第3回 数理科学ⅢA

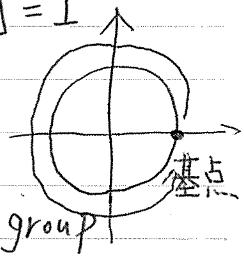
No. 5/2(火)

Date

$S^1 = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2=1\}$ の基本群の決定

$f: I = [0,1] \to S^1$ $\qquad \alpha \qquad \pi^1(S^1,(1,0))$

$f(t) = (\cos 2\pi t, \sin 2\pi t) \qquad t \in [0,1] = I$



定理 The fundamental group $\pi(S^1,(1,0))$ is an infinite cyclic group generated by α

証明。 $g: I \to S^1 \qquad g(0) = g(1) = (1,0)$

g belongs to the equivalence class α^m $\quad \alpha^0$ for some integer m

$U_1 = \{(x,y) \in S^1 \mid y > -\frac{1}{10}\}$

$U_2 = \{(x,y) \in S^1 \mid y < +\frac{1}{10}\}$

U_1, U_2 : connected open subsets of S^1

$U_1 \cup U_2 = S^1$

U_1, U_2 : homeomorphic to some open interval (開区間)

contractible

$\pi(U^1,(1,0)) = \pi(U^2,(1,0)) =$ つぶれる

$g(I) \subset U_1$ or $g(I) \subset U_2$ $\quad g \sim$ constant path

$g(I) \not\subset U_1$ and $g(I) \not\subset U_2$

No.

Date

$I \quad [0,t_1]\ [t_1,t_2], \dots, [t_{n-1},1] \qquad 0=t_0<t_1<t_2 \cdots <t_{n-1}<t_n=1$

such that

(a) $g([t_i,t_{i+1}]) \subset U_1$ or $g([t_i,t_{i+1}])$ for $0 \le i < n$

(b) $g([t_{i-1},t_i])$ and $g([t_i,t_{i+1}])$ are not both contained in the same open set U_j $(j=1,2)$

これの Lesbegue number $\quad \mathrm{diam}\, Y = \sup\{d(y_1,y_2) \mid y_1, y_2 \in Y\}$

$\{g^{-1}(U_1),\ g^{-1}(U_2)\}$ an open covering of the metric space I.

<u>Lesbegue number</u>

$\varepsilon>0$ is a Lebesgue number of a covering of a metric space X iff, any subset of X of diameter $<\varepsilon$ is contained in some set of the covering

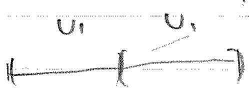
Theorem (Lebesgue)

(☆) Any open covering of a <u>compact</u> metric space has a Lebesgue number

証明略.

$<\varepsilon$

amalgamate



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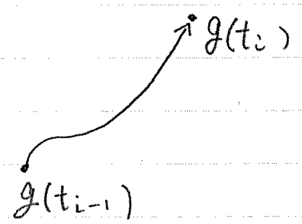
Date . .

β = the equivalence class of g

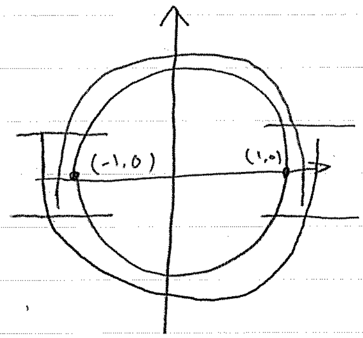
$g \mid [t_{i-1}, t_i]$ $\qquad \beta_i$



$\beta = \beta_1 \cdot \beta_2 \cdots\cdots \beta_n$ $\qquad \beta_i \qquad U_1 \qquad U_2$

$g(t_i) \in U_1 \cap U_2$

$U_1 \cap U_2$ two components



For each i, $0 < i < n$

path class γ_i in $U_1 \cap U_2$

initial point $g(t_i)$ terminal point $(1,0)$ or $(-1,0)$

$\delta_1 = \beta_1 \gamma_1$

$\delta_i = \gamma_{i-1}^{-1} \beta_i \gamma_i$ for $1 < i < n$

$\delta_n = \gamma_{n-1}^{-1} \beta_n$

$\beta = \delta_1 \delta_2 \cdots \delta_n$ $\qquad \delta_i$ path class in U_1 or in U_2 having its initial and terminal points in the set $\{(1,0), (-1,0)\}$

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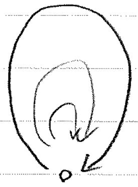
if δ_i is a closed path class, then $\delta_i = 1$ と仮定してよい。

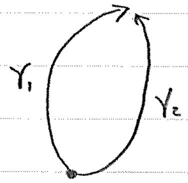
$\delta_1, \delta_2, \ldots, \delta_n$ are <u>not closed</u> paths

No.

Date

X : simply connected





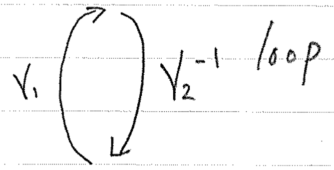
$F : I \times I \to X$

$F(0,t) = \gamma_1(t)$

$F(1,t) = \gamma_2(t)$

$F(s,0) = x_0$

$F(s,1) = x_1$



$$\gamma_1 \sim \gamma_1 \cdot (\gamma_2^{-1}\gamma_2) \sim \underbrace{(\gamma_1 \cdot \gamma_2^{-1})}_{1} \cdot \gamma_2 = \gamma_2$$

U_1 simply connected

η_1 unique path class η_1 in U_1 with initial point $(1,0)$ and terminal point $(-1,0)$

η_1^{-1} = unique path in U_1 with initial point $(-1,0)$ and terminal point $(1,0)$

η_2 : unique path in U_2 with initial point $(-1,0)$ and terminal point $(1,0)$

$\eta_1 \eta_2 = \alpha$

No.
Date

$\delta_i = \eta_1^{\pm 1}$ or $\delta_i = \eta_2^{\pm 1}$

(b) if $\delta_i = \eta_1^{\pm 1}$ then $\delta_{i+1} = \eta_2^{\pm 1}$
while if $\delta_i = \eta_2^{\pm 1}$ then $\delta_{i+1} = \eta_1^{\pm 1}$

$\beta = 1$

$\beta = \eta_1 \eta_2 \eta_1 \eta_2 \cdots \eta_1 \eta_2$

cyclic group
(巡回群)

$\beta = \eta_2^{-1} \eta_1^{-1} \eta_2^{-1} \eta_1^{-1} \cdots \eta_2^{-1} \eta_1^{-1}$