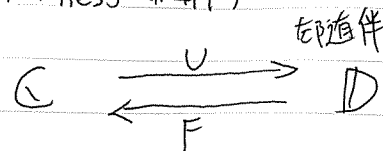


Adjointness (随伴)

 $\mathcal{C}, \mathcal{D}$  categories $U, F$ : functors

bijection

$$\mathcal{C}(FX, A) \cong \mathcal{D}(X, UA)$$

 $X$  object of  $\mathcal{D}$  $A$  object of  $\mathcal{C}$ 
 $X \text{ と } A \text{ について natural}$   
 自然
 $f: X \rightarrow X'$  = morphism in  $\mathcal{D}$ 

$$\mathcal{C}(FX, A) \cong \mathcal{D}(X, UA)$$

$$\mathcal{C}(f, A) \uparrow \quad \hookrightarrow \quad \uparrow \mathcal{D}(f, UA)$$

$$\mathcal{C}(FX', A) \cong \mathcal{D}(X', UA)$$

 $g: A \rightarrow A'$  = morphism in  $\mathcal{C}$ 

$$\mathcal{C}(FX, A) \cong \mathcal{D}(X, UA)$$

$$\mathcal{C}(FX, g) \downarrow \quad \hookrightarrow \quad \downarrow \mathcal{D}(X, Ug)$$

$$\mathcal{C}(FX, A') \cong \mathcal{D}(X, UA')$$

natural transformation

自然変換

Date

report

I

ie)  $f: X \rightarrow X'$  $X \xrightarrow{\eta_X} UFX$  $f \downarrow \quad \hookrightarrow \quad \downarrow Uf$  $X' \xrightarrow{\eta_{X'}} UFX'$ 

$$\mathcal{C}(FX, FX) \cong \mathcal{D}(X, UFX)$$

$$\text{id}_{FX} \quad \eta_X: X \rightarrow UFX$$

$$\mathcal{C}(FUA, A) \cong \mathcal{D}(UA, UA)$$

 $\text{id}_{UA}$ 

$$\epsilon_A: FUA \rightarrow A$$

$$UA \xrightarrow{\eta_{UA}} UFUA \xrightarrow{U\epsilon_A} UA = \text{id}_{UA}$$

$$FX \xrightarrow{F\eta_X} FUFUX \xrightarrow{\epsilon_{FX}} FX = \text{id}_{FX} \quad \text{(II) 同様に示す}$$

$$UA \quad \quad \quad UFUA$$

 $\eta_{UA}$  $\text{id}_{FUA}$ 

$$\mathcal{D}(UA, UFUA) \cong \mathcal{C}(FUA, FUA)$$

$$\mathcal{D}(UA, U\epsilon_A) \downarrow \quad \hookrightarrow \quad \downarrow \mathcal{C}(FUA, \epsilon_A)$$

$$\mathcal{D}(UA, UA) \cong \mathcal{C}(FUA, A)$$

 $\text{id}_{UA}$  $\epsilon_A$ 

$$\begin{array}{ccc} \eta_C & & \\ \mathcal{C} \rightarrow & UF & \mathcal{C} \\ & \downarrow Uf & \\ \phi & \downarrow & UC \end{array}$$

$$\text{逆に } \mathbb{C} \xrightleftharpoons[F]{U} \mathbb{D}$$

$$\eta_X : X \rightarrow UFX \quad \text{natural transformation}$$

$$\epsilon_A : FUA \rightarrow A$$

$$UA \xrightarrow{\eta_{UA}} UFX \xrightarrow{U\epsilon_A} UA = \text{id}_{UA}$$

$$FX \xrightarrow{F\eta_X} FUX \xrightarrow{\epsilon_{FX}} FX = \text{id}_{FX}$$

$$\mathbb{C}(FX, A) \xrightleftharpoons[\text{bijection}]{\cong} \mathbb{D}(X, UA)$$

$$FX \xrightarrow{\oplus} A \xrightarrow{\ominus} FX$$

$$X \xrightarrow{\eta_X} UFX \xrightarrow{Uf} UA$$

$$X \xrightarrow{g} UA \quad FX \xrightarrow{Fg} FUA \xrightarrow{\epsilon_A} A$$

$$\begin{array}{ccccc} FX & \xrightarrow{F\eta_X} & FUX & \xrightarrow{FUF} & FUA \\ \downarrow \epsilon_{FX} & & \downarrow \eta_X & & \downarrow \epsilon_A \\ FX & \xrightarrow{f} & A & & \end{array}$$

$$\textcircled{III} \xrightarrow{\star} \text{を示せ。}$$

$$X \xrightarrow{g} X'$$

$X \in A$  に関する natural

$$\mathbb{C}(FX, A) \cong \mathbb{D}(X, UA)$$

$$\mathbb{C}(FX', A) \uparrow \quad \hookrightarrow \quad \uparrow \mathbb{D}(X', UA)$$

$$\mathbb{C}(FX', A) \cong \mathbb{D}(X', UA)$$

$$f : FX' \rightarrow A$$

$$FX \xrightarrow{g} FX' \xrightarrow{f} A$$

$$\begin{array}{c} X \\ \downarrow g \\ X' \end{array} \xrightarrow{\eta_{X'}} UFX' \xrightarrow{Uf} UA$$

$$UFX \xrightarrow{Ug} UFX' \xrightarrow{Uf} UA$$

$$\begin{array}{ccc} \uparrow \eta_X & \hookrightarrow & \uparrow \eta_{X'} \\ X & \xrightarrow{g} & X' \end{array}$$

IV  $A$  に関する natural であることを示せ。