PAPER

A Study on a Hybrid Dilated Banyan Network

Komain PIBULYAROJANA†, **Shigetomo KIMURA**††, *and* **Yoshihiko EBIHARA**††, *Members*

SUMMARY Banyan networks are used in multiprocessor computer applications for an ATM switching. In this paper, we study the continuous blocking of the the first n-stage which makes the performance of the banyan networks decrease. We use the 2-dilated banyan networks into the banyan networks to remove the the continuous blocking of the the first n-stage. We call the new networks as the hybrid dilated banyan networks. We explain how to analyze the throughput of this networks at each stage. Based on the analysis of input rate and output rate at each stage, we can design the hybrid dilated banyan networks with the desirable output rate. The result of analysis shows the hybrid dilated banyan networks have higher performance and feasibility than the banyan networks.

key words: *banyan network, switching, switching element, self-routing networks, input rate, output rate, blocking*

1. Introduction

For reasons concerning the economy and flexibility, B-ISDN (Broadband Integrated Services Digital Network) is expected to replace the existing application-oriented communication networks. ATM (Asynchronous Transfer Mode) is a high-speed packet-switching technique [1], [2] that has emerged as the most promising technology for B-ISDN.

ATM is a high-speed connection-oriented packet-switching technique with minimal functionality in the network which differs from the conventional packet switching in many aspects. First, high-level protocol functions such as error control are performed on an end-to-end basis, not such as link-by-link basis, in ATM. Furthermore, ATM uses short fixed-length packets called *cells*. Each cell consists of 53 bytes; 48 for the payload and 5 for the header or label [3]. This choice simplifies the design of switches, and reduces delay and jitter especially for delay-sensitive services such as voice and video.

In the ATM, a switch of size N can be regarded as a box with N input ports and N output ports, which routes cells arriving at its input ports in a time-slotted fashion to their desired output ports (destined by its headers). All input lines are synchronized. The minimum slot size is equal to the transmission time of a single cell. Input and output lines are assumed to operate at the same speed. Each arriving cell carries two parts-the ATM header and the payload. Prior to entering the switching fabric, a cell is provided with a local switching header that is used within the switch under the following two parts:

1. An activity bit (a) to indicate the presence of a cell when $a = 1$.
2. An address field which contains the address of the local destination port. N is assumed to be a power of two, in this case the address field will be in a binary form $d_1 d_2 \ldots d_n$, where $n = \log_2 N$ and d_1 is the most significant bit.

In ATM switching fabrics, all cells flow through a single resource that is shared by all input and output ports. This resource may be either a common memory or a shared medium such as a ring. So that many cells may simultaneously be transmitted across the switching fabric [4].

A number of architectures of banyan networks have been proposed for an ATM switching. But there is a blocking problem in the first stage that makes the throughput of a banyan network decrease.

In this paper, we improved the throughput of a banyan network by replacing each single internal link of the first n-stages with double ones. We call this improved network as the hybrid dilated banyan network. We show that the hybrid dilated banyan network can be designed with lower blocking probabilities and the higher feasibility.

2. Background

Using ATM technology, self-routing networks [5], [6] are being considered for constructing the interconnection fabric of the switching system. Many switching systems allow distributed control of the individual switches in the network and can be advantageous over conventional designs in high-speed systems.

2.1 Banyan Networks

A popular method for realizing the self-routing switching fabric is the banyan network. A banyan network is

Manuscript received April 9, 1996.
Manuscript revised September 9, 1996.

†The author is with Doctoral Programs of Engineering, University of Tsukuba, Tsukuba-shi, 305 Japan.
††The authors are with Institute of Electronics and Information Sciences, University of Tsukuba, Tsukuba-shi, 305 Japan.

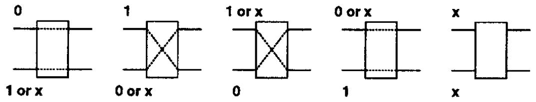

Fig. 1 2×2 switching elements in a network.

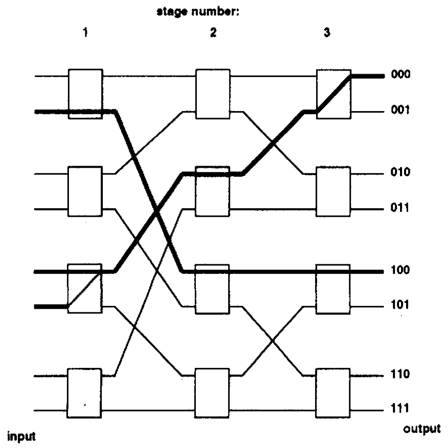

Fig. 2 8×8 banyan network with three stages.

defined as a network with a unique part between each input/output pair, where each stage performs a fixed permutation on the incoming lines, and then routes them through a column of 2×2 switching elements to the output as shown in Fig. 1.

Figure 2 shows an 8-input routing network known as the banyan network. It has three stages, each consisted of exchange permutation. The exchange is realized by the column of 2×2 switching elements which can be set to either a straight through connection or a crossed connection. The inputs to the network are labeled $d_0, d_1, \ldots, d_7$. An N-input network has $\log_2$N stages where N is a power of 2.

The routing algorithm of a banyan network is as follows: Each request contains an n-bit destination equal to the binary address of the requested output port. In the nth stage, a switch which receives a request examines the nth most significant bit in the destination and selects an output port based on the value of the bit. If the bit is a 0, the upper output port is selected, and if the bit is a 1, the lower output port is selected. If only one request selects a particular output port, it is forwarded on the single output link. When two requests arrive at a switch at the same time and select the same output port, link debation has occurred. One request is selected randomly and forwarded, and the other is blocked. The blocking in the switch makes the performance of the banyan network decreased. In this paper, we discuss how to improve the performance of the banyan network in next chapter.

Figure 2 shows the attempted routing of requests from 1 to 4, from 4 to 0 and from 5 to 4, respectively. In the case of the request from the 1st input port to the 4th output port, no blocking occurs in the routing. In the case of the request from the 4th input port to the 0st output port and from the 5th input port to the 4th output port, two requests are involved in same link debation in the first stage and only one is forwarded. If the switch network is busy, the winner of that debation from the first stage is also involved in link contention in the next stage.

2.2 Input Rate and Output Rate of Banyan Networks

We consider cell-switching networks built of 2×2 unbuffered switches with the topology of an n-stage square banyan network [7]–[9]. When several cells at the same switch require the same output, a randomly chosen one is forwarded and the remaining cells are deleted. The relevant figure of merit for such networks is the probability p_m that there are some cells on any particular input at the mth stage of the networks. By the above lemma, it is easily seen that p_m is well defined and satisfies the recurrence relation

$$p_{m+1} = 1 - \left(1 - \frac{p_m}{2}\right)^2 \tag{1}$$

With boundary condition $p_0 = p$, where p is the probability of cell arrival rate at an input port [10]. We call an input rate.

2.3 Dilated Banyan Networks

A *d-dilated* banyan network [11] is obtained from an ordinary banyan network by replacing each link by independent d links (in this paper we use 2-dilated). It can be constructed by multiplexing space of each link in the regular banyan so that it can support up to d connections.

The routing algorithm for these networks is a simple generalization of that used in unipath networks. Each request carries an n-bit destination, and a switch in stage n selects the same output port based on the nth most significant bit of the destination (as shown before). When one or two requests arrive at a switch and select the same output port, each request can be forwarded. If three or more requests arrive at a switch output port, then two requests are randomly selected and forwarded and the others are blocked.

2.4 Input Rate and Output Rate of 2-Dilated Banyan Networks

We consider the output performance of a 2-dilated banyan network by the following assumptions: 1) Every processor generates random and independent requests. 2) At the beginning of every cycle, each processor generates a new request with a probability of p_m. Thus, m is also the stage number of requests generated per cycle by each processor. 3) The requests which are blocked (unaccepted) are ignored. And the requests issued at the next cycle are independent of the blocked requests.

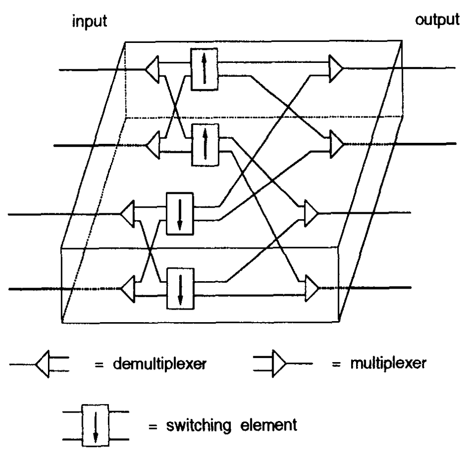

Fig. 3 A switching element for 2-dilated banyan network.

4) An input rate is defined as the probability number of cell arrival rate at an input port.

Figure 3 shows the construction of the switching elements for a 2-dilated banyan network. The demultiplexer examines the bit destination in a cell, selects a link to pass the cell based on the value of the bit. If the bit is 0, the upper link is selected, and if the bit is 1, the lower output link is selected. The switching element is shown as a box with an arrow. If only one input port of a switching element has a cell on it, the cell is forwarded to the upper output port if the arrow in the box points upward and to the lower output port if it points down. If both input ports of a switching element have cells on them, they are both passed straight through. The multiplexer can forward at least one cell to the output port of a switching element. After the arriving cells, when several cells at the same switch require the same output port, the 2-dilated banyan switch can randomly choose only two cells to be forwarded, and the remaining cells are deleted. We give the following, the probability, p_m, that an edge will contain cells per cycle. p_m represents also the input rate, and p_{m+1} is defined as the output rate after the first m stages of a 2-dilated banyan network. By the above lemma, we found that p_m is defined and satisfies the recurrence relation (see detail in Appendix A·2)

$$p_{m+1} = p_m - \frac{1}{4}{p_m}^3 + \frac{1}{16}{p_m}^4. \tag{2}$$

3. Hybrid Dilated Banyan Networks

The blocking in the first n-stages of each switching elements in banyan networks is a big problem in using the banyan networks for cell self-routing switch. Because of blocking in the first n-stages of each switching elements, many cells will be dropped before examining the destination in the banyan switch.

We consider cells arrived at the same switching elements will be blocked when the binary number of the address field is the same. An address field in the cell header which contains the address of the local destination port is a binary digit $d_1 d_2 \ldots d_n$ where d_1 is the most significant bit. For example, let's consider the blocking problems of 8×8 banyan switch.

1. When the attempted routings of requests are from 1 to 5 and from 0 to 7, the address fields of these requests are $1_1 0_2 1_3$ and $1_1 1_2 1_3$. We can predict the blocking will occur in the first stage. If we could remove the blocking at the first stage, two requests would not block each other.
2. When the attempted routings of requests are from 1 to 6 and from 0 to 7, the address fields of these requests are $1_1 1_2 0_3$ and $1_1 1_2 1_3$. We predict the blocking will occur in the first stage and the second stage. If we could remove the blocking at the first stage and the second, two requests will not block each other.
3. When the attempted routings of requests are from 1 to 7 and from 0 to 7, the address fields of these requests are $1_1 1_2 1_3$ and $1_1 1_2 1_3$. We predict the blocking will occur at each stage. Even if we could remove the blocking in the first stage and the second stage, these two requests would still be blocked together at the third stage.

From the blocking of above cases, when cells arrived at the same switching elements. We found that the probability of blocking at the 1st stage is 1/2, 2nd stage is 1/4, and 3rd is 1/8. Thus, the continuous blocking at the nth stage must be $1/2^n$. If n is too large, we don't have to get rid of the continuous blocking from the 1st stage to the nth stage. We should remove the continuous blocking of the first n-stage. Then, the performance of the banyan networks will be improved.

3.1 Using the Hybrid Dilated Banyan Networks

The performance of a banyan network can be improved by replacing each internal link of a banyan network with d links. Figure 4 shows the network structure for $d = 2$. This is known as the *dilated banyan networks.*

Hence, we replace the 2-dilated banyan switch in the first n-stages of a banyan network. Then the continuous blocking in the first n-stages is removed, and the throughput of such a banyan network can be improved. Besides, in this paper we set two kinds of re-arrangeable switches as follows.

1. Setting a 2×4 re-arrangeable input switch at the first stage to improve the effective utilization of input ports. (see detail in Appendix A·3)
2. Setting a 4×2 re-arrangeable output switch at the last stage before the stage of banyan networks to connect at the front stage of the banyan network each other. (see detail in Appendix A·4)

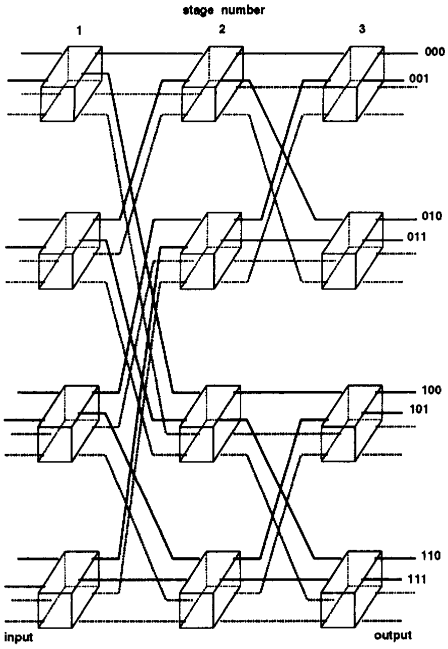

Fig. 4 2-dilated 8×8 banyan network with three stages.

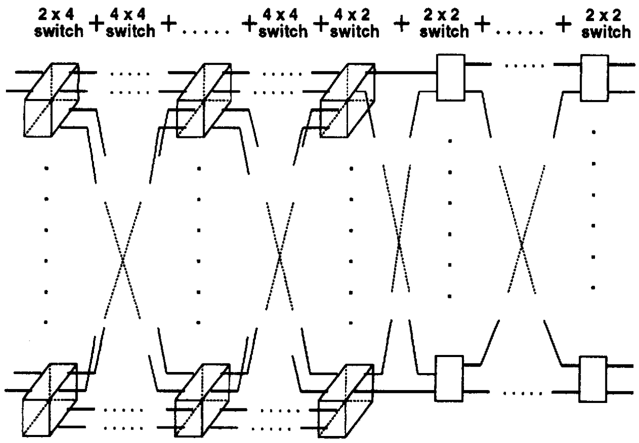

Fig. 5 The structure of hybrid dilated banyan network.

The Hybrid Dilated Banyan Networks are constructed with 2×4 re-arrangeable input switches in the first stage, the 2-dilated banyan networks of first n-stages, 4×2 re-arrangeable output switches before the connection with ordinary banyan network. This network structure is shown in Fig. 5. And the 16×16 hybrid dilated banyan network is shown in Fig. 6.

The routing algorithm of hybrid dilated banyan networks is as follows: Each request (the output address label) contains an n-bit destination. In the nth stage, a switching element which receives a request examines the nth most significant bit in the destination and selects an output port based on the value of the bit. If the bit is a 0, the upper output port is selected, and if the bit is a 1, the lower output port is selected.

We divide the hybrid banyan networks by stages into the following parts. (If the hybrid dilated banyan networks have n-stages and d is the number of stages in the proposed 2-dilated banyan network, then $n > d$.)

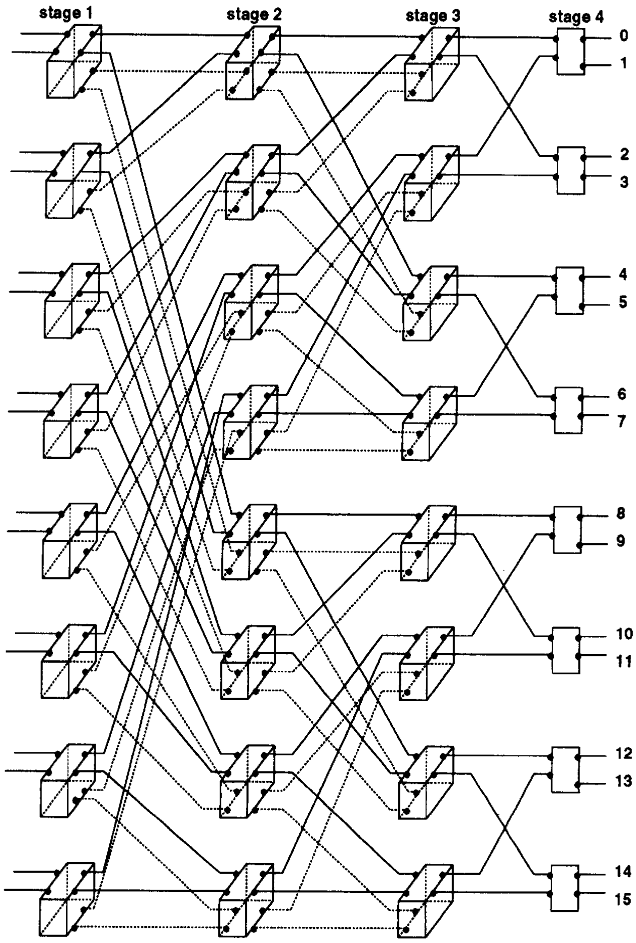

Fig. 6 The 16×16 hybrid dilated banyan network.

- In the first stage of hybrid dilated banyan networks, all requests are forwarded. Although two requests arrive at the same switch and select the same output port, two requests are randomly forwarded to the output port by the 2×4 re-arrangeable input switch, and go to the next stage by using 2 internal links. Thus, the blocking at the first stage can be removed.

- From the second stage to the $(d+1)$th stage, the 2-dilated banyan network is used. When one or two requests arrive at a switch and select the same output port, each requests can be forwarded. If more than two requests arrive at the same output port, then two requests are randomly selected to the output port. And the others are blocked.

- In the $(d+2)$th stage, We use 4×2 re-arrangeable output switch in this stage to connect a banyan network as the next stage. Here, the requests from 4 input ports are passed to 2 output ports. When requests arrive at a switch and select the same output port, link contention occurs. Only one request is selected at random and forwarded, and the others are blocked.

- From the $(d+3)$th stage to the nth stage, a banyan network is used. When two requests arrive at a switch and select the same output port, link debation occurs. One request is selected at random and forwarded, and the other is blocked.

3.2 Input Rate and Output Rate of Hybrid Dilated Banyan Networks

In this paper, we consider the input and output rate of hybrid dilated banyan networks by setting the following assumptions: 1) Every processor in a hybrid dilated banyan network generates random and independent requests. 2) At the beginning of each time cycle, each processor generates a new request with a probability of input rate for the next processor. 3) The blocked requests are neglected. Therefore, the requests issued at the next time cycle are independent of the former ones.

By the above conditions, we analyze the output rate of $2^n \times 2^n$ hybrid dilated banyan networks with n-stages. First, let's assume $p_0 = p$, where p is the probability of cell arrival rate at an input port. At the first stage, the output rate is calculated as the following relation (Appendix A·3)

$$p_1 = \frac{1}{2}p. \tag{3}$$

Secondly, we use the result of (3) as an input rate until the $(d+1)$th stage, where $d(\ d+2 < n\ )$ is the number of stages. As we use 2-dilated banyan networks, we get an output rate at the $(d+1)$th stage by using this equation (Appendix A·2)

$$p_{d+1} = p_d - \frac{1}{4}{p_d}^3 + \frac{1}{16}{p_d}^4, \tag{4}$$

with boundary condition $p_0 = p_1$, where p_1 is output rate from the output port of the first stage.

Thirdly, we use the result of (4) as an input rate of the $(d+2)$th stage. Thus, the output rate results in the following relation (Appendix A·4)

$$p_k = 2p - \frac{3}{2}p^2 + \frac{1}{2}p^3 - \frac{1}{16}p^4. \tag{5}$$

Where p is the result from (4), and p_k is the output rate of the $(d+2)$th stage.

Fourthly, we use p_k as an input rate of the $(d+3)$th stage. Under the condition $p_0 = p_k$, q is the number from 0 to $(n-d-2)$. Thus, we can find the output rate in the nth stage by using this relation (Appendix A·1)

$$p_{q+1} = p_q - \frac{1}{4}{p_q}^2. \tag{6}$$

By applying all the above steps, we can obtain the output rate of $2^n \times 2^n$ hybrid dilated banyan networks.

Figures 7, 8, 9 and 10 show the output rate of hybrid dilated banyan networks compared to that the ordinary banyan network with the arrival rate $1.0, 0.75, 0.5$ and 0.25 (input rate)

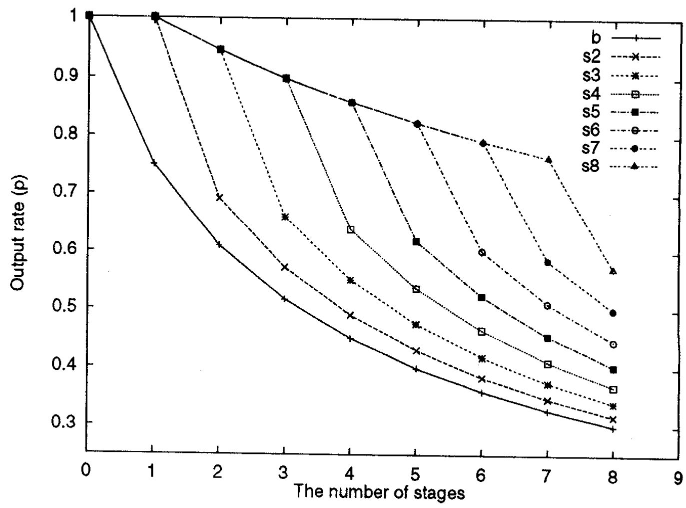

Fig. 7 Output rate vs. the number of stages at input rate = 1.

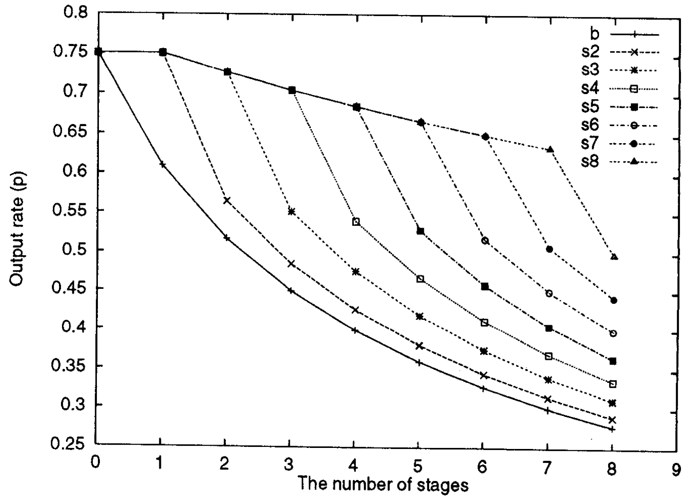

Fig. 8 Output rate vs. the number of stages at input rate = 0.75.

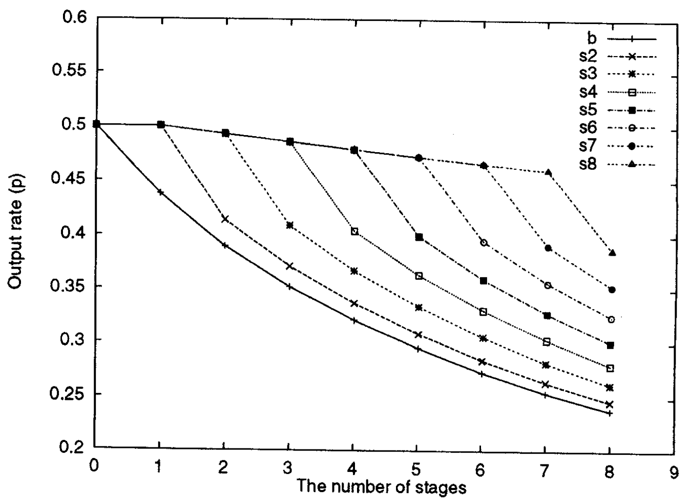

Fig. 9 Output rate vs. the number of stages at input rate = 0.50.

- b show the output rate of a banyan network.
- $s2$ shows output rate of a hybrid dilated banyan network with using 2×4 re-arrangeable input

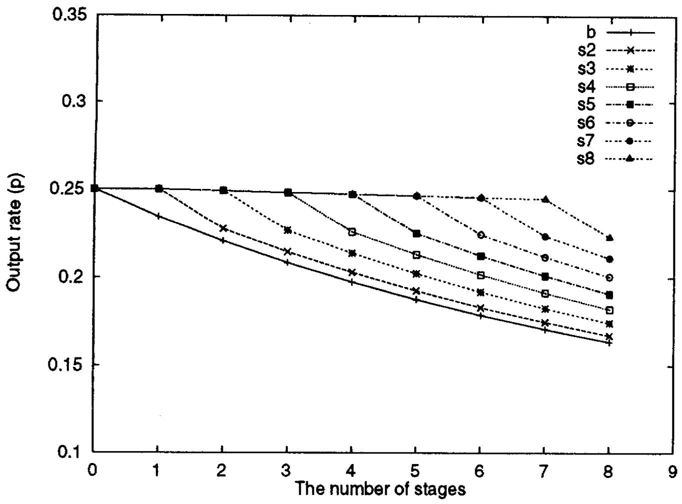

Fig. 10 Output rate vs. the number of stages at input rate = 0.25.

switches at the first stage, 4×2 re-arrangeable output switches at the second stage and banyan network from the 3th stage to the 8th.

- si (i = 3–7) shows output rate of a hybrid dilated banyan network with 2×4 re-arrangeable input switches at the first stage, ($i-2$) stages of 2-dilated banyan networks from the second stage to the ($i-1$)th stage, 4×2 re-arrangeable output switches are set at the ith stage. The rest of the hybrid dilated network from the ($i+1$)th stage to the 8th stage consists of a banyan network.

- $s8$ shows output rate of a hybrid dilated banyan network with using 2×4 re-arrangeable input switches at the first stage, six stages of 2-dilated banyan networks at the 2th to the 7th stage, 4×2 re-arrangeable output switches at the 8th stage without banyan networks connection.

From Figs. 7, 8, 9 and 10, we can conclude as follows.

1. In the network that has not much traffic (In this paper, we assigned the input rate at 0.25 and 0.5), the blocking probability is low in each stage of the hybrid dilated banyan network and the ordinary banyan network. So the using of hybrid dilated banyan network has a little bit better performance than the ordinary banyan network.

2. In the heavy traffic network (we assigned the input rate at 0.75 and 1.0), the blocking probability in each stage of the hybrid dilated banyan network are high. The hybrid dilated banyan network can remove the blocking at a level. This means that the performance of the network increases. The hybrid dilated banyan network is suitable for the heavy traffic network.

3. From the view point of the $2^n \times 2^n$ ATM switch with the given input rate, we can design and build the effective network with the suitable types of hybrid dilated banyan network. For example, if we want to design a $2^5 \times 2^5$ ATM switch with 1.0 input rate under required the output rate (throughput) more than 0.4, the ordinary $2^5 \times 2^5$ banyan network can not perform the output rate (throughput) more than 0.4. When the hybrid dilated banyan network is chosen, we can select the $s2$–$s5$ type of hybrid dilated banyan network as shown in Fig. 7 to design a network with high performance.

4. Comparing Hybrid Dilated Banyan Network with Delta, Rerouting Banyan and Tandem Network

4.1 Routing Process and Hardware Size

A delta network is an $a^n \times b^n$ switching network with n-stages, consisting of $a \times b$ crossbar modules (more cost-effective if b is a power of 2) [8], [12]. From this condition, we consider a $2^n \times 2^n$ delta network which is constructed from the 2×2 switching element and $4^n \times 4^n$ delta network which is constructed from 4×4 switching element. The 4×4 switching element has 4 input ports, 4 output ports and uses the same routing process as a 2×2 switching element. The hardware size of a 4×4 switching element is nearly 4 times as large as a 2×2 switching element. The routing process is shown as the ordinary banyan network in Sect. 2.1. Let N be 2^n then the $N \times N$ delta network has n-stages. The delta network which is constructed from the 4×4 switching element has $\frac{n}{2}$ stages, since $N = 4^{\frac{n}{2}}$. The hardware size of the $2^n \times 2^n$ delta network is $\frac{nN}{2}$ 2×2 switching elements and the $4^{\frac{n}{2}} \times 4^{\frac{n}{2}}$ delta network is $\frac{n}{2} \times \frac{N}{4}$ 4×4 switching elements. Because the 4×4 switching element hardware size is nearly 4 times as large as the 2×2 switching element. We can conclude that the $4^{\frac{n}{2}} \times 4^{\frac{n}{2}}$ delta network have the same hardware size as the $2^n \times 2^n$ delta network.

The $N \times N$ rerouting banyan network [13] has N input ports, N output ports and the through links without buffers. When two cells are blocking in a switching element of this network, one of the cell is correctly routed. The other is routed by using the other link and start a new routing at the next stage. So the rerouting banyan network needs specific switching elements for rerouting the blocking cells. We estimate the hardware size of this network. Let n be the number of stages of the ordinary banyan switching elements ($n = \log_2 N$) and m be the number of stages of the specific switching elements. Then, this network is consisted of $\frac{nN}{2}$ ordinary banyan switching elements and $\frac{mN}{2}$ specific switching elements. The total of the switching elements is $\frac{(n+m)N}{2}$.

The $N \times N$ tandem banyan network structure consists of K banyan networks connected in series(each banyan network has N input ports and N output ports).

Each output of a banyan network is connected to both an input of the subsequent banyan network and a concentrator (statistical multiplexer) [14], [15]. This network needs a link per one request from each banyan network to connect with a concentrator and requires N concentrators for N output ports. A cell would be routed to the concentrator if it reaches the correct output, otherwise to subsequent banyan network. Thus, each cell can have up to K attempts to reach its destined output. To achieve a low cell loss rate, the tandem banyan network may require many banyan networks. So we assume $K > 1$. Let us consider the hardware size of this network. The network is consisted of $\frac{nKN}{2}$ switching elements, where n is the number of switching elements in a banyan network.

The $N\times N$ hybrid dilated banyan network don't require buffers. Routing process is shown in Sect. 3.1. To estimate the hardware size of the network, let n be the number of stages in the network and x be the number of stages which we use the 2-dilated banyan network. Then, this network is consisted of $\frac{(3x+n)N}{2}$ of switching elements, $(2x+1)N$ demultiplexers and $(2x+1)N$ multiplexers.

4.2 Delay

A regular $N\times N$ banyan network is constructed from 2×2 switching elements organized into $n = \log_2 N$ stages, with each stage having $\frac{N}{2}$ switching elements. Let D_b refer to the constant time at which a cell is incurred at each stage and render D_b constant over all stages. So the switching time incurred within one banyan is nD_b.

The $N\times N$ delta network which is constructed from 2×2 switching element incurs the same switching time as the banyan network. So the switching time of $2^n\times 2^n$ delta network results in nD_b. The $N\times N$ delta network which is constructed from 4×4 switching element has $\frac{n}{2}$ stages. The switching time in the 4×4 switching element is almost 2 times as much as the 2×2 switching element. Therefore, the $4^{\frac{n}{2}}\times 4^{\frac{n}{2}}$ delta network can be assumed to has the same switching time as the $2^n\times 2^n$ delta network.

Rerouting banyan network is connected by two parts, the banyan switch and the rerouting switches in series with the through links inside. Although each specific switching element takes times more than the ordinary banyan switching element to reroute the blocking cells, we assume both of the switching elements nearly incurred the same switching time for simple calculation. When m is the number of stages that use the specific switching elements and n is the number of stages of the ordinary banyan switching elements, the delay of switching time in the rerouting banyan network is $(n+m)D_b$ (Ignoring the propagation delays between networks and delay in the through links).

Tandem banyan network is constructed from K banyan networks connected in series. It requires the concentrator at each output port. We refer the maximum switching time is $KD_b+\alpha$, where α is the incurred time in the concentrator.

Hybrid dilated banyan network is constructed from 2×4 re-arrangeable input switch, 2-dilated banyan network, 4×2 re-arrangeable output switch and the banyan network (see each switching element construction in Appendix A). We assume that the constant delay within each demultiplexer is t_d and that of each multiplexer is t_m. The incurred time of each 2×2 switching element within the 2-dilated banyan network is D_s. Under the condition of $x+2 \leqq n$, where n is the number of stages in the network and x is the number of stages which are constructed by the 2-dilated banyan network. The delay incurred with in the $N\times N$ hybrid dilated banyan network is $(x+1)t_d+xD_s+(x+1)t_m+(n-x-1)D_b$. We assume the switching time in a 2×2 switching elements nearly equals to the switching time of 2×2 switching element in the 2-dilated banyan network ($D_s \approx D_b$), then the delay time is $(x+1)t_d+(x+1)t_m+(n-1)D_b$. The 2×2 switching element can be constructed by two demultiplexers and two multiplexers. In this construction, a cell passes one demultiplexer and one multiplexer in series. So, the total of delay incurred in a demultiplexer and a multiplexer nearly equals to the switching time in a 2×2 switching element ($t_d+t_m \approx D_b$). We approximate the switching time in the hybrid dilated banyan network is $(n+x)D_b \geqq nD_b$.

4.3 Switching Efficiency

The output performance of the delta network is fixed. In these four kinds of networks, the delta network has the lowest output performance. By adding the banyan networks, the tandem network can get the highest output performance. The hybrid dilated banyan network has higher output performance than the rerouting banyan network in some cases. But if we put the stages of specific switching elements in the rerouting banyan network, the rerouting banyan network can perform higher output performance than the hybrid dilated banyan network.

For example, we compare the output performance of these networks at the $2^8\times 2^8$ ATM switch with 1.0 input rate. The $2^8\times 2^8$ delta network achieves the output rate $= 0.3003$ and the $4^4\times 4^4$ delta network achieves the output rate $= 0.3669$. Although the $4^{\frac{n}{2}}\times 4^{\frac{n}{2}}$ delta network gets better performance than the 2×2 delta network, the $4^{\frac{n}{2}}\times 4^{\frac{n}{2}}$ delta network can't be constructed if n is an odd number. The tandem banyan network (with the 5 banyan networks connected in series) performs at the output rate $= 0.9930$. The rerouting banyan network which has 1 to 6 stages of specific switching elements performs at the output rate from 0.3188 to 0.4540 and 7 to 27 stages of specific switching elements performs at the output rate from 0.4903 to 0.6431(this output rate is

Table 1 Comparing hybrid dilated banyan network with delta, rerouting banyan and tandem network.

	Delta	Rerouting banyan	Tandem	Hybrid dilated banyan
Switching efficiency ($2^8 \times 2^8$ switch with input rate 1.0)	0.3003 ($2^8 \times 2^8$ delta) and 0.3669 ($4^4 \times 4^4$ delta)	0.3188 to 0.6431	0.3003 to 0.9930	0.3168 to 0.5716
Hardware (SE. = switching elements)	$nN/2$ SE.	$(n+m)N/2$ SE.	$nKN/2$ SE.	$(3x+n)N/2$ SE. $+(2x+1)N$ demultiplexers $+(2x+1)N$ multiplexers
Delay	nD_b	$(n+m)D_b$	$nKD_b+\alpha$	$(n+x)D_b$
Flexibility	none	good	difficult	good

the maximum of the $2^8 \times 2^8$ rerouting banyan network without buffers). The hybrid dilated banyan network with 0 to 6 stages of 2-dilated banyan network performs at the output rate from 0.3168 to 0.5716.

4.4 The Flexibility of the Systems

Delta network has the same switching element and a fixed construction. This network can not be constructed under the required output rate. So this network is not flexible.

Rerouting banyan network is constructed by the banyan switches and the rerouting switches which have specific function for rerouting in the switching element. By adding a number of stages, the output performance can be increased at a level. So this network can be designed to satisfy the required performance.

Tandem network is constructed by connecting the banyan networks in series. By increasing the number of the banyan networks, this network can be constructed to get the required output rate. But the links which are connected with the concentrator at each output port are also increased. Therefore, each concentrator must be large. So the tandem network is difficult to design for the required performance.

Hybrid dilated banyan network is constructed from four kinds of switching elements. If the users require an output rate, the hybrid dilated banyan network can be constructed by referencing the analysis output performance in this paper. So this network is easy and flexible to design.

4.5 The Result of Comparison

We compared the hybrid dilated banyan network with delta, rerouting banyan and tandem network as shown in Table 1. The hybrid dilated banyan network has better switching efficiency and flexibility than the delta network. The hybrid dilated banyan network has shorter delay and better switching efficiency than the rerouting banyan network in some kinds of switching structure. The hybrid dilated banyan network has smaller delay, smaller hardware size and better flexibility than the tandem network.

5. Conclusion

We have presented in this paper how to improve the performance of banyan networks. We added many kinds of switches to design a new networks, named hybrid dilated banyan networks, which is easy to control and design. We derive the relationship between the output rate and the number of stages. When a desirable output rate or throughput is given, the reasonable number of stages is decidable from the Eq. (6). From the view point, this relationship will present a good sight to network designer. The next problem we concern are how to improve this networks without blocking.

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Appendix: Analysis Input Rate and Output Rate

A.1 Input Rate and Output Rate of Banyan Networks

Under the condition $p_0 = p$ (cell arrival rate), where p is the probability of cell creation at an input port.

Using the above model, we calculated the input rate and output rate in three cases.

- No cells arrive at any input ports. The output rate of this case is
$$(1-p)^2 \times 0 = 0.$$
- There is an arriving cell at one of the two input ports. The output rate of this case is
$$2p(1-p)\times\frac{1}{2} = p(1-p).$$
($2p(1-p)$ is the probability of the events that occur in all input ports and $\frac{1}{2}$ is the probability of the cell which is forwarded to an output port).
- There are arriving cells at both of input ports. The output rate of this case is
$$p^2\times\frac{3}{4} = \frac{3}{4}p^2.$$
(p^2 is the probability of the events that occur in all input ports and $\frac{3}{4}$ is the probability of the cell which is forwarded to an output port).

The total output rate of banyan networks per output port must be

$$p' = p - \frac{1}{4}p^2.$$

Therefore, the probability of the output rate is generally represented as follows.

$$p_{m+1} = 1 - \left(1 - \frac{p_m}{2}\right)^2,$$

where m is the number of stages [7], [10].

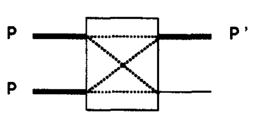


Fig. A·1 A 2 × 2 switching element.

A.2 Input Rate and Output Rate of 2-Dilated Banyan Networks

Under the condition $p_0 = p$ (cell arrival rate), where p is the probability of cell creation at an input port.

Using the above model, we calculated the input rate and output rate in the following cases.

- No cells arrive at any input ports. The output rate of this case is
$$(1-p)^4 \times 0 = 0.$$
- There is an arriving cell at one of four input ports. The output rate of this case is
$$4p(1-p)^3\times\frac{1}{4} = p(1-p)^3.$$
($4p(1-p)^3$ is the probability of the events that occur in an input port and $\frac{1}{4}$ is the probability of the cell which is forwarded to an output port).
- There are arriving cells at two of the four input ports. The output rate of this case is
$$6p^2(1-p)^2\times\frac{1}{2} = 3p^2(1-p)^2.$$
($6p^2(1-p)^2$ is the probability of the events that occur in an input port and $\frac{1}{2}$ is the probability of the cell which is forwarded to an output port).
- There are arriving cells at three of the four input ports. The output rate of this case is
$$4p^3(1-p)\times\frac{44}{64} = \frac{44}{16}p^3(1-p).$$
($4p^3(1-p)$ is the probability of the events that occur in an input port and $\frac{44}{64}$ is the probability of the cell which is forwarded to an output port).
- There are arriving cells at all of the four input ports. The output rate of this case is
$$p^4\times\frac{208}{256} = \frac{208}{256}p^4.$$
(p^4 is the probability of the events that occur in an input port and $\frac{208}{256}$ is the probability of the cell which is forwarded to an output port).

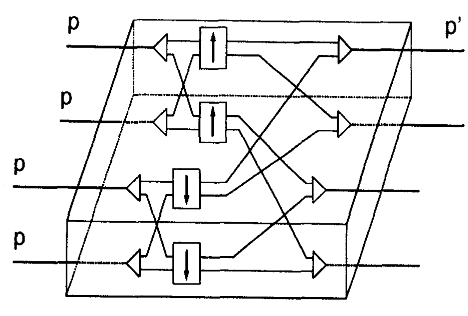


Fig. A·2 A 2-dilated banyan switching element.

The total output rate of the 2-dilated banyan networks per output port must be

$$p' = p - \frac{1}{4}p^3 + \frac{1}{16}p^4.$$

Therefore, the probability of the output rate is generally presented as follows.

$$p_{m+1} = p_m - \frac{1}{4}{p_m}^3 + \frac{1}{16}{p_m}^4,$$

where m is the number of stages.

A.3 Input Rate and Output Rate of 2×4 Re-arrangeable Input Switch

Under the condition $p_0 = p$ (cell arrival rate), where p is the probability of cell creation at an input port. Using the presented model, we calculated the input rate and output rate in the following cases.

- No cells arrive at any input ports. The output rate of this case is

  $$(1-p)^2 \times 0 = 0.$$

- There is an arriving cell at one of the two input ports. The output rate of this case is

  $$2p(1-p)\times\frac{1}{4} = \frac{1}{2}p(1-p).$$

  ($2p(1-p)$ is the probability of the events that occur in all input ports and $\frac{1}{4}$ is the probability of the cell which is forwarded to an output port).

- There are arriving cells at both of the input ports. The output rate of this case is

  $$p^2\times\frac{1}{2} = \frac{1}{2}p^2.$$

  (p^2 is the probability of the events that occur in all input ports and $\frac{1}{2}$ is the probability of the cell which is forwarded to an output port).

The total output rate of 2×4 re-arrangeable input switch per output port must be

$$p_{out} = \frac{1}{2}p_{in},$$

where p_{in} is an input rate and p_{out} is an output rate.

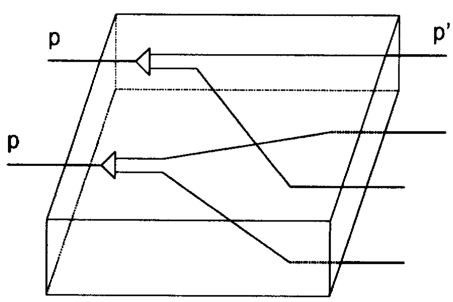


Fig. A·3 A 2 × 4 rearrangeable input switching element.

A.4 Input Rate and Output Rate of 4×2 Re-arrangeable Output Switch

Under the condition $p_0 = p$ (cell arrival rate), where p is the probability of cell creation at an input port.

Using the presented model, we calculated the input rate and output rate on the following cases.

- No cells arrive at any input ports. The output rate of this case is

  $$(1-p)^4 \times 0 = 0.$$

- There is an arriving cell at one of the four input ports. The output rate of this case is

  $$4p(1-p)^3\times\frac{1}{2} = 2p(1-p)^3.$$

  ($4p(1-p)^3$ is the probability of the events that occur in all input ports and $\frac{1}{2}$ is the probability of the cell which is forwarded to an output port).

- There are arriving cells at two of the four input ports. The output rate of this case is

  $$6p^2(1-p)^2\times\frac{3}{4} = \frac{9}{2}p^2(1-p)^2.$$

  ($6p^2(1-p)^2$ is the probability of the events that occur in all input ports and $\frac{3}{4}$ is the probability of the cell which is forwarded to an output port).

- There are arriving cells at three of the four input ports. The output rate of this case is

  $$4p^3(1-p)\times\frac{7}{8} = \frac{7}{2}p^3(1-p).$$

  ($4p^3(1-p)$ is the probability of the events that occur in all input ports and $\frac{7}{8}$ is the probability of the cell which is forwarded to an output port).

- There are arriving cells at all of the four input ports. The output rate of this case is

  $$p^4\times\frac{15}{16} = \frac{15}{16}p^4.$$

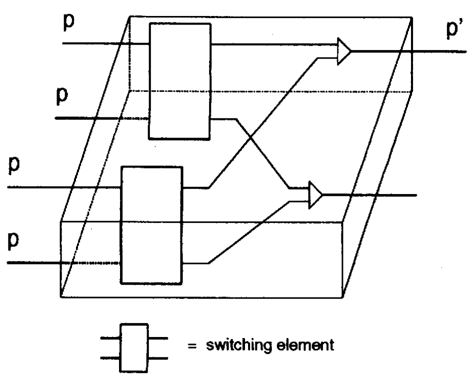


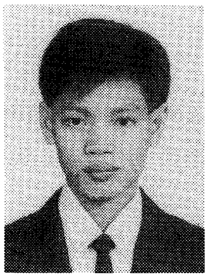
Fig. A·4 A 4 × 2 rearrangeable output switching element.

(p^4 is the probability of the events that occur in all input ports and $\frac{15}{16}$ is the probability of the cell which is forwarded to an output port).

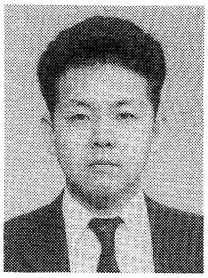
The total output rate of the 4×2 re-arrangeable output switch per output port is

$$p_{out} = 2p_{in} - \frac{3}{2}{p_{in}}^2 + \frac{1}{2}{p_{in}}^3 - \frac{1}{16}{p_{in}}^4,$$

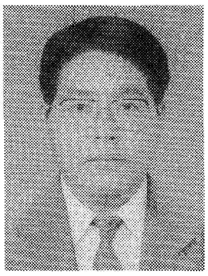
where p_{in} is the input rate and p_{out} is the output rate.



Komain Pibulyarojana was born in Bangkok, Thailand, on October 8, 1971. He received the B.E. degree in information engineering from the University of Tsukuba, Japan, in 1995. He is currently with Doctoral Programs of Engineering at the same university. His research interests are in the high speed network architecture and ATM switching networks. He is a member of IPSJ.



Shigetomo Kimura was born in 1967. He received the D. Info. degree in information science from Tohoku University, in 1995. He is currently a Lecturer of Institute of Information Sciences and Electronics, the University of Tsukuba, since 1995. His primary research interests are in the areas of algebraic formulation of concurrent processes and program synthesis by inductive inference. He is a member of IPSJ and JSSS.



Yoshihiko Ebihara was born in Ibaraki, Japan, on April 11, 1947. He received the B.E., M.E. and D.E. degrees in electrical communication engineering all from Tohoku University, in 1970, 1972 and 1978, respectively. From 1973 to 1975 he was on a staff of ALOHA system project in the University of Hawaii satellite communication network and also Hawaii node installation of ARPA computer network. In 1975 he was appointed a research assistant in the Institute of Electrical Communication, Tohoku University. He was appointed a lecturer and an Associate Professor at the Institute of Information Sciences and Electronics, the University of Tsukuba, in 1976 and 1985 respectively. He is currently a Professor of the Institute of Information Sciences and Electronics, the University of Tsukuba since 1993. His primary research interests include computer network, performance measurement, distributed data base management and digital communication system. He is a member of ISPJ.