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Extended Block-Lifting-based Lapped Transforms

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Abstract—We extend an original lapped transform (LT) and use block-lifting factorization to get an extended block-lifting-based LT (XBL-LT). The block-lifting structure maps integer input signals to integer output signals, and results in a reversible transform that reduces rounding errors by merging many rounding operations. Although other such block-lifting-based LTs (BL-LTs) have been proposed, they are forcibly constrained by the use of discrete cosine transform (DCT) matrices. In contrast, XBL-LT is DCT-unconstrained and hence also embodies the DCT-constrained form. Furthermore, it has fewer rounding operations by merging the scaling factor with block-lifting coefficients. The both DCT-constrained and unconstrained XBL-LTs perform well at lossy-to-lossless image coding which has scalability from lossless data to lossy data.

Index Terms—Block-lifting structure, lapped transform (LT), lossy-to-lossless image coding

I. INTRODUCTION

LAPPED transforms (LTs) [1] are popular subband transforms that can be used as substitutes for the discrete cosine transform (DCT) [2] used in image/video compression (image coding). Although almost all of the JPEG and H.26x series [3–5], image coding standards, use DCTs for their good energy compaction, DCT-based image codings generate unpleasant artifacts, i.e., blocking artifacts, at low bit rates due to their ignoring the continuity of the blocks. LT-based image coding solves that problem by using a processing that works over the blocks.

The lifting structure [6] is a very important technology to achieve a lossless mode in subband transform-based image coding. It maps integer input signals to integer output signals; i.e., it is an integer-to-integer transform. The 4 × 8 lifting-based LT (L-LT) [7] in JPEG XR [8], the newest image coding standard, is a time-domain LT (TDLT) [9] with simple scaling factors and lifting structures. In spite of its simple structure, it performs well at lossy-to-lossless image coding, which has scalability from lossless to lossy data. The block-lifting structure [10] is a class of lifting structures and results in a reversible transform that reduces rounding errors by merging many rounding operations. Inspired by the L-LT in JPEG XR, we have proposed block-lifting-based LTs (BL-LTs) [11], [12] that perform well with larger block size than those of the L-LT in JPEG XR. However, they are forcibly DCT-constrained and degrade coding performance at high bit rates.

Here, we extend an original LT and use block-lifting factorization to get an extended BL-LT (XBL-LT). The XBL-LT is DCT-unconstrained, unlike the BL-LTs presented in [11], [12], and hence also embodies the DCT-constrained form. Furthermore, more lifting operations than the methods described in our previous studies are removed by merging the scaling factor with block-lifting coefficients. As a result, the both DCT-constrained and unconstrained XBL-LTs perform well at lossy-to-lossless image coding.

Notation: The italic letter $M$ ($M = 2^n$, $n \in \mathbb{N}$) denotes the block size. Boldface letters $I_m$, $J_m$, $0$, and $D_m$ denote an $m \times m$ identity matrix, an $m \times m$ reversal matrix, a null matrix, and an $m \times m$ diagonal matrix with alternating ±1 entries (i.e., diag{1, −1, 1, −1, · · · }). Respectively, The superscripts $T$ and −1 respectively mean the transpose and inverse of a matrix.

II. REVIEW AND DEFINITION

A. Lapped Transform (LT)

In accordance with [11], [12], let $E(z)$ be a polyphase matrix of an $M \times 2M$ LT with a scaling factor $s$ derived from the L-LT in JPEG XR [7]:

$$
E(z) = P \begin{bmatrix} I_N & 0 \\ 0 & S_N^T C_N^{H} \end{bmatrix} \Gamma(z) \begin{bmatrix} C_N^{H} & 0 \\ 0 & C_N^{IV} J_N \end{bmatrix} \tilde{S} \tilde{W} J_M, \tag{1}
$$

where

$$
\Gamma(z) = W A(z) W, \quad A(z) = \text{diag} \{ I_N, z^{-1} I_N \}
$$
$$
W = \frac{1}{\sqrt{2}} \begin{bmatrix} I_N & I_N \\ I_N & -I_N \end{bmatrix}, \quad \tilde{W} = \frac{1}{\sqrt{2}} \begin{bmatrix} I_N & J_N \\ J_N & -I_N \end{bmatrix}
$$
$$
S = \text{diag} \{ s I_N, s^{-1} I_N \}.
$$

$C_N^{H}$ and $S_N^T$ are type-X DCT (DCT-X) and type-X discrete sine transform (DST-X) matrices, and the (i, j)-elements of $C_N^{H}$ and $C_N^{IV}$ are

$$
\begin{bmatrix} C_N^{H} \end{bmatrix}_{i,j} = \sqrt{\frac{2}{M}} c_i \cos \left( \frac{(i+j+1/2) \pi}{M} \right),
$$
$$
\begin{bmatrix} C_N^{IV} \end{bmatrix}_{i,j} = \sqrt{\frac{2}{M}} \cos \left( \frac{(i+1/2)(j+1/2) \pi}{M} \right),
$$

where $c_k = 1/\sqrt{2}$ ($k = 0$) or 1 ($k \neq 0$), respectively. The following relationships between matrices can be established:

$$
C_N^{III} = \left( C_M^{H} \right)^{-1} = \left( C_M^{H} \right)^T, \quad S_N^T = D_M C_M^{IV} J_M. \tag{2}
$$

Here, $P$ is an $M \times M$ permutation matrix. The optimal scaling $s$ in $S$ is empirically determined, e.g., $s = 0.8981$ if $M = 8$ and $0.9360$ if $M = 16$. Since the LT in Eq. (1) with $s = 1$ is completely equivalent to a lapped orthogonal transform (LOT), we will use the LT in Eq. (1) as a representative expression of LT.
and rounding operations, respectively.

**B. Block-Lifting Structure**

The block-lifting structure [10] (Fig. 1) is a class of lifting structures. The structure can be expressed as follows:

\[ y_j = x_j + \text{round}(B_0 x_i), \quad y_i = x_i + \text{round}(B_1 y_j) \]

\[ z_i = y_i - \text{round}(B_1 y_j) = x_i, \quad z_j = y_j - \text{round}(B_0 x_i) = x_j, \]

where \( x, \ y, \) and \( z \) are \( N \times 1 \) input/output vector signals, \( \text{round}(\cdot) \) is a rounding operation, and the block-lifting coefficients \( B_0 \) and \( B_1 \) are \( N \times N \) arbitrary matrices. In this case, the matrices and their inverses are expressed as lower and upper block-lifting matrices as follows:

\[
\mathcal{L}[B_0] = \begin{bmatrix} I_N & 0 \\ B_0 & I_N \end{bmatrix}, \quad \mathcal{L}[B_0]^{-1} = \mathcal{L}[-B_0]
\]

\[
\mathcal{U}[B_1] = \begin{bmatrix} I_N & B_1 \\ 0 & I_N \end{bmatrix}, \quad \mathcal{U}[B_1]^{-1} = \mathcal{U}[-B_1].
\]

Rounding errors generated by the rounding operation in each lifting step degrade coding performance. The block-lifting structure reduces such rounding errors by merging many rounding operations. A special class of block-lifting structure is expressed as [13]

\[
\begin{bmatrix} M & 0 \\ 0 & M^{-1} \end{bmatrix} = \mathcal{L}[M^{-1}] \mathcal{U}[-M] \mathcal{L}[M^{-1}] \mathcal{J}_M
\]

\[
= \mathcal{J}_M \mathcal{L}[-M] \mathcal{U}[M^{-1}] \mathcal{L}[-M],
\]

where

\[
\mathcal{J}_M = \begin{bmatrix} 0 & I_N \\ -I_N & 0 \end{bmatrix}, \quad \mathcal{J}_M = \begin{bmatrix} 0 & -I_N \\ I_N & 0 \end{bmatrix},
\]

and \( M \) is an \( N \times N \) arbitrary nonsingular matrix.

**III. EXTENDED BLOCK-LIFTING-BASED LAPPED TRANSFORM (XBL-LT)**

**Theorem:** We can extend the DCT-constrained LT in Eq. (1) to a DCT-unconstrained LT as follows:

\[
E(z) = P \begin{bmatrix} I_N & 0 \\ 0 & D_N \hat{\mathcal{V}}_{N}^{-1} \end{bmatrix} \Gamma(z) \begin{bmatrix} \hat{\mathcal{U}} & 0 \\ 0 & \hat{\mathcal{V}}_{N} \end{bmatrix} \hat{W} \mathcal{J}_M,
\]

where \( \hat{\mathcal{U}} \) and \( \hat{\mathcal{V}} \) are \( N \times N \) arbitrary nonsingular matrices. This equation can be simplified as

\[
E(z) = P \begin{bmatrix} I_N & 0 \\ 0 & \mathcal{V}^{-1} \mathcal{U}^{-1} \end{bmatrix} \Gamma(z) \begin{bmatrix} \mathcal{U} & 0 \\ 0 & \mathcal{V} \end{bmatrix} \hat{W} \mathcal{J}_M,
\]

where

\[
\mathcal{U} = \sqrt{2} \mathcal{U}, \quad \mathcal{V} = \frac{1}{\sqrt{2}} \mathcal{V}, \quad \hat{\mathcal{W}} = \begin{bmatrix} I_N & 0 \\ \frac{1}{2} I_N & -I_N \end{bmatrix},
\]

by using Eq. (2) and skipping the sign inversion matrix \( D_N \).

The scaling matrix \( S \) is being embedded in \( \mathcal{U} \) and \( \mathcal{V} \). When \( \mathcal{U} = \mathcal{C}_N^0 \) and \( \mathcal{V} = \mathcal{C}_N^1 \), the LT in Eq. (5) is completely equivalent to the LT in Eq. (1) except that the signs are different. Then, we factorize the LT in Eq. (5) into block-lifting structures as

\[
E(z) = P \mathcal{L}[B_3] \mathcal{U} \mathcal{L}[B_3] \Lambda(z) \mathcal{U} \mathcal{L}[B_2] \mathcal{U} \mathcal{L}[B_1] \mathcal{U} \mathcal{L}[B_0] \mathcal{U}
\]

\[
\cdot \mathcal{U} \begin{bmatrix} 1 \quad -1 \\ 2 \quad 1 \end{bmatrix} \mathcal{L}[J_N] \mathcal{J}_M,
\]

wherein each matrix is defined as

\[
B_0 = -\mathcal{V}^{-1}, \quad B_1 = \mathcal{V}, \quad B_2 = B_0 + B_4
\]

\[
B_3 = -\mathcal{U} \mathcal{J}_N \mathcal{V}, \quad B_4 = \mathcal{V}^{-1} \mathcal{J}_N \mathcal{U}^{-1}.
\]

Actually, the block-lifting matrices \( \mathcal{U} \mathcal{L}[B_3] \) on both sides of the delay matrix \( \Lambda(z) \) are collectively implemented, as shown in Fig. 2.

**Proof:** We can perform an easy matrix manipulation as follows:

\[
\begin{bmatrix} M_0 & 0 \\ 0 & M_1 \end{bmatrix} \begin{bmatrix} I_N & N_0 \\ 0 & N_1 \end{bmatrix} = \begin{bmatrix} I_N & \frac{M_0 N_0 M_1^{-1}}{N_1} \\ M_1 N_1 M_0^{-1} & I_N \end{bmatrix} \begin{bmatrix} 0 & M_1 \\ 0 & M_1 \end{bmatrix},
\]

where \( M_0 \) and \( N_1 \) are an \( N \times N \) arbitrary nonsingular matrix and \( N \times N \) arbitrary matrix, respectively. First, \( \Gamma(z) \) in Eq. (5) can easily be represented by

\[
\Gamma(z) = \mathcal{L}[I_N] \mathcal{U} \begin{bmatrix} 1 \quad -1 \\ 2 \quad 1 \end{bmatrix} \Lambda(z) \mathcal{U} \begin{bmatrix} 1 \quad -1 \\ 2 \quad 1 \end{bmatrix} \mathcal{L}[-I_N].
\]

Next, \( \mathcal{U}^{-1} \) in Eq. (5) is moved to the right side of \( \Gamma(z) \) and simplified as

\[
\mathcal{F}(z) \triangleq \begin{bmatrix} I_N & 0 \\ 0 & \mathcal{U}^{-1} \end{bmatrix} \Gamma(z) \begin{bmatrix} \mathcal{U} & 0 \\ 0 & I_N \end{bmatrix}
\]

\[
= \mathcal{L}[\mathcal{U}^{-1}] \mathcal{U} \begin{bmatrix} 1 \quad -1 \\ 2 \quad 1 \end{bmatrix} \Lambda(z) \mathcal{U} \begin{bmatrix} 1 \quad -1 \\ 2 \quad 1 \end{bmatrix} \mathcal{L}[-\mathcal{U}^{-1}] \mathcal{U} \begin{bmatrix} 0 & 0 \\ 0 & \mathcal{U}^{-1} \end{bmatrix}
\]

\[
= \mathcal{L}[\mathcal{U}^{-1}] \mathcal{U} \begin{bmatrix} 1 \quad -1 \\ 2 \quad 1 \end{bmatrix} \Lambda(z) \mathcal{U} \begin{bmatrix} 1 \quad -1 \\ 2 \quad 1 \end{bmatrix} \mathcal{L}[-\mathcal{U}^{-1}] \mathcal{J}_M,
\]

because the block diagonal matrix \( \text{diag}\{\mathcal{U}, \mathcal{U}^{-1}\} \) in \( \mathcal{F}(z) \) can be factorized into block-lifting structures as in Eq. (3). Then, \( \mathcal{U}^{-1} \mathcal{J}_N \) in Eq. (5) is moved to the right side of \( \mathcal{F}(z) \) and
where \( k \) is the index of \( \{ k \} \), because the block diagonal matrix \( \text{diag}(\{ \mathcal{W}^{-1}, \mathcal{V} \}) \) in \( \Omega(z) \) can also be factorized into block-lifting structures as in Eq. (4). Finally, the residual part \( \bar{\mathcal{W}}J_M \) in Eq. (5) and \( \text{diag}(\{ J_N, J_N \}) \) in \( \Omega(z) \) are collectively factorized as

\[
\begin{bmatrix}
J_N & 0 \\
0 & J_N
\end{bmatrix}
\bar{\mathcal{W}}J_M = \mathcal{U}
\begin{bmatrix}
1/2 & J_N \\
0 & \mathcal{L}[J_N]
\end{bmatrix}J_M,
\]

where \( \mathcal{W} = \mathcal{U}J_N\mathcal{V} \), because the block diagonal matrix \( \text{diag}(\{ \mathcal{W}^{-1}, \mathcal{V} \}) \) in \( \Omega(z) \) can also be factorized into block-lifting structures as in Eq. (4). Finally, the residual part \( \bar{\mathcal{W}}J_M \) in Eq. (5) and \( \text{diag}(\{ J_N, J_N \}) \) in \( \Omega(z) \) are collectively factorized as

\[
\begin{bmatrix}
J_N & 0 \\
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\end{bmatrix}
\bar{\mathcal{W}}J_M = \mathcal{U}
\begin{bmatrix}
1/2 & J_N \\
0 & \mathcal{L}[J_N]
\end{bmatrix}J_M,
\]

where \( \mathcal{W} = \mathcal{U}J_N\mathcal{V} \), because the block diagonal matrix \( \text{diag}(\{ \mathcal{W}^{-1}, \mathcal{V} \}) \) in \( \Omega(z) \) can also be factorized into block-lifting structures as in Eq. (4). Finally, the residual part \( \bar{\mathcal{W}}J_M \) in Eq. (5) and \( \text{diag}(\{ J_N, J_N \}) \) in \( \Omega(z) \) are collectively factorized as

\[
\begin{bmatrix}
J_N & 0 \\
0 & J_N
\end{bmatrix}
\bar{\mathcal{W}}J_M = \mathcal{U}
\begin{bmatrix}
1/2 & J_N \\
0 & \mathcal{L}[J_N]
\end{bmatrix}J_M.
\]

TABLE I

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<th>Block Size</th>
<th>DCT-Constrained</th>
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<tr>
<td>8</td>
<td>9.4475</td>
<td>9.4538</td>
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<td>16</td>
<td>9.8455</td>
<td>9.8621</td>
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TABLE II

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<tbody>
<tr>
<td>8</td>
<td>48</td>
<td>28</td>
<td>32</td>
</tr>
<tr>
<td>16</td>
<td>96</td>
<td>56</td>
<td>48</td>
</tr>
<tr>
<td>( M )</td>
<td>( 2M )</td>
<td>( 3M/2 )</td>
<td>( 3M )</td>
</tr>
</tbody>
</table>

simplified as

\[
\Omega(z) \triangleq \begin{bmatrix} I_N & 0 \\ 0 & \mathcal{V}^{-1}J_N \end{bmatrix} \Psi(z) \begin{bmatrix} I_N & 0 \\ 0 & \mathcal{V}J_N \end{bmatrix}
\]

\[
= \mathcal{L}[\mathcal{W}^{-1}] \mathcal{U} \begin{bmatrix} 1/2 \mathcal{W} \end{bmatrix} \Lambda(z) \mathcal{U} \begin{bmatrix} 1/2 \mathcal{W} \end{bmatrix}
\]

\[
= \mathcal{L}[\mathcal{W}^{-1}] \mathcal{U} \begin{bmatrix} 1/2 \mathcal{W} \end{bmatrix} \Lambda(z) \mathcal{U} \begin{bmatrix} 1/2 \mathcal{W} \end{bmatrix}
\]

\[
= \mathcal{L}[\mathcal{W}^{-1} - \mathcal{V}^{-1}] \mathcal{U} \mathcal{V} \mathcal{L}[-\mathcal{V}^{-1}] \begin{bmatrix} J_N & 0 \\ 0 & J_N \end{bmatrix},
\]

where \( \mathcal{W} = \mathcal{U}J_N\mathcal{V} \), because the block diagonal matrix \( \text{diag}(\{ \mathcal{W}^{-1}, \mathcal{V} \}) \) in \( \Omega(z) \) can also be factorized into block-lifting structures as in Eq. (4). Finally, the residual part \( \bar{\mathcal{W}}J_M \) in Eq. (5) and \( \text{diag}(\{ J_N, J_N \}) \) in \( \Omega(z) \) are collectively factorized as

\[
\begin{bmatrix}
J_N & 0 \\
0 & J_N
\end{bmatrix}
\bar{\mathcal{W}}J_M = \mathcal{U}
\begin{bmatrix}
1/2 & J_N \\
0 & \mathcal{L}[J_N]
\end{bmatrix}J_M.
\]

IV. EXPERIMENTAL RESULTS

A. Coding Gain, Frequency Response, and Number of Rounding Operations

We designed \( 8 \times 16 \) and \( 16 \times 32 \) XBL-LTs by optimizing the coding gain (CG) [14]

\[
\text{CG [dB]} = 10 \log_{10} \frac{\sigma^2}{\prod_{k=0}^{M-1} \sigma^2_{x_k} \| f_k \|^2},
\]

where \( \sigma^2 \) is the variance of the input signal, \( \sigma^2_{x_k} \) is the variance of the \( k \)-th subbands and \( \| f_k \|^2 \) is the norm of the \( k \)-th synthesis filter. To simplify the design in DCT-unconstrained case, we set \( \bar{\mathcal{U}} \) and \( \bar{\mathcal{V}} \) in Eq. (6) as \( N \times N \) arbitrary unitary matrices, where \( \bar{\mathcal{U}} \) is designed such that it has structural one-degree regularity [10] to achieve good image coding. Table I compares of the CGs[dB] of the XBL-LTs in the DCT-constrained and unconstrained cases. In addition, Fig. 3 shows the frequency responses of the analysis and synthesis parts of the XBL-LTs for the same cases as in Table I. In Table I and Fig. 3, the DCT-unconstrained case had slightly better results than the DCT-constrained case. Table II shows the numbers of rounding operations of BL-LTs. It is clear that the XBL-LTs have fewer rounding operations than those of conventional BL-LTs.

B. Lossy-to-Lossless Image Coding

For convenience regarding the number of pages, lossy-to-lossless image coding was implemented in only the \( M = 8 \) case. We used L-LTs in [7], [9], [11], [12] as the conventional L-LTs. The L-LT in [7] is the \( 4 \times 8 \) L-LT for JPEG XR. The L-LT in [9] is the \( 8 \times 16 \) TDLT with the pre-filtering part indicated by Fig. 5 in Table V in [9] and the DCT part indicated by [15]. The L-LTs in [11], [12] are the DCT-constrained BL-LTs. After the images were transformed by the L-LTs and periodic extension at the boundaries, the transformed coefficients were rearranged from the subband mode to the multiresolution mode similar to the wavelet transform. They were encoded with a common wavelet-based zerotree.

Fig. 2. XBL-LT (black and white circles mean adders and rounding operations, respectively).

Fig. 3. Frequency responses of analysis and synthesis parts of XBL-LTs (dashed and solid lines indicate DCT-constrained and unconstrained cases, respectively): (top) \( 8 \times 16 \) XBL-LT, (bottom) \( 16 \times 32 \) XBL-LT.
In [11], we developed an XBL-LT with fewer rounding operations. It is DCT-unconstrained and hence can be DCT-constrained as well; i.e., it can be considered to be a more general structure than other BL-LTs. Although we constrained the design by using unitary matrices in this paper, the DCT-unconstrained structures have the potential to achieve better coding.

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