{i=1}^{d-1} \partial_i f(\tilde{x}, x_d) y_i + \frac{1}{2} \sum_{i,j=1}^{d-1} \partial_{ij}^2 f(\tilde{x} + \theta \tilde{y}, x_d) y_i y_j.$$  

and $|\partial_d^2 f(\tilde{x} + \theta \tilde{y}, x_d)| \leq C x_d$ for $i, j \leq d$. Moreover note that by the symmetric property in $y_i$ we have

$$\int_{R_{d-1}} d\tilde{y} \int_{x_d}^{\infty} \sum_{i=1}^{d-1} \partial_i f(\tilde{x}, x_d) y_i I(|y| < 1) \frac{dy_d}{|y|^{d+\alpha}}$$

$$= \sum_{i=1}^{d-1} \partial_i f(\tilde{x}, x_d) \int_{x_d}^{1} dy_d \int_{|y| < \sqrt{1-y_d^2}} \frac{y_i d\tilde{y}}{|y|^{d+\alpha}} = 0.$$  

Let $0 < \epsilon < 2 - \alpha$ and set $\alpha := \alpha + \epsilon \in (0, 2)$, then $|y|^{d+\epsilon} \geq |\tilde{y}|^{d-1-\epsilon} |y_d|^{1+\alpha}$ by $\epsilon$. Thus we can get the following: by $x_d \leq y_d$,

$$|J_3(x)| \leq c \int_{|\tilde{y}| < 1} d\tilde{y} \int_{x_d}^{1} \frac{dy_d}{|y|^{d+\alpha}}$$

$$+ c \int_{|\tilde{y}| < 1} d\tilde{y} \int_{x_d}^{1} \frac{1}{2} \sum_{i,j=1}^{d-1} |\partial_{ij}^2 f(\tilde{x} + \theta \tilde{y}, x_d) y_i y_j| \frac{dy_d}{|y|^{d+\alpha}}$$

$$\leq C x_d \int_{|\tilde{y}| < 1} \frac{d\tilde{y}}{|\tilde{y}|^{d-1-\epsilon}} \int_{x_d}^{1} (y_d^2 + x_d y_d + x_d^2) \frac{dy_d}{|y_d|^{1+\alpha}}$$

$$+ c \int_{|\tilde{y}| < 1} \frac{1}{2} C x_d |\tilde{y}|^2 \frac{dy_d}{|y|^{d+\alpha}}$$

$$\leq 3 C x_d \frac{1}{\epsilon} \int_{0}^{1} y_d^{1-\alpha} dy_d + \frac{c C}{2} x_d \int_{|\tilde{y}| < 1} d\tilde{y} |\tilde{y}|^2 \int_{0}^{1} \frac{dy_d}{|y|^{d+\alpha}}.$$  

For the second term if $0 < \alpha < 1$, then by $|y|^{d+\epsilon} \geq |\tilde{y}|^{d} y_d^\beta$,

$$\int_{|\tilde{y}| < 1} d\tilde{y} |\tilde{y}|^2 \int_{0}^{1} \frac{dy_d}{|y|^{d+\alpha}} \leq \int_{|\tilde{y}| < 1} |\tilde{y}|^2 \int_{0}^{1} dy_d = \frac{1}{1 - \alpha},$$
or if $1 \leq \alpha < 2$, then by $|y|^{d+\alpha} \geq |\tilde{y}|^{d+1-\alpha} y_{\tilde{d}}^{\alpha-1}$,

$$\int_{|\tilde{y}|<1} d\tilde{y} |\tilde{y}|^2 \int_0^1 \frac{dy_{\tilde{d}}}{|\tilde{y}|^{d+\alpha}} \leq \int_{|\tilde{y}|<1} \frac{|\tilde{y}|^2}{|\tilde{y}|^{d+1-\alpha}} d\tilde{y} \int_0^1 \frac{dy_{\tilde{d}}}{y_{\tilde{d}}^{\alpha-1}} = \frac{1}{c(2-\alpha)}. $$

Therefore

$$|J_3(x)| \leq \begin{cases} 
\left( \frac{3C}{c(2-x)} + \frac{6C}{2(1-x)} \right) x_d & (0 < \alpha < 1) \\
\left( \frac{6C + 2C}{2(1-x)} \right) x_d & (1 \leq \alpha < 2)
\end{cases}
= C'' x_d.$$

Therefore we have $|A^{-f}(x)| \leq C'' x_d$.

Next in order to show $Q^{-f}(x) \leq Cg_1(x)$, it suffices to prove that there is a constant $C > 0$ such that for $0 < x_d \leq 1$, if $0 < \alpha < 1$, then $Q^{-f}(x) \leq C x_d$, if $\alpha = 1$, then $Q^{-f}(x) \leq C x_d \log(1/x_d)$ if $1 < \alpha < 2$, then $Q^{-f}(x) \leq C x_d^{2-\alpha}$. We use the first formula of (4.2). In the following we decompose as $Q^{-f} = (R_1 + R_2) + (S_1 + S_2)$ and we shall show each $R_1$, $R_2$, $S_1$ has order of $x_d^3$, $x_d^{2-\alpha}$, $x_d$ respectively, and the main parts is $S_2$. In the first term of the right hand side of (4.2), we divide the integral area of $R^{d-1}$ to $\{|\tilde{y}| \geq 1\}$, $\{|\tilde{y}| < 1\}$ and denote the corresponding terms by $R_1(x)$, $R_2(x)$ respectively. By $f(x) \leq C x_d$, we have

$$R_1(x) = c \int_{|\tilde{y}| \geq 1} d\tilde{y} \int_{-x_d}^{x_d} [f(x+y) - f(x)]^2 \frac{dy_{\tilde{d}}}{|\tilde{y}|^{d+\alpha}} 
\leq 2c \int_{|\tilde{y}| \geq 1} \frac{d\tilde{y}}{|\tilde{y}|^{d+\alpha}} \int_0^{x_d} C^2 (2x_d + y_d)^2 dy_d
= C x_d^3.$$

For $R_2$, by $|\tilde{\partial}_i f(x)| \leq C x_d$ if $i \neq d$,

$$R_2(x) = 2c \int_{|\tilde{y}| \leq 1} d\tilde{y} \int_0^{x_d} [f(x+y) - f(x)]^2 \frac{dy_{\tilde{d}}}{|\tilde{y}|^{d+\alpha}} 
\leq 2c \int_{|\tilde{y}| \leq 1} d\tilde{y} \int_0^{x_d} \left[ C(x_d + y_d) \sum_{i=1}^{d-1} y_i + \|\tilde{\partial}_d f\|_\infty y_d \right]^2 \frac{dy_{\tilde{d}}}{|\tilde{y}|^{d+\alpha}}
\leq C \int_{|\tilde{y}| \leq 1} d\tilde{y} \int_0^{x_d} \left[ (x_d^2 + y_d^2)|\tilde{y}|^2 + y_d^2 \right] \frac{dy_{\tilde{d}}}{|\tilde{y}|^{d+\alpha}}.$$


In the above we first consider the last term (which is the main term), i.e.,
\[ \int_{|y|<1} d\tilde{y} \int_0^{x_d} y_d^2 \frac{dy_d}{|y|^{d+\alpha}} = \int_0^{x_d} dy_d \int_{|y|<1} \left( \int_{|y|<1} + \int_{y_d \leq |y| < 1} \right) \frac{d\tilde{y}}{|y|^{d+\alpha}} =: R_{2,1}(x) + R_{2,2}(x). \]

For \( R_{2,1}, \) let \( \alpha_\varepsilon = \alpha + \varepsilon < 2 \) be the same as before. By \( |y|^{d+\alpha} \leq |y|^{d-1-\varepsilon} \),
\[ \int_{|y|<1} \frac{d\tilde{y}}{|y|^{d-1-\varepsilon}} = \int_0^{x_d} y_d^{d-1} \, dr = \frac{y_d^d}{\varepsilon}. \]

Hence
\[ R_{2,1}(x) \leq \int_0^{x_d} \frac{y_d^2}{y_d^{d+2+\varepsilon}} \frac{dy_d}{\varepsilon} = \frac{1}{\varepsilon} \int_0^{x_d} y_d^{1-\alpha} \, dy_d = \frac{1}{(2-\alpha)\varepsilon} x_d^{2-\alpha}. \]

For \( R_{2,2}, \) by \( |y|^{d+\alpha} \geq |y|^{d-2} \) and
\[ \int_{y_d \leq |y| < 1} \frac{d\tilde{y}}{|y|^{d-1+\alpha}} = \int_{y_d}^{1} dr = \frac{1}{1 + \alpha} (y_d^{-1} - 1) \leq \frac{1}{1 + \alpha} y_d^{-1-\alpha}. \]

Hence
\[ R_{2,2}(x) \leq \int_0^{x_d} \frac{y_d^2}{1 + \alpha} y_d^{1-\alpha} \, dy_d = \frac{1}{1 + \alpha} \int_0^{x_d} y_d^{1-\alpha} \, dy_d = \frac{1}{(1 + \alpha)(2-\alpha)} x_d^{2-\alpha}. \]

Furthermore we can show more easily that the other terms of \( R_2 \) are \( o(x_d^2) \). In fact, by \( |y|^{d+\alpha} \geq |y|^{d-1+\alpha} |y_d|^{1-\varepsilon} \),
\[ \int_{|y|<1} \frac{d\tilde{y}}{|y|^{d+1+\alpha}} \leq \int_{|y|<1} \frac{|\tilde{y}|^2}{|y|^{d+1+\alpha}} d\tilde{y} \int_0^{x_d} (y_d^2 + y_d^2) \frac{dy_d}{|y|^{1-\varepsilon}} = C x_d^{2+\varepsilon}. \]

Therefore we have \( R_2(x) \leq C x_d^{2-\varepsilon} \) for all \( 0 < \alpha < 2 \).

In the second term of the right hand side of (4.2), we divide the integral area to \( \{|y| \geq 1\}, \{|y| < 1\} \) and denote the corresponding terms by \( S_1(x), S_2(x) \) respectively. For \( S_1, \) by
\[ (4.3) \quad |f(x + y) - f(\tilde{y} + x_d, y_d - x_d)| \leq 2x_d \| \partial_d f \|_\infty, \]
we have
\[ S_1(x) \leq (2x_d \| \partial_d f \|_\infty \cdot 3 \| f \|_\infty + 2C x_d^2) \int_{|y| \geq 1} \frac{dy_d}{|y|^{d+\alpha}} \leq C x_d. \]

For \( S_2, \) by (4.3) and by \( |f(x)| \leq C x_d, \)
\[
\left\{ \begin{array}{c}
(f(x + y) - f(x + \bar{x}, y_d - x_d)) \{ f(x + y) + f(x + \bar{x}, y_d - x_d) - 2f(x) \} + 2f(x)^2 \\
\leq 2\| \partial_d f \|_\infty x_d \cdot C(x_d + y_d) + 2Cx_d^2 \\
\leq Cx_d(x_d + y_d).
\end{array} \right.
\]

Hence, noting that \( \{ |y| < 1 \} = \{ |\bar{y}| < 1 \} \times \{ |y_d| < 1 \} \),

\[
|S_2(x)| \leq Cx_d \int_{|y| < 1} \left( \int_{x_d}^{x^1} (\partial_d y) \frac{dy_d}{\| y \|^{d+\alpha}} \right) \leq 2Cx_d \int_{|y| < 1} \left( \int_{x_d}^{x^1} \frac{dy_d}{\| y \|^{d+\alpha}} \right).
\]

By the same way as in R2, we can show the desired estimate as follows. Let

\[
\int_{x_d}^{x^1} \frac{dy_d}{\| y \|^{d+\alpha}} =: (S_{2,1}(x) + S_{2,2}(x)).
\]

Then \( |S_2(x)| \leq Cx_d(S_{2,1}(x) + S_{2,2}(x)) \). Let \( 0 < \epsilon < 2 - \alpha \). By \( |y|^{d+\alpha} \geq |\bar{y}|^{d-1-\epsilon} |y_d|^{1+\alpha} \),

\[
S_{2,1}(x) \leq \int_{x_d}^{x^1} \frac{dy_d}{\| y \|^{d+\alpha}} \int_{|y| < 1} \left( \int_{x_d}^{x^1} \frac{dy_d}{\| y \|^{d+\alpha}} \right) \leq 1 \int_{x_d}^{x^1} \frac{dy_d}{\| y \|^{d+\alpha}} = 1 \int_{x_d}^{x^1} \frac{dy_d}{\| y \|^{d+\alpha}}.
\]

That is, if \( 0 < \alpha < 1 \), then \( S_{2,1}(x) \leq C \), if \( \alpha = 1 \), then \( S_{2,1}(x) \leq C \log(1/x_d) \), if \( 0 < \alpha < 1 \), then \( S_{2,1}(x) \leq Cx_d^{-1-\alpha} \). Moreover for \( S_{2,2} \), as in R2, by

\[
\int_{x_d}^{x^1} \frac{dy_d}{\| y \|^{d+\alpha}} \leq y_d^{-1-\alpha}/(1 + \alpha),
\]

\[
S_{2,2}(x) \leq \int_{x_d}^{x^1} \frac{dy_d}{\| y \|^{d+\alpha}} \int_{|y| < 1} \frac{dy_d}{\| y \|^{d+\alpha}} \leq \int_{x_d}^{x^1} \frac{dy_d}{1 + \alpha} y_d^{-1-\alpha} dy_d = 1 \int_{x_d}^{x^1} y_d^{-1-\alpha} dy_d.
\]

Thus \( S_{2,2} \) satisfies the same estimates as \( S_{2,1} \). By \( |S_2(x)| \leq Cx_d(S_{2,1}(x) + S_{2,2}(x)) \), we have if \( 0 < \alpha < 1 \), then \( S_2(x) \leq Cx_d \), if \( \alpha = 1 \), then \( S_2(x) \leq Cx_d \log(1/x_d) \) if \( 1 < \alpha < 2 \), then \( S_2(x) \leq Cx_d^{-2-\alpha} \). These imply our desired result.

By \( P_{t}^\alpha C_{d}^\infty \subset C_{p,0}^3 \), the following result for martingale problem is obtained by the same way as in \( d = 1 \).

**Theorem 6.** Let \( \mu \in \mathcal{M}_{p,0} \). The martingale problems for \((\mathcal{L}_0, \mathcal{D}_0, \mu)\), \((\mathcal{L}_1, \mathcal{D}_0, \mu)\) associated with absorbing stable motion on \( H \) are well-posed.

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