Correction to the Automorphism Group of a Cyclic $p$-gonal Curve

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CORRECTION TO THE AUTOMORPHISM GROUP OF A CYCLIC $p$-GONAL CURVE

By

Naonori Ishii and Katsuaki Yoshida

In our paper [1], we have made an error about the conditions on which our arguments were built. More precisely, we presented a wrong assertion as Lemma 2.1 (ii) in [1]. Necessarily the assertion Lemma 2.1 (iii) that $V$ is contained in the center of $G$ is not correct either. In order to carry the whole argument through the paper, we have to assume that $V$ is in the center of $G$, and we should rewrite Lemma 2.1 as follows.

The authors would like to thank Professor Aaron Wootton who pointed out this error.

**Lemma 2.1.** Here the notations are same as in [1].

(i) The group $H$ acts on $\mathcal{S}$.

The following two conditions are equivalent.

(ii) Let $a_i$ and $a_j$ be in $\mathcal{S}$. If there exists an element $T \in G$ satisfying $T a_i = a_j$, then we have $r_i = r_j$. Here we define $r_{s+1}$ by $r_{s+1} \equiv -\sum_{i=1}^{s} r_i \pmod{p}$ and $0 < r_{s+1} < p$ when $\sum_{i=1}^{s} r_i \neq 0 \pmod{p}$.

(iii) The automorphism $V$ is contained in the center of $G$.

**Proof.** The statement (i) and the implication (ii) $\Rightarrow$ (iii) have actually been proved in [1].

*Proof of* (ii) $\iff$ (iii).

Assume $\tilde{T}^* x = \zeta_n x = \xi x$. Moreover assume $\mathcal{S} \cap \{0, \infty\} = \emptyset$. Let

$$\mathcal{S} = \langle \tilde{T} \rangle b_1 \cup \cdots \cup \langle \tilde{T} \rangle b_l = \bigcup_{k=1}^{l} \{ b_k, \zeta_n b_k, \zeta_n^2 b_k, \ldots, \zeta_n^{n-1} b_k \}$$

be the decomposition of $\mathcal{S}$ by the action of $\langle \tilde{T} \rangle$. Then $M$ is defined by

$$y^p = \prod_{k=1}^{l} (x - b_k)^{u_{k,0}} (x - \zeta_n b_k)^{u_{k,1}} \cdots (x - \zeta_n^{n-1} b_k)^{u_{k,n-1}}, \quad 1 \leq u_{k,j} \leq p - 1. \quad (1)$$

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By acting $T^*$ on (1), we have

$$(T^*y)^p = \prod_{k=1}^{t}(T^*x - b_k)^{u_k,0}(T^*x - \zeta^n b_k)^{u_k,1}(T^*x - \zeta^{2n} b_k)^{u_k,2} \cdots (T^*x - \zeta^{n-1} b_k)^{u_k,n-1}$$

$$= \zeta_n^C \prod_{k=1}^{t}(x - \zeta^n b_k)^{u_k,0}(x - b_k)^{u_k,1}(x - \zeta b_k)^{u_k,2} \cdots (x - \zeta^{n-2} b_k)^{u_k,n-1},$$

where $C = \sum_{k=1}^{t} \sum_{j=0}^{n-1} u_{k,j}$.

By the assumption that $V$ is in the center, $\frac{T^*y}{y}$ is invariant under the action of $V^*$. Then

$$\frac{T^*y^p}{y^p} = \xi_n^C \prod_{k=1}^{t}(x - \zeta^n b_k)^{u_k,0}(x - b_k)^{u_k,1}(x - \zeta b_k)^{u_k,2} \cdots (x - \zeta^{n-2} b_k)^{u_k,n-1}$$

$$= \xi_n^C \prod_{k=1}^{t}(x - b_k)^{u_k,0} \cdots (x - \zeta b_k)^{u_k,1} \cdots (x - \zeta^{n-2} b_k)^{u_k,n-2}$$

$$= \xi_n^C \prod_{k=1}^{t} (x - b_k)^{u_k,0} \cdots (x - \zeta b_k)^{u_k,1} \cdots (x - \zeta^{n-2} b_k)^{u_k,n-1}$$

is $p$-th power of the rational function $\frac{T^*y}{y} \in \mathbb{C}(x)$. Therefore we have

$$u_{k,0} \equiv \cdots \equiv u_{k,n-1} \mod p.$$ As $u_{k,j} \leq p - 1$, we have $u_{k,0} = \cdots = u_{k,n-1}$. In case $\mathcal{S} \cap \{0, \infty\} \neq \emptyset$, we can carry the same argument as above. ⊓⊔

According to this revised lemma, we have to correct the results in [1] as follows:

(1) we add the assumption that $V$ is in the center of $G$ to Theorem 2.1 [1];
(2) the curves listed in Theorems 3.1 and 5.1 are those with the condition that $V$ is in the center of $G$.

References