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Enriched Category Theory (豊穣圏)

例) Mod_R : the category of left modules over a fixed, not necessarily commutative ring R.

$\mathrm{Mod}_R(A,B)$ abel 群

$(Ab, \otimes_{\mathbb{Z}}, \mathbb{Z})$ monoidal category

→ admits the structure of a category enriched over

The base for enrichment

symmetric monoidal category $(V, \times, *)$

V : category

$- \times - : V \times V \to V$ bifunctor called the monoidal product

$* \in V$ called the unit object

$v \times w \cong w \times v$ $\quad u \times (v \times w) \cong (u \times v) \times w$

$* \times v \cong v \times *$

V : complete

cocomplete

Enriched Categories

V-category $\underline{D}$

- a collection of objects $x, y, z \in \underline{D}$
- for each pair $x, y \in \underline{D}$, a hom-object $\underline{D}(x,y) \in V$
- for each $x \in \underline{D}$, a morphism $id_x : * \to \underline{D}(x,x)$ in V
- for each triple $x, y, z \in \underline{D}$, a morphism

$\circ : \underline{D}(y,z) \times \underline{D}(x,y) \to \underline{D}(x,z)$ in V

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such that the following diagrams commute for all $x, y, z, w \in \underline{D}$

$$\begin{tikzcd}
\underline{D}(z,w) \times \underline{D}(y,z) \times \underline{D}(x,y) \arrow[r, "1\times\circ"] \arrow[d, "\circ\times 1"'] & \underline{D}(z,w) \times \underline{D}(x,z) \arrow[d, "\circ"] \\
\underline{D}(y,w) \times \underline{D}(x,y) \arrow[r, "\circ"'] & \underline{D}(x,w)
\end{tikzcd}$$

associativity

$$\begin{tikzcd}
\underline{D}(x,y) \times * \arrow[r, "1\times id_x"] \arrow[dr, "\cong"'] & \underline{D}(x,y) \times \underline{D}(x,x) \arrow[d, "\circ"] \\
 & \underline{D}(x,y)
\end{tikzcd}$$

$$\begin{tikzcd}
\underline{D}(y,y) \times \underline{D}(x,y) \arrow[d, "\circ"'] & * \times \underline{D}(x,y) \arrow[l, "id_y \times 1"'] \arrow[dl, "\cong"] \\
\underline{D}(x,y) &
\end{tikzcd}$$

one object monoid

例) $Ab = (Ab, \otimes_{\mathbb{Z}}, \mathbb{Z})$ base

one object Ab-category — a ring with identity

$\underline{D}$ x

$\underline{D}(x,x) \otimes_{\mathbb{Z}} \underline{D}(x,x) \to \underline{D}(x,x)$

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例) Topological spaces are naturally enriched over gropoids.

category

$\underline{Top}(X, Y)$ — object: ~~con~~ continuous maps from X to Y

morphism: homotopy classes of homotopies between these maps

例) V has copowers of the unit object that are preserved by the monoidal product in each variable.

$\mathbb{C}$: 任意の category は それに付随する free V-category を持つ.

$a, b \in \mathbb{C}$ $\quad \coprod_{\mathbb{C}(a,b)} *$

$$\left(\coprod_{\mathbb{C}(b,c)} *\right) \times \left(\coprod_{\mathbb{C}(a,b)} *\right) \cong \coprod_{\mathbb{C}(b,c)} \left(* \times \left(\coprod_{\mathbb{C}(a,b)} *\right)\right)$$

$$\cong \coprod_{\mathbb{C}(b,c)} \left(\coprod_{\mathbb{C}(a,b)} * \times *\right) \cong \coprod_{\mathbb{C}(b,c) \times \mathbb{C}(a,b)} *$$

定義 (closed monoidal categories)

$\_ \times v : V \to V$ functor

right adjoint ~~V(u,~~ $\underline{V}(v, -)$

$\underline{V}(-,-)$: bifunctor

$V(u \times v, w) \cong V(u, \underline{V}(v, w))$ $\forall u, v, w \in V$

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命題

V is enriched over itself

$\underline{V}(v, w) \times \underline{V}(u, v) \to \underline{V}(u, w)$ composition law 定義

$- \times u \dashv \underline{V}(u, -)$

adjunct

$\underline{V}(v, w) \times \underline{V}(u, v) \times u \xrightarrow{1 \times \epsilon} \underline{V}(v, w) \times v \to w$

V-category

$\Downarrow$

underlying category

G : fixed discrete group

$\underline{Top} X, Y$