Corruption in a repeated psychological game with imperfect monitoring☆

Masakazu Fukuzumi*

ABSTRACT

The aim of this note is to examine the effect of imperfect monitoring on bribery and corrupt practices in public administration. Our findings suggest that if we would reconstruct public service system to exterminate its bribery and corrupt practices, the rate of turnover of the bureaucrat and the amount of the noise in monitoring its behavior should be in the suitable regions respectively. We introduce imperfectness of monitoring the behaviour of the bureaucrat into the infinitely repeated game model developed by Balafoutas (2011). Supposing that the players adopt strategies with two-phases, the corruption and punishment phase, we derive the extent of the amount of the bribe for the bureaucrat and the lobby to sustain collusion in the corruption phase. Moreover, we show a sufficient condition for the two-phase strategies to constitute a psychological Nash equilibrium.

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Key words: corruption, repeated game, imperfect monitoring, psychological game

1. Introduction

We consider corruption in public administration in the context of a psychological repeated game. Balafoutas (2011) develops an elegant model of corruption, which is an infinitely repeated game with perfect information. The set of players of the game consists of the bureaucrat, the lobby, and the public in a nation. In this paper, we introduce imperfectness of monitoring the behavior of the bureaucrat into the model.

In each stage of the repeated game, the bureaucrat chooses (or manages to realize) either one policy *H* or the other policy *L*. The monetary payoff of the bureaucrat in the policy *H* is assumed to be greater than that in the policy *L*. The monetary payoff of the public in the policy *H* is assumed to be greater than that in the policy *L*, also. In contrast, the monetary payoff of the lobby in the policy *H* is assumed to be *less* than that in the policy *L*. The lobby would like to realize the policy L. However, the sum of all of the three players' monetary payoffs in the

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policy *H* is assumed to be greater than that in the policy *L*, that is, the policy *H* is efficient in the nation.

The public is assumed to have an expectation that is the probability that the bureaucrat chooses the policy *H*. This expectation is called the belief of the public. The bureaucrat anticipates this belief of the public. This anticipation is called the second-order belief of the bureaucrat. The stronger second-order belief the bureaucrat has, the payoff of the bureaucrat is assumed to get smaller in choosing the policy *L*. This reflects the bureaucrat's emotion of guilty aversion.[1]

For all the second order belief of the bureaucrat, if this game would be one-shot, then by backwards-induction it would turn out that the bureaucrat would choose the policy *H* whether the lobby would bribe the bureaucrat to choose the policy *L* or not. However, this stage game is assumed to be infinitely repeated. Hence, as a Nash equilibrium outcome the lobby could bribe the bureaucrat and the policy L could be chosen by the bureaucrat.

Balafoutas (2011) assumes that in each stage *t* the lobby can perfectly observe the action that the bureaucrat has chosen at the previous stage *t-1*. That is, the lobby knows perfectly whether or not the bureaucrat has rewarded the lobby's bribe with choosing the policy *L* in the previous period. In reality, however, the policy decision in the government often needs to follow many procedures. The effect of bribery might be obscure. Therefore, we relax the assumption of the perfect observability for the bureaucrat's action. We assume that even if the bureaucrat chooses the policy *L*, then another policy *H* is realized with positive probability. Our model becomes a psychological repeated game with imperfect monitoring.[2] To simplify the analysis, we assume that if the bureaucrat chooses the efficient policy *H*, then the policy *H* is surely realized.

Supposing that the bureaucrat and the lobby adopt the two-phase strategies, the corruption phase and the punishment one, we explore the effect of introducing such imperfectness of information into the infinitely repeated game of Balafoutas (2011). It is shown that the bureaucrat and the lobby can sustain collusion in each period of the corruption phase if and only if the amount of bribe falls in a range that depends on the second order belief of the bureaucrat. Moreover, we find a sufficient condition for sustaining their collusion with the second order belief that is consistent with the actual play of the bureaucrat. The profile of each player's optimal strategy and the consistent second-order belief is called a psychological Nash equilibrium.

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[1] The model of guilty aversion is formulated in a psychological game (Dufwenberg, 2002). The theory of psychological games was developed in Geanakoplos et al., (1984).

[2] Using the framework of such a repeated game with imperfect monitoring, Green and Porter (1984) showed that firms in an oligopoly market can sustain to collude. The formulation of our model is analogous to the model of efficiency wages developed in Shapiro and Stiglitz (1984). For a survey of the theory of the repeated games with imperfect monitoring, see Mailath and Samuelson (2006).

The rest of this note is organized as follows: In Section 2, our model is specified. In Section 3, we define the two-phase strategies and our results are proved. In Section 4, we state an implication of our results for the reform of civil service systems and argue the remaining topic for future research.

2. Model

We consider an infinitely repeated game of which players are a bureaucrat, a lobby (or a firm), and the public in a nation. It is assumed that the bureaucrat and the lobby have a common discount factor $\delta \in [0,1)$. The stage game of the repeated game is as follows. First, the lobby decides an amount of monetary transfer to the bureaucrat, which means the amount of bribery. The amount of this transfer is denoted by a non-negative real number k. Second, the bureaucrat observes the value of k chosen by the lobby and manages to realize either a policy H or a policy L. Whenever the bureaucrat manages to realize the policy H, the policy in the nation is sure to be H. However, if the bureaucrat manages to realize the policy L, then the policy in the nation becomes L with probability $1-\varepsilon$ but H with probability ε.

For each policy $i \in \{L, H\}$, a_i, b_i, and c_i denotes the corresponding monetary payoff of the bureaucrat, of the lobby, and of the public, respectively. Given the amount of transfer k, the bureaucrat's monetary payoff becomes $a_i + k$, and the lobby's monetary payoff $b_i - k$. We assume that $a_H > a_L, b_H < b_L, c_H > c_L$ and $a_H + b_H + c_H > a_L + b_L + c_L$, that is, only the lobby prefers the policy L to the policy H, but the policy H is more efficient than the policy L in the nation. Moreover, we assume that $a_H - a_L < b_L - b_H$, that is, there is scope of the bribe from the lobby to the bureaucrat.

The payoff of the bureaucrat is assumed to comprise an emotional factor, the sense of guilt averse, as follows. Let p denote the probability that the bureaucrat will choose H, and let π denote the public's expectation of p. Finally, let q denote the bureaucrat's expectation of π, that is the second-order belief of the bureaucrat. When the policy L is realized, which is the inefficient policy, the payoff of the bureaucrat is altered to $a_L + k - \gamma q$, where a non-negative parameter γ measures the intensity of guilty aversion of the bureaucrat.

3. Analysis

3.1. Subgame-perfect outcome of the stage game

It is easy to check the equilibrium outcome of the stage game by backwards-induction. Since $a_H + k > a_L + k$ for each k, it is optimal for the bureaucrat to choose H. The policy H is surely realized and the payoff of the lobby is given by $b_H - k$. Consequently, it is optimal for the lobby to choose the amount of transfer $k = 0$. Thus, no corruption would happen in the short-run relationship.

3.2. Subgame-perfect equilibrium of our infinitely repeated game

Assuming that the bureaucrat and the lobby adopt two-phase strategies defined below and we investigate the condition that these strategies compose the subgame-perfect Nash equilibrium in the infinitely repeated game. The two-phase strategies involve an amount of transfer $k^* > 0$ that is determined later and a parameter T which means the length of the punishment periods.

Corruption phase:

・ In this phase the lobby chooses a positive amount of transfer $k^* > 0$. The lobby stays in this phase if the transfer has been the positive k^* and the realized policy has been L from the first period of the game and/or from the first period in this phase, or right after the last period of the punishment phase. In all the other cases than those above, the lobby shifts to the punishment phase.

・ In this phase the bureaucrat chooses the policy L. The bureaucrat stays in this phase if the transfer has been the positive k^* from the first period of the game and/or from the first period in this phase (including the current period), or right after the last period of the punishment phase. In all the other cases than those above, the bureaucrat shifts to the punishment phase.

Punishment phase:

In this phase the lobby chooses no transfer k=0 and the bureaucrat chooses the policy H for T periods. After those T periods both players shift to the corruption phase.

We prove the following proposition by the same line of the arguments instructed in Fujiwara-Greve (2015). Let S_b and S_l denote the two-phase strategy defined above of the bureaucrat and that of the lobby, respectively.

Proposition For each value of the second order belief $q \in [0,1]$ of the bureaucrat, (S_b, S_l) is a subgame-perfect equilibrium of our infinitely repeated game if and only if in the corruption phase the lobby pays an amount k^* of bribe such that

$$k^* \in \left[\frac{(1-\delta^{T+1})}{(1-\delta^{T})} \times \frac{[(a_H - a_L) + \gamma q]}{\delta},\ (1-\varepsilon)(b_L - b_H) - \varepsilon\delta^{T} b_H\right]. \tag{1}$$

Proof: Whenever the bureaucrat is in the punishment phase, the lobby's action in no way influences the continuation strategy of the bureaucrat. Hence, the realized policy is always H at most *T* periods, so that the lobby's payoff in each period of this phase is $b_H - k,\ k \in [0, \infty)$. Thus, no transfer k=0 is optimal for the lobby at each period of this phase. Similarly, whenever the lobby is in the punishment phase, the bureaucrat's action in no way influences the continuation strategy of the lobby. If the bureaucrat follows the strategy S_b, the bureaucrat chooses H in this phase and yields the payoff a_H in each period of this phase. If the bureaucrat would choose L in a period of this phase, the bureaucrat's payoff in the period would be $(1-\varepsilon)(a_L - \gamma q) + \varepsilon a_H$. Since $a_H > a_L$ and $\varepsilon \in [0,1]$, we have $a_H > (1-\varepsilon)(a_L - \gamma q) + \varepsilon a_H$. H is the optimal choice for the bureaucrat in each period of the punishment phase.

Sequentially, we consider the situations that performing one-period deviation in the corruption phase, the player does not earn higher expected payoff.

Consider the long-run payoff of the bureaucrat at a period of the corruption phase in the case that the bureaucrat and the lobby follow (S_b, S_l). When they stick to (S_b, S_l), denote by V_b the expected present-value of the bureaucrat's total payoff, and by W_b the expected present-value of the bureaucrat's total payoff at the beginning of the punishment phase. V_b and W_b are the solutions of the following system of recursive equations:

$$\begin{cases} V_b = (1-\varepsilon)\{(a_L + k^* - \gamma q) + \delta V_b\} + \varepsilon\{(a_H + k^*) + \delta W_b\} & (2) \\ W_b = (1 + \delta + \delta^2 + \cdots + \delta^{T-1})a_H + \delta^T V_b & (3) \end{cases}$$

The first term of RHS of (2) is the bureaucrat's payoff in the case that the policy L is realized with probability $1-\varepsilon$ in the current period. Then, the bureaucrat earns the payoff $a_L + k^* - \gamma q$ in this period and continues to be in the corruption phase in the next period. The bureaucrat goes back to such a same situation as in this period and he (she) earns the payoff δV_b that is the one-period discounted continuation payoff. The second term of RHS of (2) is the bureaucrat's payoff when the policy H is realized with probability ε in this period. The bureaucrat earns the payoff $a_H + k^*$ in this period and both players shift to the punishment phase in the next period. The bureaucrat earns the payoff δW_b that is the one-period discounted continuation payoff. The second equation (3) shows that W_b is the sum of the present values of a_H that the bureaucrat continues to get for *T* periods in the punishment phase and the payoff $\delta^T V_b$ that sequentially going back to the corruption phase after *T* periods the bureaucrat get. Solving this system of (2) and (3) for V_b, we have

$$V_b = \frac{1}{1-(1-\varepsilon)\delta - \varepsilon\delta^{T+1}}\left[(1-\varepsilon)(a_L - \gamma q) + k^* + \frac{1-\delta^{T+1}}{1-\delta}\varepsilon a_H\right]. \quad (4)$$

If choosing the policy H for one period, the bureaucrat deviates from S_b, then the bureaucrat yields the payoff

$$(a_H + k^*) + (\delta + \delta^2 + \cdots + \delta^T)a_H + \delta^{T+1}V_b. \tag{5}$$

The first term of (5) is the bureaucrat's payoff where the lobby has given the transfer k^* in this period and the policy *H* is realized certainly. This $a_H + k^*$ is the maximal payoff in this period to the bureaucrat. After this deviation, both players shift to the punishment phase. Consequently, the bureaucrat continues to earn the payoff a_H for T periods, which is represented by the second term of (5). After the punishment phase the situation goes back to the corruption phase and bureaucrat earns $\delta^{T+1}V_b$. Therefore, in the corruption phase it is optimal for the bureaucrat to conform to the strategy S_b if and only if

$$V_b \geq (a_H + k^*) + (\delta + \delta^2 + \cdots + \delta^T)a_H + \delta^{T+1}V_b. \tag{6}$$

Substituting (4) into (6) above and arranging it, we yield

$$k^* \geq \frac{(1-\delta^{T+1})}{(1-\delta^T)} \times \frac{[(a_H - a_L) + \gamma q]}{\delta}, \tag{7}$$

in which the RHS of this inequality (7) is the minimum value of the interval (1) of this proposition. The RHS of (7) is the smallest bribe for the bureaucrat to maintain collude in the corruption phase.

We turn to confirming the lobby's strategy S_l to be optimal in the corruption phase, which is similar to the above argument.

Consider the long-run payoff of the lobby at a period of the corruption phase in the case that these players follow (S_b, S_l). We denote by V_l the expected present-value of the lobby's total payoff when they continue to follows (S_b, S_l), and by W_l the expected present-value of the lobby's total payoff of the punishment phase. V_l and W_l are the solutions of the following system of recursive equations:

$$\begin{cases} V_l = (1-\varepsilon)\{(b_L - k^*) + \delta V_l\} + \varepsilon\{(b_H - k^*) + \delta W_l\} & (8) \\ W_l = (1 + \delta + \delta^2 + \cdots + \delta^{T-1})b_H + \delta^T V_l & (9) \end{cases}$$

The first term of RHS of (8) is the lobby's total payoff in the case that the policy *L* is realized with probability $1 - \varepsilon$ in the current period. When the policy *L* is realized, the lobby gets the current payoff $b_L - k^*$ and he (she) continues to be in the corruption phase in the next period. The lobby goes back to such a same situation as in the current period. Hence, the lobby yields the one-period discounted continuation payoff δV_l. The second term of RHS of (9) is the bureaucrat's payoff in the case that the policy *H* is realized with probability ε in the current period. The lobby yields the current payoff $b_H - k^*$, and both players shift to the punishment phase in the next period. In the next period the lobby yields the one-period discounted continuation payoff δW_l. The second equation (9) means that W_l is the sum of the payoff b_H that the lobby continues to get for *T* periods in the punishment phase and the payoff $\delta^T V_l$ that the lobby get at *T* periods later in the corruption phase. Solving this system of (8) and (9) for V_l, we have

$$V_l = \frac{1}{1-(1-\varepsilon)\delta - \varepsilon\delta^{T+1}}\left[(1-\varepsilon)b_L - k^* + \frac{1-\delta^T}{1-\delta}\varepsilon b_H\right]. \tag{10}$$

No bribe $\mathrm{k}=0$ is the most profitable deviation from S_l for one period. This deviation gives the lobby the total payoff,

$$b_H + (\delta + \delta^2 + \cdots + \delta^T)b_H + \delta^{T+1}V_l. \tag{11}$$

The first term of (11) is the lobby's payoff at the situation in which the bureaucrat chooses the policy *H* in this period and the policy *H* is certainly realized. The sum of the second term $(\delta + \delta^2 + \cdots + \delta^T)b_H$ and the third term $\delta^{T+1}V_l$ of (11) is equal to δW_l, which is the one-period discounted total payoff of the punishment phase.

Therefore, it is optimal for the lobby to adopt the strategy S_l in the corruption phase if and only if

$$V_l \geq b_H + (\delta + \delta^2 + \cdots + \delta^T)b_H + \delta^{T+1}V_l. \tag{12}$$

Substituting (10) into (12) above and arranging it, we yield a following condition:

$$k^* \leq (1-\varepsilon)(b_L - b_H) - \varepsilon\delta^T b_H, \tag{13}$$

in which the RHS of this inequality (13) is the maximum amount of bribe for the lobby to maintain collusion in the corruption phase. Combing (7) and (13), we get the possible interval of k^*, (1), in the proposition, for both players to sustain collusion. ■

So far, we have assumed that the second-order belief q has an arbitrary value in the interval [0,1]. A *psychological Nash equilibrium*, however, requires that each player adopts his (her) own optimal strategy and the second order belief must consist with the actual play (Geanakoplos et al. (1989)). Figure 1 provides a graphic illustration of such an equilibrium play, the payoff of the lobby, and the sum of monetary payoffs among all players that are induced by the two-phase strategy.

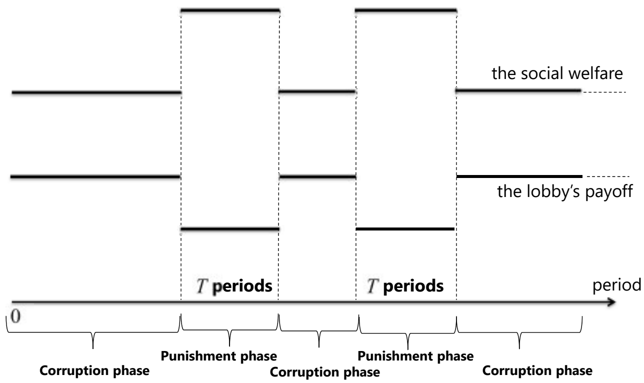

Figure 1. An equilibrium play of the two phase strategy, the lobby's payoff and the sum of monetary payoffs among all players.

Corollary Suppose that

$$\frac{(1-\delta^{T+1})}{(1-\delta^{T})}\times\frac{[(a_H-a_L)+\gamma]}{\delta}\leq(1-\varepsilon)(b_L-b_H)-\varepsilon\delta^{T}b_H.$$

Then (S_b, S_l) composes a psychological Nash equilibrium of our infinitely repeated game where

· $q=0$ in every period of the corruption phase, and

· $q=1$ in every period of the punishment phase.

Proof: From (7) the smallest value of the bribe to sustain collusion is given by $\frac{(1-\delta^{T+1})}{(1-\delta^{T})}\times\frac{[(a_H-a_L)+\gamma q]}{\delta}$. Since $q\leq 1$, if the assumption

$$\frac{(1-\delta^{T+1})}{(1-\delta^{T})}\times\frac{[(a_H-a_L)+\gamma]}{\delta}\leq(1-\varepsilon)(b_L-b_H)-\varepsilon\delta^{T}b_H$$

of this corollary holds, then for each q there is some value of the bribe k^* such that the two-phase strategies (S_b, S_l) above can be the subgame-perfect equilibrium. Knowing this, the public is convinced that the lobby and the bureaucrat will collude in the corruption phase. Since in the psychological Nash equilibrium it is required that the belief of the public is correct, q is actually zero in the corruption phase, and the bureaucrat confirms this expectation by choosing *L*. Similarly, the public is convinced that the lobby and the bureaucrat will not collude in the punishment phase. In the psychological Nash equilibrium, the second-order belief q must be actually one, and the bureaucrat confirms this expectation by choosing H. ■

4. Concluding remarks

We see that the interval (1) of k^* in Proposition above narrows as the noise ε increases and vanishes at some level of the noise. Furthermore, as the length of the punishment phase T shortens, the interval of k^*narrows. These findings suggest that if we would reconstruct public service system to exterminate its bribery and corrupt practices, the the rate of turnover of the bureaucrat and the amount of the noise in monitoring its behavior should be in the suitable regions respectively.

The psychological Nash equilibrium requires that the belief of the public coincides with the actual behaviors of players. Balafoutas (2011) investigates a dynamic adaptation process of the belief of the public such that the process converges to the actual behaviors of the bureaucrat and the lobby. We have not been able to provide such an explicit adaptation process of the belief. It would be an important subject of future research to investigate such a dynamic process of beliefs in the game with imperfect monitoring.

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