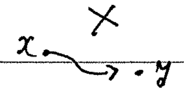
No. 5/23(火)

第6回数理科学IIIA

Seifert（独立） − van Kampen の定理　商集合



$t(0)=x$
$t(1)=y$

U, V arcwise connected open subsets

$t: I \to X$

$X = U \cup V$（和集合）

$U \cap V$（共通部分） nonempty and arcwise connected

base point（基点） $x_0 \in U \cap V$

$$\begin{tikzcd} & \pi(U) \arrow[dr, "\psi_1"] \arrow[drr, bend left, dashed] & & \\ \pi(U \cap V) \arrow[ur, "\varphi_1"] \arrow[dr, "\varphi_2"'] & & \pi(X) \arrow[r, dashed] & H \\ & \pi(V) \arrow[ur, "\psi_2"'] \arrow[urr, bend right] & & \end{tikzcd}$$

自由

前回
Theorem $U \cap V$ simply connected
$\Downarrow$
$\pi(X)$: free product of $\pi(U)$ and $\pi(V)$

∞

今回　V : simply connected
$\Downarrow$
kernel
$\psi_1 : \pi(U) \to \pi(X)$ epimorphism（全射）

kernel は the smallest normal subgroup of $\pi(U)$ containing the image $\varphi_1(\pi(U \cap V))$

応用 (applications)

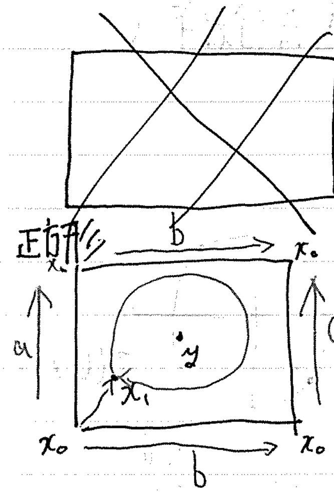
No.

Date

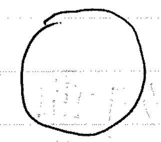
torus $T = S^1 \times S^1$　　$\pi(S^1)$

無限巡回群

$\pi(T) = \pi(S^1) \times \pi(S^1)$
$\mathbb{Z} \times \mathbb{Z}$



$U = T - \{y\}$



V : the interior image of

homeomorphic.

deformation retract of the whole square minus a point

U, V　T の open subsets

$U \wedge V$　arcwise connected

V : simply connected

$\psi_1 : \pi : \pi(U, x_1) \to \pi(T, x_1)$ epimorphism

$\varphi_1 : \pi(U \cap V, x_1) \to \pi(U, x_1)$ 正規部分群 $\begin{smallmatrix}\alpha & a\\ \beta & b\end{smallmatrix}$

the union of the two circles ~~of~~ a and b is a deformation retract of U.

$\pi(U, x_1)$: the free group on two generations.
$\pi(U, x_1)$

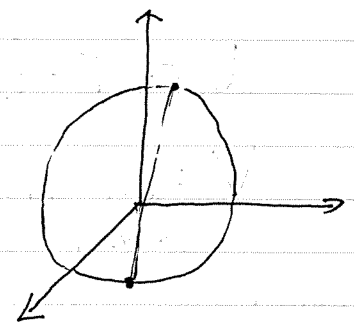
$\alpha' = \delta^{-1} \alpha \delta$　自由群　非可換
$\beta' = \delta^{-1} \beta \delta$

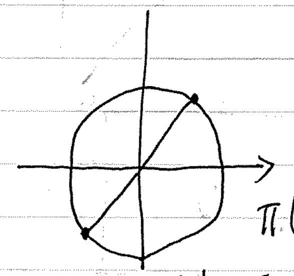
U∩V circle $\underline{\gamma}$

$\varphi_1(\gamma) = \alpha' \beta' \alpha'^{-1} \beta'^{-1}$ Abel化

結論 $\pi(T)$ α'とβ'で生成される自由群を
交換子 $\alpha' \beta' \alpha'^{-1} \beta'^{-1}$で生成される正規部分群で
割ったもの.

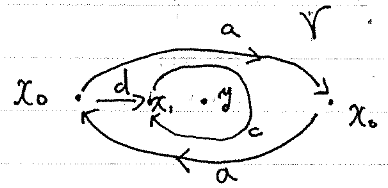
射影平面 P_2
直線





$\pi(U, x_1)$

$\sigma^{-1} \alpha \sigma$



$U = P_2 - \{y\}$ $\varphi_1(\gamma) = \alpha'^2$

V : the image of the interior $\boxed{\text{図2}}$
of the <u>polygon</u> under the
identification.

the circle a is a deformation retract of U
$\pi(U, x_0)$ <u>αで生成される無限巡回群</u> α a