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Title:
Stable Detonation Wave Propagation in Rectangular-Cross-Section Curved Channels

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Abstract

The detonation propagation phenomena in curved channels were experimentally studied in order to determine the stable propagation condition. A stoichiometric ethylene-oxygen mixture gas and five types of rectangular-cross-section curved channels with different inner radii of curvature were employed. The detonation waves propagating through the curved channels were visualized using a high-speed video camera. Multi-frame short-time open-shutter photography (MSOP) was developed in the present study to simultaneously observe the front shock shape of the detonation wave and the trajectories of triple points on the detonation wave. The detonation wave became more stable under the conditions of a higher filling pressure of the mixture gas and/or a larger inner radius of curvature of the curved channel. The critical condition under which the propagation mode of the detonation wave transitioned from unstable to stable was having an inner radius of curvature of the curved channel (\( r_i \)) equivalent to 21 to 32 times the normal detonation cell width (\( \lambda \)). In the stable propagation mode, the normal detonation velocity (\( D_n \)) increased with the distance from the inner wall of the curved channel and approached the velocity of the planar detonation propagating through the straight section of the curved channel (\( D_{str} \)). The smallest \( D_n \) was observed on the inner wall and decreased with decreasing \( r_i/\lambda \). The distribution of \( D_n \) on the detonation wave in the stable mode was approximately formulated. The approximated \( D_n \) given by the formula agreed well with the experimental results. The front shock shape of the detonation wave could be reconstructed accurately using the formula. The local curvature of the detonation wave (\( \kappa \)) nondimensionalized by \( \lambda \)
decreased with increasing distance from the inner wall. The largest $\lambda\kappa$ was observed on the inner wall and increased with increasing $r_i/\lambda$. $D_n/D_{at}$ decreased with increasing $\lambda\kappa$. This nondimensionalized $D_n-\kappa$ relation was nearly independent of $r_i/\lambda$. (300 words)

Keywords

Detonation wave; curved channel; cell width; triple point trajectory; curvature

Nomenclature

$D$ Detonation velocity

$m$ Exponent in Eq. (12)

$p$ Pressure

$r$ Distance from polar coordinate origin

$t$ Time

$T$ Temperature

Greek symbols

$\phi$ Angular difference between rotational direction and tangential direction of detonation wave

$\kappa$ Curvature of detonation wave
\( \lambda \) Detonation cell width

\( \theta \) Angle from initial line

\( \omega \) Angular velocity of detonation wave

Subscripts

\( 0 \) Initial

\( \text{asy} \) Asymptotic

\( \text{CJ} \) Chapman-Jouguet state

\( i \) Inner wall of curved section of flow channel

\( k \) Index of position on detonation wave

\( n \) Normal

\( \text{str} \) Straight section

\( \text{trans} \) Transient
1. Introduction

A detonation wave is a self-sustained combustion wave which propagates at supersonic speed and instantaneously generates a high pressure and temperature gas. The adiabatic compression due to the passage of the front shock of the detonation increases the temperature of the combustible mixture above the ignition limit, and the chemical reaction of the gas is induced. The chemical reaction results in a heat energy release, and the expansion of the burned gas drives the front shock forward. The detonation wave propagates self-sustainingly due to the mutual interaction between the ignition of the combustible gas initiated by the front shock and the expansion of the burned gas.

A rotating detonation engine (RDE) is a propulsion system which generates thrust by propagating a continuous detonation wave in an annular combustor (Braun et al. [1], Meredith et al. [2], Kindracki et al. [3], Hishida et al. [4], Falempin and Daniau [5]). Since the RDE cycle operates with near-constant volume combustion, RDE theoretically has a higher thermal efficiency than other conventional propulsion systems. The RDE needs only one ignition:deflagration-to-detonation-transition (DDT) process in principle, since the RDE utilizes a detonation wave that propagates continuously. The increase of the thrust density of the RDE is also considered to be easy since the propellant can be supplied continuously into the combustor chamber. For the purpose of RDE design, it is important that the conditions for stable detonation wave propagation in an annular combustor be well understood. However, the study of the stability of the detonation wave
propagating through a curved channel, which is a basic element of an annular combustor, is limited.

Several experimental and numerical studies on unsteady detonation waves in a curved channel (90 deg or 180 deg bend with a rectangular or circular cross-section) have been performed (Tsuboi et al. [6], Liang et al. [7, 8], Edwards et al. [9], Thomas and Williams [10], Frolov et al. [11-14]). By contrast, only a few studies of steady detonation waves in a curved channel have been conducted. Kudo et al. [15] visualized the detonation waves of a stoichiometric ethylene-oxygen mixture gas propagating through the rectangular-cross-section curved channels with constant inner/outer radii of curvature shown in Fig. 1. They experimentally demonstrated that detonation waves could propagate steadily and stably through the curved channels. Sasamoto et al. [16] also showed the existence of a detonation wave propagating stably in a curved channel by two-dimensional numerical simulation.

Kudo et al. [15] observed the propagation modes of the detonation waves in the curved channels. Using the normal detonation velocity on the inner wall of a curved channel \(D_{n,i}\) and CJ detonation velocity \(D_{CJ}\), they categorized the propagation modes into three types: the stable mode \(D_{n,i}/D_{CJ} \geq 0.8\), the critical mode \(D_{n,i}/D_{CJ} \geq 0.6\), and the unstable mode \(D_{n,i}/D_{CJ} < 0.6\). They also showed that the critical condition under which the propagation mode of the detonation wave in a curved channel transitioned from unstable to stable was having an inner radius of curvature of the curved channel equivalent to 14 to 40 times the normal detonation cell width. They used the detonation cell width from the Detonation Database (Kanesige and Shepherd [17], Abid et al. [18], Knystautas [19],
Strehlow [20]) of the California Institute of Technology to reduce their experimental data. They have also proposed a formula which gives a geometrical front shock shape of a detonation wave propagating through a curved channel at the CJ detonation velocity.

A simultaneous observation of the front shock shape of the detonation wave and the trajectories of triple points on the detonation wave is ideal as a way to experimentally investigate the conditions of a stable detonation wave propagating through a curved channel. We developed a new visualization method to perform such a simultaneous observation and examined the conditions of the stable detonation wave propagation in a curved channel by alternating the inner radius of the curved channel and the filling pressure of the mixture gas parametrically. We also propose a formula which gives a rigorous geometrical front shock shape for a detonation wave propagating stably through a curved channel and examine the relation between the normal detonation velocity and the local curvature of the detonation wave.

2. Geometric front shock shape of stable detonation wave in a curved channel

2.1 Geometric shape proposed by Kudo et al.

As shown in Fig. 1, the center of the inner/outer radius of curvature of a curved channel is defined as the origin, and the boundary between the curved section and the straight section of the curved channel is defined as the initial line in a two-dimensional polar coordinate system. The variable $r$
represents the distance from the origin, and $\theta$ is the angle from the initial line. If we assume that the angular velocity of the detonation wave propagating stably through a curved channel is constant everywhere on the detonation wave and the angular velocity is time-unvarying, the detonation wave propagates through the curved channel while maintaining a specific shape [15]. If $dr$, $dt$, and $d\theta$ are sufficiently small, one can consider the gray-colored areas in Fig. 1 to be right triangles. The geometric relations can then be established at an arbitrary point $P(r, \theta)$ on the detonation wave,

\[
\sin \phi = \frac{D_n}{r \omega}, \quad (1)
\]

\[
\tan \phi = \frac{\sin \phi}{\cos \phi} = \frac{1}{\sqrt{1 - \sin^2 \phi}} = \frac{1}{r} \frac{dr}{\omega^2} \quad (2)
\]

where $\phi$ is the angular difference between the rotational direction and the tangential direction of the detonation wave at $P(r, \theta)$, $D_n$ is the normal detonation velocity at $P(r, \theta)$, and $\omega$ is the angular velocity of the detonation wave. The following differential equation is derived from Eq. (1) and Eq. (2):

\[
\frac{d\theta}{dr} = -\frac{\sqrt{\rho^2 - D_n^2}}{D_n r}. \quad (3)
\]

If $D_n$ is equal to the CJ detonation velocity ($D_{CJ}$) everywhere on the detonation wave and the detonation wave is perpendicular to the inner wall of the curved channel, $D_n$ is equal to the product of the inner radius of curvature of the curved channel ($r_i$) and $\omega$. Thus, the integration of Eq. (3) gives a geometrical front shock shape of the detonation wave propagating through a curved channel.
at a constant normal detonation velocity of $D_{CJ}$ in terms of the relation between $r$ and $\theta$ as follows [15]:

$$\theta - \theta_i = -\sqrt{\left(\frac{r}{r_j}\right)^2 - 1} \tan^{-1}\left(\sqrt{\left(\frac{r}{r_j}\right)^2 - 1}\right),$$  \hspace{1cm} (4)

where $\theta_i$ is the angle from the initial line to the detonation wave front on the inner wall of the curved channel. The front shock shape of the detonation wave given by Eq. (4) becomes independent of $D_n$ since $D_n$ is assumed to be constant everywhere on the detonation wave.

The experiment by Kudo et al. [15] showed that $D_n$ on the inner wall of the curved channel is lower than $D_{CJ}$ due to the expansion waves from the inner wall, even though the detonation wave propagates stably in the curved channel. Hence, Eq. (4) is applicable only under the limited condition of a sufficiently high filling pressure of mixture gas and a large inner radius of curvature of the curved channel in which $D_n$ becomes almost equivalent to $D_{CJ}$. Some modifications are considered to be necessary in Eq. (4) to reconstruct the front shock shape of the detonation wave propagating stably through a curved channel in other conditions. In order to derive a formula which gives a rigorous geometrical front shock shape of the detonation wave under a wide range of conditions, the deficit of $D_n$ caused by the expansion waves from the inner wall should be taken into account.
2.2 Geometric shape proposed in the present study

If the composition and temperature of the mixture gas are fixed, the velocity of the planar detonation wave propagating through the straight section of a curved channel ($D_{str}$) and the normal detonation cell width ($\lambda$) can be determined. Therefore, only the four variables, $r_i$, $\lambda$, $r$, and $D_{str}$ are considered to be the physical quantities which determine the value of $D_n$ of the detonation wave propagating stably through a curved channel. Therefore, $D_n$ is expressed as a function of these parameters as follows:

$$D_n = f_1(r_i, \lambda, r, D_{str}) \quad (5)$$

By using $r_i$, $\lambda$ and $r$, we can define two significant dimensionless variables, $r_i/\lambda$ and $r/r_i$. The variable $r_i/\lambda$ represents the ratio of the representative physical length scales of the flow field and detonation wave, and the variable $r/r_i$ represents the normalized front shock position of the detonation waves. Considering that the physical dimensions of both sides of an equation need to be equivalent, Eq. (5) may be reduced into the following dimensionless form by introducing these two dimensionless variables:

$$\frac{D_n}{D_{str}} = f_2\left(\frac{r_i}{\lambda}, \frac{r}{r_i}\right) \quad (6)$$

If $f_2$ is determined, the following relation between $r$ and $\theta$, which gives the front shock shape of the detonation wave propagating stably through a curved channel, is derived by substituting Eq. (6) into Eq. (3) and integrating Eq. (3):
\[ \theta - \theta_1 = -\int_{r_i}^{r} \frac{\sqrt{\left(r \omega\right)^2 - D_n^2}}{D_n} dr. \]  

(7)

Since \( D_{n,i} \) is equal to \( r_i \omega \), Eq. (7) can be reduced into the following equation:

\[ \theta - \theta_1 = -\int_{r_i}^{r} \left\{ \left( \frac{r}{r_i} \right)^2 \left( \frac{D_{n,i}}{D_{at}} \right)^2 \left( \frac{D_n}{D_{at}} \right)^{-2} - 1 \right\} \left\{ \frac{1}{r} \right\} dr \]

\[ = -\int_{r_i}^{r} \left\{ \left( \frac{r}{r_i} \right)^2 \right\} \left\{ f_2 \left( \frac{r_i}{\lambda}, \frac{r}{r_i} \right) \right\} \left\{ f_2 \left( \frac{r_i}{\lambda}, \frac{r}{r_i} \right) \right\}^{-1} \left\{ \frac{1}{r} \right\} dr \]

(8)

That is to say, the front shock shape of the detonation wave can be determined if \( f_2, r_i \) and \( \lambda \) are given.

Generally, the local curvature of the detonation wave (\( \kappa \)) can be calculated from the following equation:

\[ \kappa = r^2 + 2 \left( \frac{dr}{d\theta} \right)^2 - r \frac{d^2 r}{d\theta^2} \left( \frac{dr}{d\theta} \right)^2 + r^2 \]

(9)

From Eq. (8), we can see that \( dr/d\theta \) and \( d^2 r/d\theta^2 \) become a product of \( r \) and a function of \( r_i/\lambda \) and \( r_i/r \).

Therefore, by substituting \( dr/d\theta \) and \( d^2 r/d\theta^2 \) derived from Eq. (8) into Eq. (9), Eq. (9) can be reduced into a function of \( r_i/\lambda \), \( r_i/r \) and \( r \) as follows:

\[ \kappa = g_1 \left( \frac{r_i}{\lambda}, \frac{r_i}{r}, \frac{1}{r} \right). \]  

(10)

Eq. (10) can be reduced into the following dimensionless form by multiplying both sides of Eq. (10) by \( \lambda \):
The following equation is derived by solving Eq. (11) for $r/r_i$ and substituting the equation into Eq. (6):

$$
\frac{D_n}{D_{str}} = h\left(\frac{r}{\lambda}, \lambda \kappa \right).
$$

The bottom line is that the front shock shape of the detonation wave and the relation between $D_n/D_{str}$ and $r_i/\lambda$ or between $D_n/D_{str}$ and $\lambda \kappa$ can be derived if $f_2$ is determined.

The expansion waves from the inner wall of the curved channel decrease $D_n$ since the expansion waves diffract the detonation wave. The influence of the expansion waves is the strongest on the inner wall and becomes negligible at a distance sufficiently far away from the inner wall. Therefore, we can suppose that $D_n$ becomes the smallest on the inner wall of the curved channel where the influence of the expansion waves becomes the strongest. We can also suppose that $D_n$ increases as if to approach asymptotically to a certain value at a distance sufficiently far from the inner wall since the influence of the expansion waves decrease with distance from the inner wall. As a formula that gives approximately such $D_n$ characteristics, we propose the following formula which is a phenomenological dependence:

\[
\frac{D_n}{D_{str}} = \frac{D_{n,asy}}{D_{str}} - \left( \frac{D_{n,asy}}{D_{str}} D_{n,i} \left( \frac{r}{r_i} \right)^m \right),
\]

(13)

where \(D_{n,asy}\) and \(m\) are constant, and \(D_{n,asy}\) is the asymptotic value of \(D_n\) at a distance sufficiently far from the inner wall. The influence of \(r_i\) and \(\lambda\) on \(D_n\) is expressed by these three constants, \(D_{n,asy}\), \(D_{n,i}\), and \(m\). \(D_n\) increases from \(D_{n,i}\) to \(D_{n,asy}\) asymptotically with increasing \(r\) in Eq. (13). \(D_{n,i}\) and \(D_n\) at arbitrary points on the detonation wave are preliminarily acquired at a given \(r_i\) and \(\lambda\) in an experiment. \(D_{n,asy}\) and \(m\) are determined by applying the least-square method to the experimental \(D_{n,i}\) using Eq. (13). Although one can assume that \(D_{n,asy}/D_{str} = 1\) at a distance sufficiently far from the inner wall, we determined \(D_{n,asy}\) by the least-square method in order to increase the flexibility and accuracy of the approximation in the vicinity of the inner wall in the present study.

If a detonation wave propagates stably through a curved channel while maintaining a specific shape at a constant angular velocity, \(D_n\) can be determined from Eq. (3). The following equation is obtained by solving Eq. (3) for \(D_n\):

\[
D_n = -\omega \left( \frac{1}{r} \frac{d r}{d \theta} \right) + \sqrt{\left( \frac{1}{r} \frac{d r}{d \theta} \right)^2 + 1}. 
\]

(14)

The value of \(d r/d \theta\) in Eq. (14) is approximately obtained by the Taylor series expansion of \(r\) with respect to \(\theta\) and by neglecting high-order terms. By taking into account the terms to the second-order, \(d r/d \theta\) is discretely approximated by the following equation:

\[
\left( \frac{dr}{d\theta} \right)_k \approx \left\{ r_{k+1} - r_k - \frac{1}{2} \left( \frac{d^2r}{d\theta^2} \right)_k \left( \theta_{k+1} - \theta_k \right)^2 \right\} / \left( \theta_{k+1} - \theta_k \right). \tag{15}
\]

d\frac{d^2r}{d\theta^2} can also be discretely approximated using the following equation:

\[
\left( \frac{d^2r}{d\theta^2} \right)_k \approx \left( \frac{r_{k+1} - r_k - r_k - r_{k-1}}{\theta_{k+1} - \theta_k - \theta_k - \theta_{k-1}} \right) / \left( \frac{\theta_{k+1} - \theta_k}{2} + \frac{\theta_k - \theta_{k-1}}{2} \right). \tag{16}
\]

If the interval of \( \theta \) is constant, Eq. (16) is the so-called second-order accurate central-difference of \( r \) with respect to \( \theta \). By recording the detonation wave shape at an arbitrary time interval and dividing it at a regular interval as shown in Fig. 1, one can pick up the coordinate values of the points of division and obtain the \( \omega \) of each point by observing the alternation of their positions from moment to moment. If the \( \omega \) values of each point are equivalent to each other and are time-unvarying, the values of \( D_n \) on the points of division are determined from Eqs. (14)-(16) using the \( \omega \) and the coordinate values of the points of division.

Since Eq. (7) and Eq. (13) give the front shock shape of the detonation wave as a continuous function of \( r \) and \( \theta \), \( \kappa \) can be calculated from Eq. (9). \( dr/d\theta \) and \( d^2r/d\theta^2 \) are derived from Eq. (3) and the derivative of Eq. (3) with respect to \( \theta \), respectively.

### 3. Visualization method

#### 3.1 Short-time open-shutter photography (SOP)

The front shock shape of the detonation wave propagating through a curved channel and the
trajectories of triple points on the detonation wave were simultaneously recorded by Short-time Open-shutter Photography (SOP). The concept of SOP is shown in Fig. 2. A detonation wave is propagating through a curved channel from the lower right to the upper left and is recorded by SOP at an exposure time of $\Delta t$ in Fig. 2. By limiting the exposure time to a few microseconds, only the triple point trajectories within the area swept by the detonation wave front are recorded in the exposure time. The forward or backward edges of the triple point trajectories recorded in the SOP image give the front shock shape of the detonation wave of the moment. The influence of the luminescence of burned gas can also be minimized in the SOP image by setting the exposure time appropriately.

3.2 Multi-frame short-time open-shutter photography (MSOP)

By performing SOP for each individual frame of a high-speed camera, the image of the overall triple point trajectories in a curved channel is visualized from the superimposition of these SOP images. This new visualization method is named Multi-frame Short-time Open-shutter Photography (MSOP) in the present study.

4. Experimental setup and conditions

The schematics of the curved channels and observation chamber used in the present study are
shown in Fig. 3. Five types of curved channels with different inner radii of curvature (5 mm, 10 mm, 20 mm, 40 mm, and 60 mm) were used. The cross-section of these channels is rectangular and the width is 20 mm. The depth of these channels is set very thin (1 mm) so that the structures of the detonation waves propagating through the channels become two-dimensional. Thus, the trajectories of the triple points can be recorded clearly by MSOP.

The observation chamber consists of circular-cross-section tubes of 25.8 mm in diameter, rectangular-cross-section tubes of 20 mm x 16 mm, and a curved channel. The mixture gas filled in the observation chamber is ignited by a spark plug mounted at the closed end of the circular-cross-section tube below the curved channel. A deflagration wave transitions to a detonation wave within the Shchelkin spiral section mounted in the tube. The detonation wave enters the curved channel via the rectangular-cross-section tube. A low-vacuum dump tank of 0.037 m$^3$ is connected to the outlet port of the observation chamber, and a mylar film separates the dump tank and the observation chamber. The detonation wave passing through the curved channel ruptures the film, and the high pressure and temperature gas generated by the detonation wave is caught in the dump tank.

A stoichiometric ethylene-oxygen mixture gas was used in the present study. The mixture gas is filled at a given pressure into the observation chamber in which the air is evacuated. The temperature of the mixture gas is equal to the room temperature. The experimental conditions are summarized in Table 1, where $p_0$ is the filling pressure of the mixture gas and $T_0$ is the temperature of mixture gas.
The images of the detonation waves propagating through the curved channels were taken by a high-speed video camera (Shimadzu HPV-2) through an observation window of 120 mm in diameter. All the images were taken at 4-µs time intervals in each experiment. The spatial resolution of the images is 0.3 mm. The output voltage of the piezoelectric transducer mounted near the closed-end of the circular-cross-section tube below the curved channel was used as a trigger to start shooting by the high-speed camera.

5. Stable and unstable detonation waves in a curved channel

5.1 Planarity and speed of detonation wave in a straight section

The positions of the detonation wave propagating through the straight section of a curved channel were determined from an MSOP image. It was verified that the detonation wave in the straight section was planar since the forward edge of the triple point trajectories recorded was quite straight. Figure 4 shows the measurement results of $D_{str}$. The symbols are the averages of three values measured at 4-µs time intervals and the solid line is $D_{CJ}$ calculated using CEA [21]. The error bars represent the ranges of minimum-to-maximum deviation and the systematic error of the measured values. All the values of measured $D_{str}$ were 5% lower than $D_{CJ}$. This velocity deficit was considered to have been caused by the thin depth (1 mm) of the curved channels, which forces the detonation structure to be two-dimensional.
5.2 Detonation cell width

The value of $\lambda$ of the detonation wave propagating through the straight section of a curved channel was measured from an MSOP image. The measurement of $\lambda$ was conducted under conditions in which the filling pressure of the mixture gas was relatively low; that is to say, the detonation cell width became large (Shot No. 15, Shot No. 16, Shot No. 24). Figure 5 shows the measurement results of $\lambda$. The values of the detonation cell width on the Detonation Database of the California Institute of Technology [17-20] are also shown in Fig. 5. In Fig. 5, the measured value in the present study is the average of ten values measured arbitrarily within the straight section. The error bars represent the ranges of minimum-to-maximum deviation of the measured values. The measurement results of $\lambda$ in the present study did not contradict those of other studies [17-20]. The relation of $\lambda = 70.020 x p_0^{-1.1270}$ could be obtained by applying the least-square method to all the data shown in Fig. 5.

5.3 Detonation propagation characteristics in a curved channel

The value of $D_{n,i}$ was measured from the alternation of the forward edge position of the luminescence region on the inner wall as determined using the MSOP image. The propagation of the detonation wave through a curved channel was categorized based on how high $D_{n,i}$ was. The typical histories of $D_{n,i}$ are shown in Fig. 6. $D_{n,i}$ is nondimensionalized by $D_{str}$ in Fig. 6. The error bars of the
rightmost symbols represent the ranges of systematic error of measurement. In the present study, the propagation mode that consistently satisfies the relation of $D_{n,i}/D_{str} \geq 0.8$ is defined as the stable mode, the mode that cannot satisfy the relation of $D_{n,i}/D_{str} \geq 0.8$ but can consistently satisfy the relation of $D_{n,i}/D_{str} \geq 0.6$ is defined as the critical mode, and the mode in which $D_{n,i}/D_{str} < 0.6$ even just once is defined as the unstable mode. In Fig. 6, circles correspond to the stable mode, triangles to the critical mode, and crosses to the unstable mode.

As shown in Fig. 6, the variation range of $D_{n,i}/D_{str}$ is narrow in the case of the stable mode, therefore we can consider the detonation wave in the stable mode to be the so-called steady detonation wave. On the other hand, the variation range of $D_{n,i}/D_{str}$ is wide in the case of the unstable mode. Therefore, we can consider the detonation wave in the unstable mode to be the so-called unsteady detonation wave. The detonation wave in the critical mode may be the intermediate between the steady and unsteady detonation waves.

The typical MSOP images taken at a given $r_i$ ($r_i = 40$ mm) are shown in Fig. 7. The images in Fig. 7 were taken in the same experiments represented in Fig. 6. The arrows in Fig. 7 show the propagation direction of the detonation wave. Fig. 7(a) corresponds to the stable mode, Fig. 7(b) to the critical mode, and Fig. 7(c) to the unstable mode. At a given $r_i$, a higher $p_0$ makes the detonation wave in a curved channel more stable.

The detonation cell in the vicinity of the inner wall of the curved channel is enlarged soon after the
detonation wave enters the curved section of the channel due to the expansion waves from the inner wall as shown in Fig. 7(a). When the cell has enlarged to about twice its normal size, new cells are generated smoothly within the enlarged cells in the vicinity of the inner wall. The smooth detonation wave front can be maintained consistently in the stable mode due to this smooth cell generation. It is possible that $D_{n,i}/D_{str}$ is maintained consistently above 0.8 in the stable mode in Fig. 6 due to this smooth cell generation. The detonation wave propagates while maintaining a specific shape, and the interval of the detonation wave front recorded in the MSOP image is constant within $\theta_i = 45-90$ deg.

In the critical mode, the cell in the vicinity of the inner wall is also enlarged soon after the detonation wave enters the curved section as shown in Fig. 7(b). Although the collapse of the detonation cell structure has not yet been observed, the cell has increased to about three times its normal size within $\theta_i = 60-90$ deg. $D_{n,i}/D_{str}$ may fall below 0.8 in the critical mode in Fig. 6 due to this significant cell enlargement in the vicinity of the inner wall. The interval of the detonation wave front recorded in the MSOP image is not constant in the region where significant cell enlargement is observed.

A collapse of the detonation cell structure occurs in the unstable mode. A temporary collapse of the detonation cell structure is observed in the vicinity of the inner wall around $\theta_i = 35$ deg in Fig. 7(b). Although the detonation cell structure in the vicinity of the inner wall is recovered from around $\theta_i = 45$ deg by the generation of new fine cells, the detonation cell structure collapses completely
within $\theta_i = 70-90$ deg. It is possible that the detonation wave has once transitioned to the deflagration wave in such a region and that $D_{n,i}/D_{str}$ falls below 0.4 in the unstable mode in Fig. 6 due to this complete collapse of the detonation cell structure.

The typical MSOP images taken at a given $p_0$ ($p_0 = 31.0 \pm 1.0$ kPa) are shown in Fig. 8. Fig. 8(a) corresponds to the stable mode, Fig. 8(b) to the critical mode, and Figs. 8(c)-(e) to the unstable mode. At a given $p_0$, a larger $r_i$ makes the detonation wave more stable. That is to say, the increase of $r_i$ has the same effect as the increase of $p_0$ on the stability of the detonation wave in a curved channel.

5.4 Condition of stable detonation propagation in a curved channel

The relation between $r_i$ and $\lambda$ is shown in Fig. 9 in terms of the propagation mode of the detonation wave propagating through a curved channel. At a given $\lambda$, a larger $r_i$ makes the detonation wave more stable. A smaller $\lambda$ also makes the detonation wave more stable at a given $r_i$. From this result, we can see that the smaller the inner radius of the annular combustor of RDE is, the smaller $\lambda$ (or the higher $p_0$) is required for the stable operation of RDE. The stable operation range of $p_0$ for RDE may be expanded by increasing the inner radius of the annular combustor of RDE.

In Fig. 9, one can see that the stable mode and critical mode as well as the critical mode and unstable mode may be separated by a line of $r_i/\lambda = \text{const}$. Therefore, we defined the stable zone, transitional zone, and unstable zone based on how high $r_i/\lambda$ was. The zone where the stable mode is
consistently attained above a line of $r_i/\lambda = \text{const.}$ is defined as the stable zone and the line is the threshold of the stable zone. The zone where no unstable mode is observed above a line of $r_i/\lambda = \text{const.}$ but below the threshold of the stable zone is defined as the transitional zone and the line is the threshold of the transitional zone. The remaining zone is defined as the unstable zone. We altered the value of $r_i/\lambda$ at the interval of 1 and found that $r_i/\lambda = 32$ and 21 were the thresholds of the stable zone and transitional zone, respectively. Therefore, the propagation mode of the detonation wave is considered to transition to the stable mode from the unstable mode within $21 \leq r_i/\lambda \leq 32$ in the present study.

6. $D_n$-$\kappa$ relation of stable detonation wave in a curved channel

The motion of the detonation wave propagating through a curved channel is analyzed under the condition of $r_i/\lambda \geq 32$ where the propagation mode of the detonation wave is consistently stable. The cases of the analysis are shown in Table 2. The detonation cell width of these cases is 0.6 mm. The front shock shape of the detonation wave is quantified by dividing it into 15 parts at a regular interval as shown in Fig. 1 and picking up the coordinate values of the points of division. The ranges of $\theta_i$ where the transition from a planar detonation wave to a fully-developed curved detonation wave was observed ($\theta_{\text{trans}}$) are also shown in Table 2. If $\theta_i$ becomes larger than $\theta_{\text{trans}}$, the detonation wave takes a specific shape and the shape becomes steady. Therefore, three images in which the
The coordinate values in order to analyze the motion of the detonation wave of which the shape was quite steady.

6.1 Angular velocity of stable detonation wave in a curved channel

The relation between $\omega$ and $r-r_i$ is shown in Fig. 10. The error bars of the symbols represent the ranges of systematic error of measurement. $\omega$ increases with decreasing $r_i$. In all of the cases in the analysis, $\omega$ is constant everywhere on the detonation wave and is time-unvarying.

Since a detonation wave is self-sustaining, the detonation wave which is once decelerated by an expansion wave has an ability to recover its velocity again to $D_{CJ}$ if the detonation wave is not quenched by the expansion wave. In the case of the detonation wave propagating stably through a curved channel, expansion waves are continuously generated from the inner wall of the curved channel as long as the detonation wave continues propagating, and they forces the detonation wave to decrease its velocity. From the results shown in Fig. 10, we can suppose that these two effects, the accelerating effect which the decelerated detonation wave has and the decelerating effect by expansion waves, balances on the inner wall of the curved channel since the value of $\omega$ becomes time-invariant.

Figure 11 shows the schematic of the behavior of the stable detonation wave in a curved channel.
In figure 11, the solid line represents the front shock of the detonation wave, the dashed line represents the particle path which is perpendicular to the front shock, and the dashed-dotted line represents the expansion wave, respectively. On the inner wall of the curved channel, \( D_n = r_i \omega \) and the particle path is along the inner wall. Now, we focus our attention on the point \( P(r_I + \delta r, \theta - \delta \theta) \) at a distance infinitely close to the inner wall (\( \delta r, \delta \theta << 1 \)). Since the point \( P \) is not on the inner wall and \( \omega \) is time-invariant, the particle path from the point \( P \) is not along the inner wall and \( D_n \) is nearly equal to \( r_i \omega \) at the point \( P \) regardless of \( \theta_I \). The particle path gradually gets away from the inner wall as the detonation wave propagates, and finally it reaches the outer wall of the curved channel. \( D_n \) increases gradually from the value of \( r_i \omega \) along the particle path. Since \( r_i \) is constant, the conditions of expansion waves are also constant regardless of \( \theta_i \). Therefore, the degree of the influence of the expansion waves on the particle path is constant at any \( \theta_i \), and thus the shape of the particle path and the distribution of \( D_n \) on the particle path do not change with the value of \( \theta_i \). Consequently, the value of \( D_n \) and its direction become constant on the line of \( r = \text{const} \). This property may make the front shock shape of the detonation wave steady. Therefore, the value of \( \omega \) becomes constant everywhere on the detonation wave as shown in Fig. 10.

6.2 Normal propagation speed of stable detonation wave in a curved channel

The relation between \( D_n/D_{str} \) and \( r - r_i \) is shown in Fig. 12. The symbols represent the average of the
measured values of $D_n$ of each point of division on the three detonation wave fronts corresponding to each $\theta_i$ shown in Table 2. The error bars of the symbols represent the ranges of minimum-to-maximum deviation of the measured values. $D_{n,i}$ becomes the smallest among the normal detonation velocities on the detonation wave and decreases with decreasing $r_i/\lambda$. $D_n$ increases as if to approach asymptotically to a certain value with increasing $r-r_i$. The solid line in Fig. 12 is drawn by Eq. (13) and is in good agreement with the measured $D_n$. Hence, Eq. (13) is considered to be an appropriate formula which approximately gives the relation between $D_n$ and $r$ on the detonation wave at a given $r_i$ and $\lambda$. The relations between $D_{n,asy}$ and $r_i/\lambda$ and between $m$ and $r_i/\lambda$ are shown in Fig. 13. $D_{n,asy}$ is nondimensionalized by $D_{str}$ in Fig. 13. Since $D_{n,asy}$ is almost equivalent to $D_{str}$ under the condition of $r_i/\lambda \geq 67.3$, one can consider $D_{n,asy}/D_{str} = 1$ if $r_i/\lambda$ is sufficiently large. However, as $D_{n,asy}$ tends to be smaller than $D_{str}$ under the condition of a smaller $r_i/\lambda$, one should determine $D_{n,asy}$ by the least-square method in such a condition. The parameter $m$ increases with increasing $r_i/\lambda$ in a linear manner.

The experimental and reconstructed front shock shapes of the detonation waves, which are expressed by $r$-$\theta$ relations, are shown in Fig. 14. The symbols represent the average positions of each point of division on the three detonation wave fronts corresponding to each $\theta_i$ shown in Table 2. The error bars represent the ranges of minimum-to-maximum deviation and systematic error of the measured values. The solid line is the front shock shape of the detonation wave reconstructed using
Eq. (7) and Eq. (13). The dashed-dotted line is the one reconstructed by using Eq. (4). The shape of the detonation wave reconstructed by using Eq. (7) and Eq. (13) agrees much better with the experimental result in all of the analysis cases than the one reconstructed by using Eq. (4) does. The result shown in Fig. 14 also shows that Eq. (13) is an appropriate formula for giving $D_n$ approximately.

6.3 $D_n$-$\kappa$ relation

The relation between $\lambda\kappa$ and $(r-r_i)/\lambda$ derived from the reconstructed front shock shape of the detonation wave is shown in Fig. 15. $\lambda\kappa$ decreases with increasing $(r-r_i)/\lambda$. The largest $\lambda\kappa$ is observed on the inner wall of the curved channel and increases with increasing $r_i/\lambda$.

The relation between $D_n/D_{str}$ and $\lambda\kappa$ is shown in Fig. 16. $D_n/D_{str}$ decreases with increasing $\lambda\kappa$. This nondimensionalized $D_n$-$\kappa$ relation is nearly independent of the $r_i/\lambda$. The $D_n$-$\kappa$ relation, in which $D_n$ and $\kappa$ are nondimensionalized by $D_{CJ}$ and the induction zone length scale respectively, has been examined theoretically by steady or quasi-steady one-dimensional analyses [22-25]. These analyses have shown that $D_n$ is a function of only $\kappa$ in one-dimensional ZND detonation. The present study has experimentally shown that $D_n/D_{str}$ of a stoichiometric ethylene-oxygen mixture gas is also a function of only $\lambda\kappa$; as in the theoretical studies.

However, the evaluation of the $D_n$-$\kappa$ relation in the present study is limited to a certain high filling
压力条件下的混合气。因此，$D_n-\kappa$ 关系尚未被证明是普遍的。

进一步评估$D_n-\kappa$ 关系在整个稳定区（图 9）是必要的，以确认$D_n-\kappa$ 关系是否是普遍的。然而，很难精确定义混合气在低充填压力条件下的前向冲击波形状。因此，我们需要使用另一种方法进行可视化实验，以进行进一步的评估$D_n-\kappa$ 关系。

6. Conclusions

 detonation propagation phenomena in curved channels were experimentally studied in order to examine the stable propagation condition. A stoichiometric ethylene-oxygen mixture gas and five types of rectangular-cross-section curved channels with different inner radii of curvature were used. The inner radius of curvature of the curved channel ($r_i$) and the filling pressure of the mixture gas ($p_0$), that is to say, the normal detonation cell width ($\lambda$), were parametrically altered. The front shock shapes of detonation waves propagating through the curved channels and the trajectories of triple points on the detonation waves were simultaneously observed using Multi-frame Short-time Open-shutter Photography (MSOP), and the following results were obtained.

When the detonation wave propagation in a curved channel was in the stable mode ($r_i/\lambda \geq 32$),
new detonation cells were generated smoothly within the enlarged cells in the vicinity of the inner wall of the curved channel. This smooth cell generation consistently maintained the smooth detonation wave front in the stable mode. At a given $r_i$, a higher $p_0$ made the detonation wave more stable. And at a given $p_0$, a larger $r_i$ also made the detonation wave more stable. The propagation mode of the detonation wave was considered to transition to the stable mode from the unstable mode within $21 \leq r_i/\lambda \leq 32$.

The normal detonation velocity ($D_n$) increased with distance from the inner wall and approached the velocity of the planar detonation entering the curved section of the curved channel ($D_{str}$). The smallest $D_n$ was observed on the inner wall of the curved channel and decreased with decreasing $r_i/\lambda$. The distribution of $D_n$ on the detonation wave in the stable mode was approximately formulated. The approximated $D_n$ given by the formula agreed well with the experimental results. The front shock shape of the detonation wave could be reconstructed accurately using the formula.

The local curvature of the detonation wave ($\kappa$) nondimensionalized by $\lambda$ decreases with increasing distance from the inner wall of the curved channel. The largest $\lambda \kappa$ was observed on the inner wall of the curved channel and increased with increasing $r_i/\lambda$. $D_n/D_{str}$ decreased with increasing $\lambda \kappa$. This nondimensionalized $D_n-\kappa$ relation was nearly independent of $r_i/\lambda$.

In the present study, the stable condition of the detonation wave propagating through curved channels was determined by using the two-dimensional curved channels. Since the actual

propagation of the detonation wave in the annular combustor of RDE is three-dimensional and another curvature may exist on the front shock of the detonation wave, its application to RDE may be limited even if a stoichiometric ethylene-oxygen mixture gas is employed as its propellant. The local curvature of the detonation wave may affect the stability of the detonation wave, however the relation between the local curvature and the stability of the detonation wave has not been revealed yet in the present study. Therefore, we need further experimental investigation in order to elucidate this relation.

Acknowledgements

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References


Table 1. Experimental conditions ($T_0 = 298 \pm 1$ K).

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Table 2. Analysis cases of the motion of the stable detonation wave in a curved channel.

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Fig. 1. Geometric relationship in a stable detonation wave propagating through a rectangular cross-section curved channel with constant inner/outer curvature radii.
Fig. 2. Schematic illustrating the concept of Short-time Open-shutter Photography (SOP).

Fig. 3. Schematic of the channels and observation chamber.
Fig. 4. Velocity of the planar detonation wave propagating through the straight section of a curved channel.
Fig. 5. Detonation cell width of the planar detonation wave propagating through the straight section of a curved channel.
Fig. 6. Typical histories of the normal detonation velocity on the inner wall of a curved channel.
Fig. 7. Typical images of Multi-frame Short-time Open-shutter Photography (MSOP) at a given inner radius of a curved channel ($r_i = 40$ mm).
Fig. 8. Typical images of Multi-frame Short-time Open-shutter Photography (MSOP) at a given filling pressure of mixture gas \((p_0 = 31.0 \pm 1.0 \text{ kPa})\).
Fig. 9. Relation between the inner radius of the curved channel and the normal detonation cell width in terms of the propagation mode of the detonation wave propagating through a curved channel.
Fig. 10. Distribution of the angular velocity on the detonation wave in the stable propagation mode.
Fig. 11. Schematic of the behavior of the detonation wave in the stable propagation mode.
Fig. 12. Distribution of the normal detonation velocity on the detonation wave in the stable propagation mode.
Fig. 13. Variations of $D_{n,asy}$ and $m$ with $r_i/\lambda$. 

$D_{str} = 2315.9 \text{ m/s}$

$D_{str} = 2277.6 \text{ m/s}$

$D_{str} = 2272.7 \text{ m/s}$
Fig. 14. Front shock shape of the detonation wave in the stable propagation mode.
Fig. 15. Relation between $\lambda \alpha$ and $(r-r_i)/\lambda$ of the detonation wave in the stable propagation mode.
Fig. 16. Relation between $D_w/D_{str}$ and $\lambda\kappa$ of the detonation wave in the stable propagation mode.