

Polarized DIS in $\mathcal{N} = 4$ SYM: Where is spin at strong coupling?

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ABSTRACT: Using the AdS/CFT correspondence, we calculate the polarized structure functions in strongly coupled $\mathcal{N} = 4$ supersymmetric Yang–Mills theory deformed in the infrared. We find that the flavor singlet contribution to the g_1 structure function is vanishingly small, while the flavor non-singlet contribution shows the Regge behavior at small- x with an intercept slightly less than 1. We explicitly check that the latter satisfies the moment sum rule. We discuss the ‘spin crisis’ problem and suggest that at strong coupling the spin of a hadron entirely comes from the orbital angular momentum.

KEYWORDS: Deep Inelastic Scattering, AdS–CFT correspondence.

Contents

1. Introduction	1
2. Review of polarized DIS in QCD	3
3. Polarized DIS in $\mathcal{N} = 4$ SYM	5
4. The g_1 structure function at strong coupling	10
4.1 Reggeized photon exchange in AdS	10
4.2 A pragmatic approach	13
5. Discussions	18
5.1 ‘Asymptotic’ spin decomposition	18
5.2 Small- x behavior	19
A. OPE at weak coupling	19

1. Introduction

The question of how the spin of the proton is distributed among quarks and gluons has been one of the central themes in QCD. It all started in the late 1980s when the European Muon Collaboration (EMC) announced data for polarized deep inelastic scattering (DIS) which apparently suggested that a disturbingly small fraction of the proton’s total spin is accounted for by the helicity of quarks [1]. After more than twenty years of vigorous experimental and theoretical efforts since then, the problem of the missing spin—or the ‘spin crisis’—has now become much less mysterious than it was initially thought. [See [2] for the latest review of the subject.] Nevertheless, there still remains a great deal of controversy regarding the nature of the decomposition of the total spin

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_z, \quad (1.1)$$

where $\Delta\Sigma$ and ΔG are contributions from the helicity of quarks and gluons, respectively, and L_z is their orbital angular momentum. Among the three terms in (1.1), $\Delta\Sigma$ is relatively well-constrained by a wealth of DIS data supplemented by the NLO global QCD analysis. The value often quoted in the literature is $\Delta\Sigma \sim 0.25$. This is larger than the original EMC result, but is still significantly smaller than predictions based on the naive quark model $1 \geq \Delta\Sigma \gtrsim 0.6$. On the other hand, extraction of ΔG is more difficult as it is not directly measurable in DIS. While there have been several other experiments which are better suited to this purpose, at present the data are not yet sufficient (especially in the

low- x region) for an accurate determination of ΔG . However, one may notice that some of the recent analyses [3–9] prefer a small value of ΔG consistent with zero, albeit with large uncertainties of order 0.5.

The present status of the sum rule (1.1) just described is a persistent reminder of fundamental theoretical questions in spin physics: (i) While the evolution with Q^2 of the helicity distributions $\Delta\Sigma(Q^2)$ and $\Delta G(Q^2)$ are calculable in perturbation theory at large $Q^2 \gg \Lambda_{QCD}^2$, they are essentially nonperturbative quantities. Can one evaluate them in the strong coupling regime where the perturbative approach breaks down? (ii) Why is $\Delta\Sigma$ ‘unnaturally’ small, and what carries the rest of the total spin? If ΔG is small as suggested by some recent works, it may well be that the orbital angular momentum L_z is the dominant component. Existing models [10–13] that can give rise to a large value of L_z include the nonperturbative dynamics of QCD in one way or another, so again the key to understand this question resides in the strong coupling sector. (iii) How do the polarized parton densities and structure functions behave at small values of the Bjorken x variable? In addition to its phenomenological impact in fitting the data at low- x , the question is of great conceptual interest because one expects, as in the unpolarized case, an interplay between ‘hard’ and ‘soft’ Regge exchanges [14].

In this paper, we try to address these questions in the framework of the AdS/CFT correspondence [15] which has emerged as a powerful tool to analyze the strong coupling regime of certain non-Abelian gauge theories. Specifically, we shall study polarized DIS in $\mathcal{N} = 4$ supersymmetric Yang–Mills (SYM) theory at large ’t Hooft coupling $\lambda \gg 1$ and compute structure functions in the small- x region. Given the many dissimilarities between $\mathcal{N} = 4$ SYM and QCD, *a priori* it is not clear whether such a study, though interesting in its own right, yields useful insights into the corresponding QCD problems. However, it turns out that somewhat to our surprise the overall picture we shall arrive at is qualitatively in line with the current experimental situation in QCD.

In the context of gauge/string duality, DIS was first formulated in [16] for the unpolarized case (see, also [17–22]), and recently generalized to the polarized case in [23]. The analysis of [23] is focused on the large- x region which corresponds to the supergravity approximation on the string theory side. However, the internal degrees of freedom (‘partons’) of hadrons are visible only at extremely small values of x of order $\sim e^{-\sqrt{\lambda}}$ which in fact is the Regge regime of the theory [16]. Thus, the very existence of spin physics at strong coupling is intrinsically tied to high energy Regge scattering, and this is what we are going to explore.

In $\mathcal{N} = 4$ SYM, there are four ‘flavors’ of Weyl fermions associated with the $SU(4)$ \mathcal{R} -symmetry. We shall find that the flavor singlet contribution to the g_1 structure function is simply *vanishing* for all values of $Q^2 > \Lambda^2$ where Λ is an infrared scale (analog of Λ_{QCD}) necessary to break conformal symmetry

$$\Delta\Sigma = 0. \tag{1.2}$$

Interestingly, the same conclusion has been reached in a Skyrme model calculation [10]. [Note that both approaches use the large- N_c approximation.] As a matter of fact, in our case (1.2) follows rather trivially from general arguments. Instead, our main focus is the

flavor non-singlet contribution which shows the expected power-law increase in $1/x$ as $x \rightarrow 0$. In the context of gauge/string duality, this of course is attributed to the usual Regge behavior of the string S-matrix.

In Section 2, we review the standard perturbative approach to polarized DIS in QCD. In Section 3 we consider the same process in $\mathcal{N} = 4$ SYM and identify the dominant t -channel process that survives at strong coupling. The actual calculation of the structure function is performed in Section 4 where we present an explicit expression of the g_1 function (4.36) which exactly satisfies the moment sum rule. Some comparisons between our results for $\mathcal{N} = 4$ SYM and the known results in QCD will be made at the end.

2. Review of polarized DIS in QCD

In this section we briefly review the basics of the operator product expansion (OPE) approach to polarized structure functions in QCD.¹

We start with the forward Compton amplitude off a polarized hadron (usually a proton) target

$$T^{\mu\nu} = \frac{i}{2\pi} \int d^4y e^{iqy} \langle PS | T \{ J^\mu(y) J^\nu(0) \} | PS \rangle = T_{sym}^{\mu\nu} + iT_{asym}^{\mu\nu}, \quad (2.1)$$

where P^μ and S^μ are the momentum and spin vectors, respectively, and $J^\mu = \sum_f e_f \bar{q}_f \gamma^\mu q_f$ is the electromagnetic current of quarks. The subscript *sym/asym* refers to the symmetric/antisymmetric part under the exchange of Lorentz indices $\mu \leftrightarrow \nu$. On the other hand, the structure functions are the components of the following hadronic tensor

$$\begin{aligned} W^{\mu\nu} &= \frac{1}{2\pi} \int d^4y e^{iqy} \langle PS | J^\mu(y) J^\nu(0) | PS \rangle = \frac{1}{2\pi} \int d^4y e^{iqy} \langle PS | [J^\mu(y), J^\nu(0)] | PS \rangle \\ &= W_{sym}^{\mu\nu} + iW_{asym}^{\mu\nu}. \end{aligned} \quad (2.2)$$

After some manipulations one can show that $W_{sym}^{\mu\nu}$ and $W_{asym}^{\mu\nu}$ are real, and that

$$2 \operatorname{Im} T_{sym}^{\mu\nu} = W_{sym}^{\mu\nu}, \quad (2.3)$$

$$2 \operatorname{Im} T_{asym}^{\mu\nu} = W_{asym}^{\mu\nu}, \quad (2.4)$$

which represents the optical theorem. The polarized structure functions g_1 and g_2 are contained in $W_{asym}^{\mu\nu}$. From (2.1) and (2.4), one has, for the antisymmetric part, ($\epsilon_{0123} = -\epsilon^{0123} = +1$)

$$\begin{aligned} \operatorname{Im} \frac{1}{2\pi} \int d^4y e^{iqy} \langle PS | T \{ J^\mu(y) J^\nu(0) \} | PS \rangle \Big|_{asym} &= \frac{1}{2} W_{asym}^{\mu\nu} \\ &= \epsilon^{\mu\nu\alpha\beta} q_\alpha \left(\frac{S_\beta}{P \cdot q} (g_1(x, Q^2) + g_2(x, Q^2)) - \frac{q \cdot S P_\beta}{(P \cdot q)^2} g_2(x, Q^2) \right), \end{aligned} \quad (2.5)$$

where $S^2 = -M^2$ with M being the hadron mass.

¹In accordance with the majority of the QCD literature, we shall employ the ‘mostly minus’ metric signature $(+ - - -)$ for the Minkowski space and also in the gravity calculations in later sections.

The usual strategy to evaluate the structure function is to perform an OPE for the product JJ

$$\begin{aligned}
& \int d^4y e^{iqy} \langle PS | T \{ J^\mu(y) J^\nu(0) \} | PS \rangle \Big|_{asym} \\
&= \frac{2}{Q^2} \epsilon^{\mu\nu\alpha\beta} q_\alpha \sum_{j=1}^{odd} \frac{2q^{\mu_1} \dots 2q^{\mu_{j-1}}}{Q^{2(j-1)}} \sum_f e_f^2 \langle PS | \Theta_{\{\beta, \mu_1, \dots, \mu_{j-1}\}}^f + \Theta_{[\beta\{\mu_1\}\mu_2, \dots, \mu_{j-1}\}}^f | PS \rangle \\
&= \epsilon^{\mu\nu\alpha\beta} \frac{q_\alpha}{P \cdot q} \sum_j^{odd} \left(\frac{a_j + (j-1)d_j}{j} \frac{S_\beta}{x^j} + \frac{(j-1)(a_j - d_j)}{j} \frac{S \cdot q}{P \cdot q} \frac{P_\beta}{x^j} \right), \tag{2.6}
\end{aligned}$$

where we defined the operators

$$\Theta_{\{\beta, \mu_1, \dots, \mu_{j-1}\}}^f = \frac{1}{j} \bar{q}_f \gamma^5 \gamma_{\{\beta} i D_{\mu_1} \dots i D_{\mu_{j-1}\}} q_f, \tag{2.7}$$

$$\Theta_{[\beta\{\mu_1\}\mu_2, \dots, \mu_{j-1}\}}^f = \frac{1}{j} \sum_i^{j-1} \left(\bar{q}_f \gamma^5 \gamma_{\beta} i D_{\{\mu_1} \dots i D_{\mu_{j-1}\}} q_f - \bar{q}_f \gamma^5 \gamma_{\mu_i} i D_{\{\mu_1} \dots i D_{\beta} \dots i D_{\mu_{j-1}\}} \right), \tag{2.8}$$

and a_j, d_j are the coefficients in their matrix elements. [The brackets $\{\dots\}$ ($[\dots]$) denote the (anti-)symmetrization and trace-subtraction of Lorentz indices.]

Although each term in (2.6) is real, one finds an imaginary part after summing over j and analytically continuing in x to the physical region $0 \leq x \leq 1$. The result is

$$\begin{aligned}
g_1(x) &= \frac{1}{2} \frac{P^+}{4\pi S^+} \sum_f e_f^2 \left(\int dy^- e^{ixP^+y^-} \langle PS | \bar{q}_f(0) \gamma_5 \gamma^+ W[0, y^-] q_f(y^-) | PS \rangle \right. \\
&\quad \left. + \int dy^- e^{-ixP^+y^-} \langle PS | \bar{q}_f(0) \gamma_5 \gamma^+ W[0, y^-] q_f(y^-) | PS \rangle \right) \\
&\equiv \frac{1}{2} \sum_f e_f^2 (\Delta q_f(x) + \Delta \bar{q}_f(x)), \tag{2.9}
\end{aligned}$$

where $W[0, y^-]$ is the Wilson line operator that assures gauge invariance and in the second equality we have introduced the polarized quark and antiquark distributions.

Taking x -moments of the structure functions, one obtains sum rules

$$\int_0^1 dx x^{j-1} g_1(x, Q^2) = \frac{a_j}{4}, \tag{2.10}$$

$$\int_0^1 dx x^{j-1} g_2(x, Q^2) = -\frac{(j-1)}{4j} (a_j - d_j), \tag{2.11}$$

which can be inverted to give a well-known relation [24]

$$g_2(x, Q^2) = -g_1(x, Q^2) + \int_x^1 \frac{dz}{z} g_1(z, Q^2) + \bar{g}_2(x, Q^2). \tag{2.12}$$

The last term \bar{g}_2 comes from the operator (2.8) and represents the twist-three contribution. (2.12) shows that the twist-two part of g_2 is related to g_1 . For $j = 1$, the sum rule (2.10)

measures the axial charge a of the target. One finds

$$\int_0^1 dx g_1(x, Q^2) = \frac{1}{4S^+} \sum_f e_f^2 \langle PS | \bar{q}_f \gamma_5 \gamma^+ q_f | PS \rangle = \frac{1}{9} a^{(0)} + \frac{1}{12} a^{(3)} + \frac{1}{36} a^{(8)}, \quad (2.13)$$

where $a^{(0)}$ is the flavor singlet axial charge and $a^{(3,8)}$ are the octet (non-singlet) charges. [To leading order, the coefficient functions are set to 1.] In massless QCD, the flavor octet axial currents are exactly conserved, so $a^{(3,8)}$ are independent of Q^2 . However, the flavor singlet axial current is not conserved due to anomaly. This induces a weak Q^2 -dependence in $a^{(0)}$ in the NLO approximation [25] where one also finds $\mathcal{O}(\alpha_s)$ corrections in the coefficient functions [26]. One can eliminate $a^{(0,8)}$ by taking a difference between the proton and neutron structure functions

$$\int_0^1 dx (g_1^p(x, Q^2) - g_1^n(x, Q^2)) = \frac{1}{12} (a_p^{(3)} - a_n^{(3)}) = \frac{g_A}{6}, \quad (2.14)$$

where $g_A \approx 1.25$ is the isovector axial charge of the nucleon. (2.14) is known as the Bjorken sum rule [27] and has been very well tested in experiment.

The singlet axial charge $a^{(0)}$ is nothing but the quark's helicity contribution to the total spin

$$a^{(0)} = \sum_f (\Delta q_f + \Delta \bar{q}_f) = \sum_f (q_f^+ - q_f^- + \bar{q}_f^+ - \bar{q}_f^-) = \Delta \Sigma, \quad (2.15)$$

where $\Delta q_f = \int_0^1 dx \Delta q_f(x)$ and the superscript $+/-$ means that the helicity is parallel/antiparallel to the proton spin. As already mentioned in Introduction, roughly the spin crisis is tantamount to the unnaturally small value of $a^{(0)} = \Delta \Sigma$ as compared to unity and also to the non-singlet axial charges.

3. Polarized DIS in $\mathcal{N} = 4$ SYM

From this section on we discuss polarized DIS in $\mathcal{N} = 4$ supersymmetric Yang-Mills theory at large values of the 't Hooft coupling $\lambda \gg 1$. Due to the AdS/CFT correspondence, the problem reduces to the scattering of closed strings in the product space of five-dimensional anti-de Sitter space AdS_5 and the 5-sphere

$$ds^2 = \frac{2dy^+ dy^- - dy_1^2 - dy_2^2 - dz^2}{z^2} - d\Omega_5^2, \quad (3.1)$$

where we set the common radius R of AdS_5 and S^5 equal to unity. In this unit, the string tension becomes $\alpha' = \frac{1}{\sqrt{\lambda}}$. Since $\mathcal{N} = 4$ SYM is not confining, one has to break conformal symmetry to generate spin-1/2 hadrons (such as the state dual to the dilatino) to be used as the target in DIS. On the string theory side, this can be done by deforming the ten-dimensional metric such that it approaches (3.1) near the Minkowski boundary $z \rightarrow 0$, but differs from it far away from the boundary, around $z \sim 1/\Lambda$. The characteristic scale Λ then sets the mass scale of hadrons. In this way one can ensure that the theory is identical

to $\mathcal{N} = 4$ SYM in the ultraviolet (UV). This is essential to our discussion since we shall heavily rely on the OPE which of course is a property in the UV. On the other hand, there is an important caveat regarding possible forms of the deformation in the infrared region $z \sim 1/\Lambda$. We shall comment on this as we go along.

The basic strategy to calculate the structure functions has been laid out in [16] for the unpolarized case and in [23] for the polarized case. Here we briefly recapitulate the essential features at strong coupling and then discuss the extension to the polarized case. In the $\mathcal{N} = 4$ theory, the analog of the electromagnetic current would be the 15-component exactly conserved current J_a^μ ($a = 1, \dots, 15$) associated with the global $SU(4)$ \mathcal{R} -symmetry. [Its explicit form is given in Appendix A.] One then introduces a photon field by gauging a $U(1)$ subgroup of $SU(4)$ which, for definiteness, is taken to correspond to the generator $t^{a=3} = \text{diag}(1/2, -1/2, 0, 0)$. As for the target, one employs a spin- $\frac{1}{2}$ hadron which has mass $M \sim \Lambda$ and charge under $SU(4)$. Similarly to (2.5), one can then define structure functions

$$\begin{aligned} & \text{Im} \frac{1}{2\pi} \int d^4 y e^{iqy} \langle PS | T \{ J_3^\mu(y) J_3^\nu(0) \} | PS \rangle \Big|_{\text{asym}} \\ &= \epsilon^{\mu\nu\alpha\beta} q_\alpha \left(\frac{S_\beta}{P \cdot q} (g_1(x, Q^2) + g_2(x, Q^2)) - \frac{q \cdot S P_\beta}{(P \cdot q)^2} g_2(x, Q^2) + \frac{P_\beta}{2P \cdot q} F_3(x, Q^2) \right) \end{aligned} \quad (3.2)$$

where the parity-violating structure function F_3 is expected to arise since the theory is chiral.

For large values of the Bjorken- x , the physical picture is completely different from QCD. Instead of the twist-two operators (2.7) which acquire a large anomalous dimension of order $\lambda^{1/4}$, the OPE is dominated by the double trace operators which create and annihilate the entire hadron. These operators are nominally of higher twist, but since their dimension is protected, at moderately large Q^2 they give a larger contribution to the structure function than the twist-two operators. Thus in this regime it is not possible to probe the internal structure of hadrons, but rather the hadron appears as a pointlike particle (see, however, [22]).

When x is exponentially small, $x \sim e^{-\sqrt{\lambda}}$, the scattering starts to look more like the situation in QCD at least from the viewpoint of the OPE. In the unpolarized case, the scattering is dominated by the graviton exchange which is dual to the energy momentum tensor. This is a twist-two operator with spin $j = 2$, and its dimension is protected. Because of the curvature of the AdS space, the relevant value of j is slightly shifted away from 2. However, the anomalous dimension remains of order unity in the vicinity of $j = 2$, and thus the twist-two operators give the leading contribution.

Similarly, in the polarized case one expects that some protected operator in the $J_3 J_3$ OPE gives the more important contribution than the double trace operators considered in [23]. We observe that this operator is the \mathcal{R} -current itself. Our argument is the following: There exists an exact (i.e., valid for arbitrary λ) result for the three-point function of the

\mathcal{R} -current operator² [28, 29]

$$\lim_{y \rightarrow 0} \langle J_a^\mu(y) J_b^\nu(0) J_c^\rho(z) \rangle = \frac{N^2}{8\pi^6} d^{abc} \epsilon^{\mu\nu}{}_{\alpha\beta} \frac{y^\alpha}{y^4(y-z)^6} \left(\eta^{\beta\rho} - 2 \frac{(y-z)^\beta (y-z)^\rho}{(y-z)^2} \right). \quad (3.3)$$

This is related to the axial anomaly in the triangle diagram. In the dual string theory description, it arises from the Chern–Simons term in the effective supergravity action. Also the two-point function is exactly known.

$$\langle J_{c'}^\beta(y) J_c^\rho(z) \rangle = \delta_{cc'} \frac{3N^2}{8\pi^4} \frac{1}{(y-z)^6} \left(\eta^{\beta\rho} - 2 \frac{(y-z)^\beta (y-z)^\rho}{(y-z)^2} \right). \quad (3.4)$$

One can deduce from (3.3) and (3.4) an OPE

$$J_a^\mu(y) J_b^\nu(0) = d^{abc} \epsilon^{\mu\nu}{}_{\alpha\beta} \frac{y^\alpha}{3\pi^2 y^4} J_c^\beta(0) + \dots, \quad (3.5)$$

which should be valid for all values of λ . In Appendix A we shall explicitly verify it in the free theory $\lambda = 0$. Using (3.5), one finds

$$\begin{aligned} \int d^4y e^{iqy} \langle PS | T \{ J_3^\mu(y) J_3^\nu(0) \} | PS \rangle \Big|_{asym} &= d^{33c} \epsilon^{\mu\nu}{}_{\alpha\beta} \frac{2q^\alpha}{3Q^2} \langle PS | J_c^\beta(0) | PS \rangle \\ &= d^{33c} \epsilon^{\mu\nu}{}_{\alpha\beta} \frac{q^\alpha}{3P \cdot q} \frac{1}{x} \langle PS | J_c^\beta(0) | PS \rangle. \end{aligned} \quad (3.6)$$

The matrix element has the form

$$\langle PS | J_c^\beta(0) | PS \rangle = \mathcal{Q}_c (A S^\beta + B P^\beta), \quad (3.7)$$

where \mathcal{Q}_c is the \mathcal{R} -charge. The first and second terms come from the fermionic and bosonic parts of the \mathcal{R} -current, respectively. The coefficients A and B are identified with the elastic form factors at zero momentum transfer. In principle, they can be computed once one specifies an AdS/QCD model. However, an important point is that in order to obtain a nonzero A (B is in general nonzero), it is necessary to employ models which are dual to the $\mathcal{N} = 4$ theory in the UV, and at the same time contain the massless Nambu–Goldstone modes of spontaneously broken \mathcal{R} -symmetry. Otherwise, the current conservation law immediately implies $A = 0$, as was observed in a ‘hardwall’ model calculation in [23]. An exemplary model which meets this requirement is the one constructed in [30]. Keeping such a model in mind, in the following we simply assume that $A \neq 0$. Though this extra assumption is not essential to our paper, it is in any case useful in order to make some contact with QCD.

(3.6) is just the first term in the OPE, and as such, it does not contribute to the structure function defined as the imaginary part of the correlator. Rather, it is related to the first moment of g_1 and F_3 as in (2.10)

$$\int_0^1 dx g_1(x, Q^2) = \frac{d^{33c} \mathcal{Q}_c}{12} A, \quad \int_0^1 dx F_3(x, Q^2) = \frac{d^{33c} \mathcal{Q}_c}{6} B. \quad (3.8)$$

²In fact there are also terms proportional to f^{abc} . We simply ignore them since they do not contain the epsilon tensor $\epsilon^{\mu\nu\alpha\beta}$. Besides, they vanish if one sets $a = b$.

In QCD, apart from the global constraint (3.8), the first term in the OPE does not tell anything about the actual x -dependence of g_1 . Its small- x behavior has then to be studied quite independently, using the perturbative QCD evolution [31–33] or perhaps in the framework of Regge theory [34]. However, in strongly coupled $\mathcal{N} = 4$ SYM it turns out that they are closely related. This can be readily understood by looking at the unpolarized case where one takes the energy momentum tensor in the JJ OPE

$$JJ \sim \frac{1}{x^2} T. \quad (3.9)$$

The $1/x^2$ dependence can be interpreted as arising from the exchange of a graviton which has spin $j = 2$ in the t -channel. Once the graviton is Reggeized, a nonzero imaginary part arises, hence the structure function

$$F_1(x, Q^2) \sim \left(\frac{1}{x}\right)^{2-\mathcal{O}(1/\sqrt{\lambda})}. \quad (3.10)$$

In the polarized case, the $1/x$ dependence in (3.6) is due to the t -channel exchange of a Kaluza–Klein photon which is dual to the \mathcal{R} -current operator. In the next section, we shall demonstrate that the Reggeization of the photon generates the polarized structure functions which indeed behave as

$$g_1(x, Q^2), F_3(x, Q^2) \sim \left(\frac{1}{x}\right)^{1-\mathcal{O}(1/\sqrt{\lambda})}, \quad (3.11)$$

in the regime $x \sim e^{-\sqrt{\lambda}}$.

As for the $g_2(x, Q^2)$ structure function, we argue that it is much smaller than g_1 . To see this, note that the derivation of the relation (2.12) hinges only on the existence of the OPE and hence it is valid even at strong coupling. Moreover, the twist-three contribution \bar{g}_2 is expected to be negligible because of a large anomalous dimension of order $\sim \lambda^{1/4}$. Now suppose that g_1 behaves as

$$g_1(x, Q^2) \sim \frac{c}{x^{1-\epsilon}}, \quad (3.12)$$

where $\epsilon \sim \mathcal{O}(1/\sqrt{\lambda})$ is a small positive number. Plugging this into (2.12) one finds

$$g_2(x) \sim -\frac{c}{1-\epsilon} + \frac{\epsilon c}{(1-\epsilon)x^{1-\epsilon}}, \quad (3.13)$$

and

$$\int_0^1 dx g_2(x, Q^2) = 0. \quad (3.14)$$

Thus g_2 has the same x dependence as g_1 , but the coefficient is suppressed by a factor of $\epsilon \sim 1/\sqrt{\lambda}$.

We have so far only considered the $SU(4)$ \mathcal{R} -currents J_a^μ that appear in the JJ OPE. On the other hand, from the relation among the $SU(4)$ generators

$$t^a t^b = \frac{\delta^{ab}}{2N_f} + \frac{d^{abc}}{2} t^c + i \frac{f^{abc}}{2} t^c, \quad (3.15)$$

with $N_f = 4$, it is clear that the OPE also involves the singlet axial current

$$J_{singlet}^\mu = \sum_{i=1}^4 \bar{\psi}_i \bar{\sigma}^\mu \psi_i, \quad (3.16)$$

associated with the $U_A(1)$ part of the putative $U(4)=U_A(1)\times SU(4)$ \mathcal{R} -symmetry. As we mentioned already, the singlet axial current measures the fermions' helicity contribution to the total spin (cf., the $a^{(0)}$ term in (2.13)). The problem with this current in $\mathcal{N} = 4$ SYM is that the $U_A(1)$ (or $U(4)$) is not a symmetry group of the theory, even classically (except in the free theory), due to the Yukawa coupling $\sim f_{abc}\phi_a^{ij}\psi_b^i\psi_c^j$ in the Lagrangian. [The scalars cannot have charges under $U_A(1)$.] Moreover, quantum mechanically, there is an axial anomaly in the divergence of the current

$$\partial_\mu J_{singlet}^\mu = -\frac{\lambda}{4\pi^2} F_{\mu\nu}^a \tilde{F}_a^{\mu\nu} + \dots, \quad (3.17)$$

where \dots denotes the 'classical anomaly' from the Yukawa coupling. Due to the strong violation of the 'conservation law', the singlet current is not protected under renormalization. The operator (3.16) and its superdescendants (the so-called Konishi multiplet) are dual to the string states in the first excited level. Consequently, the singlet current acquires a large anomalous dimension of order $\lambda^{1/4}$ at strong coupling,³ which makes it completely irrelevant to the present discussion. Thus, as announced in Introduction, we are lead to a drastic conclusion that the helicity of Weyl fermions does not contribute at all to the total spin for all values of $Q^2 > \Lambda^2$.

What about the gluon helicity contribution ΔG ? In the light-cone OPE on which our analysis is based, the singlet current (3.16) that appears in the g_1 sum rule does not mix with gluonic operators since there is no local, gauge-invariant spin-1 gluonic operator [36]. Still, one can define ΔG as the first moment of a gauge-invariant non-local operator and consider the evolution

$$\frac{\partial}{\partial \ln Q^2} \begin{pmatrix} \Delta\Sigma \\ \Delta G \end{pmatrix} = \frac{\lambda}{16\pi^2} \begin{pmatrix} -6 & 0 \\ 3 & 0 \end{pmatrix} \begin{pmatrix} \Delta\Sigma \\ \Delta G \end{pmatrix}, \quad (3.18)$$

where, as an illustration, we show the one-loop result from [37]. Unlike in QCD, the (1,1) entry is nonzero because the singlet current is not conserved even classically. The anomalous dimension matrix has a left eigenvector

$$\Delta\Sigma + 2\Delta G, \quad (3.19)$$

with zero eigenvalue. Due to superconformal invariance, the linear combination (3.19) has vanishing anomalous dimension to all orders [37]. Combining this fact with the anomalous dimension of $\Delta\Sigma$ at strong coupling, $\gamma \approx -\lambda^{1/4}$, one finds

$$\Delta\Sigma(Q^2) = \tilde{C} \left(\frac{\Lambda^2}{Q^2} \right)^{\lambda^{1/4}}, \quad \Delta G(Q^2) = -\frac{\tilde{C}}{2} \left(\frac{\Lambda^2}{Q^2} \right)^{\lambda^{1/4}} + C, \quad (3.20)$$

³For a detailed study of the Konishi multiplet and the anomaly in $\mathcal{N} = 4$ SYM, see, e.g., [35]. We thank M. Bianchi for answering our questions.

where C and \tilde{C} are integration constants (independent of Q^2), and therefore,

$$\Delta\Sigma(Q^2) + \Delta G(Q^2) = \frac{\tilde{C}}{2} \left(\frac{\Lambda^2}{Q^2} \right)^{\lambda^{1/4}} + C \approx C. \quad (3.21)$$

We see that there is a very rapid transfer of angular momentum from the helicity degrees of freedom to the orbital motion. In practice, these two contributions have constant values C and $\frac{1}{2} - C$, respectively, for all $Q^2 > \Lambda^2$.

Finally in this section, we comment on the physical meaning of the g_1 sum rule in the $\mathcal{N} = 4$ theory. Setting $a = b = 3$ in (3.15), we get

$$\frac{d^{33c}}{2} t^c = \frac{1}{8} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \quad (3.22)$$

Therefore,⁴

$$g_1(x) \sim \sum_{i=1,2} (\psi_i^+(x) - \bar{\psi}_i^-(x)) - \sum_{i=3,4} (\psi_i^+(x) - \bar{\psi}_i^-(x)) + (\text{bosonic contributions}). \quad (3.24)$$

Thus the g_1 sum rule (3.8) is a direct analog of the Bjorken sum rule (2.14) (no need to subtract the singlet part since there is none) which simply represents the conservation of the axial, or rather the \mathcal{R} -charge corresponding to the generator (3.22).

4. The g_1 structure function at strong coupling

The goal of this section is to derive (3.11) in its precise form. We first consider an approach based on the string S-matrix in AdS_5 following [38] (see also, [17,39,40]). Based on this, we then develop a pragmatic approach suited for our problem and construct the g_1 structure function which exactly satisfies the sum rule (3.8).

4.1 Reggeized photon exchange in AdS

Scattering amplitudes in type IIB string theory in the background of $\text{AdS}_5 \times \text{S}^5$ show the usual Regge behavior at small momentum transfer $\tilde{s} \gg |\tilde{t}|$

$$\mathcal{A} \sim \tilde{s}^{2 + \frac{\alpha' \tilde{t}}{2}}, \quad (4.1)$$

where we put a tilde on the Mandelstam variables in order to distinguish them from the corresponding quantities in the four-dimensional boundary gauge theory. The \tilde{s}^2 -dependence

⁴Note that Weyl fermions contain only two degrees of freedom. This is why there are only two terms (as opposed to four, cf., (2.15)) in (3.24) for each flavor. Also, (3.24) is valid if the helicity of the hadron is $h = +\frac{1}{2}$. If $h = -\frac{1}{2}$, then one should rather write

$$g_1(x) \sim \sum_{i=1,2} (\bar{\psi}_i^+(x) - \psi_i^-(x)) - \sum_{i=3,4} (\bar{\psi}_i^+(x) - \psi_i^-(x)). \quad (3.23)$$

is due to the exchange of the (Reggeized) graviton G_{++} which has spin $j = 2$ in the light-cone direction. On the other hand, the photon associated with the \mathcal{R} -current operator is, from the ten-dimensional point of view, a component of the graviton with one of its indices polarized in the S^5 direction

$$G_{+a} = A_+ K_a, \quad (4.2)$$

where K_a ($a = 1, 2, 3, 4, 5$) is a Killing vector. Since the momenta in the S^5 direction carried by the external states are at most of order $1/R$ ($R = 1$ is the common radius of AdS_5 and S^5), the photon exchange gives an amplitude smaller than (4.1) by a factor of \tilde{s}

$$\mathcal{A} \sim \tilde{s}^{1+\frac{\alpha' \tilde{t}}{2}}. \quad (4.3)$$

As suggested in [16, 38], the momentum transfer \tilde{t} in (4.1) and (4.3) can be interpreted as the Laplacian operator acting on the eight-dimensional transverse space. The form of the Laplacian is determined by looking at the equation of motion obeyed by the field exchanged in the t -channel. Generically, it takes the form

$$(\Delta_j + f(j))\Phi_{j+} = 0. \quad (4.4)$$

for a spin- j field

$$\Phi_{j+} \equiv \underbrace{\Phi_{++ \dots +}}_{j \text{ indices}}. \quad (4.5)$$

In (4.4) $f(j)$ is a c-number function and the differential operator Δ_j is defined as

$$\Delta_j \equiv \frac{1}{z^j} \Delta_0 z^j, \quad (4.6)$$

where Δ_0 is the scalar Laplacian in AdS_5

$$\Delta_0 = -z^2(\partial_z^2 + \partial_\perp^2) + 3z\partial_z. \quad (4.7)$$

Note that we keep only the derivative $\partial_\perp^2 \equiv \partial_{y_1}^2 + \partial_{y_2}^2$ in the transverse direction of the Minkowski space. At high energy this factor is essentially identified with the momentum transfer in the boundary gauge theory $\partial_\perp^2 \rightarrow t$.

For example, the graviton G_{++} obeys the linearized Einstein equation in AdS_5 which reduces to

$$\Delta_2 G_{++} = (-z^2 \partial_z^2 - z^2 \partial_\perp^2 - z\partial_z + 4)G_{++} = 0. \quad (4.8)$$

Thus one sees that $f(2) = 0$. The amplitude (4.1) then becomes an operator

$$\mathcal{A} \sim \tilde{s}^{2-\alpha' \Delta_2/2}, \quad (4.9)$$

acting on the hyperbolic space H_3 parameterized by (z, \vec{y}_\perp) . Going to the basis which diagonalizes the operator Δ_2 , one finds that

$$\mathcal{A} \sim \tilde{s}^{2-2/\sqrt{\lambda}}. \quad (4.10)$$

Thus the graviton intercept is slightly shifted from 2 due to the curvature of AdS_5 , and this can be identified with the Pomeron intercept [38, 41]. In unpolarized DIS, (4.10) is essentially the origin the behavior (3.10) due to the correspondence $\tilde{s} \leftrightarrow 1/x$.

Similarly, the photon field obeys the five-dimensional Maxwell equation

$$\begin{aligned} D_M F^{M-} &= z^2(-z^2 \partial_z^2 - z^2 \partial_\perp^2 + z \partial_z) A_+ \\ &= z^2(\Delta_1 - 3) A_+ = 0. \end{aligned} \quad (4.11)$$

Thus $f(1) = -3$ in this case, meaning that the field $z A_+$ behaves as a scalar with negative mass squared $m^2 = -3$. The amplitude (4.3) becomes

$$\mathcal{A} \sim \tilde{s}^{1-\alpha'(\Delta_1-3)/2}. \quad (4.12)$$

Though it is straightforward to diagonalize the operator Δ_1 , in order to make contact with the OPE (2.6) it is convenient at this point to express (4.12) as a contour integral in the complex j -plane

$$\begin{aligned} \langle u | \tilde{s}^{1-\alpha'(\Delta_1-3)/2} | u' \rangle &= \int \frac{dj}{2\pi i} \langle u | \frac{\tilde{s}^j}{j-1+\alpha'(\Delta_j-3)/2} | u' \rangle \\ &= 2\sqrt{\lambda} \int \frac{dj}{2\pi i} \tilde{s}^j \sqrt{G'} (G'^{+-})^j D_{j+j-}(u, u'), \end{aligned} \quad (4.13)$$

where we collectively denoted $u \equiv (z, y^\mu)$ and used the relation $\alpha' = 1/\sqrt{\lambda}$ which holds in the present units. In the second equality, we have defined the t -channel propagator D_{j+j-} of the photon whose spin is analytically continued away from $j = 1$. It satisfies the following relation

$$\left(\Delta_j - 3 + 2\sqrt{\lambda}(j-1) \right) D_{j+j-}(u, u') = \frac{\delta^{(5)}(u-u')}{(G'^{+-})^j \sqrt{G'}}. \quad (4.14)$$

The solution is, for $t = \partial_\perp^2 = 0$,

$$D_{j+j-}(u, u') = (zz')^{2-j} \int \frac{d\nu}{\pi} \frac{e^{-i\nu(\rho-\rho')}}{4\nu^2 + 1 + 2\sqrt{\lambda}(j-1)} \delta^{(4)}(y-y'), \quad (4.15)$$

where we have defined $z^2 = e^{-\rho}$. Then (4.13) can be evaluated as

$$\begin{aligned} \langle u | \tilde{s}^{1-\alpha'(\Delta_1-3)/2} | u' \rangle &\approx \tilde{s}^{j_A} (zz')^{2-j_A} \int \frac{d\nu}{\pi} \tilde{s}^{-D\nu^2} e^{-i\nu(\rho-\rho')} \delta^{(4)}(y-y') \\ &= \tilde{s}^{j_A} (zz')^{2-j_A} \frac{e^{-\frac{(\rho-\rho')^2}{4D\tau}}}{\sqrt{\pi D\tau}} \delta^{(4)}(y-y'), \end{aligned} \quad (4.16)$$

where $D \equiv 2/\sqrt{\lambda}$ is the ‘diffusion’ parameter, and $\tau \equiv \ln \tilde{s}$ is the rapidity. The Reggeized photon intercept

$$j_A \equiv 1 - \frac{1}{2\sqrt{\lambda}}, \quad (4.17)$$

is slightly shifted from 1 due to the nonzero curvature of AdS_5 . (4.16) implies that the structure functions indeed exhibit the behavior anticipated in (3.11). It also shows that the interaction is nonlocal in the fifth dimension due to diffusion: In a complete calculation (4.16) should be convoluted with the target wavefunctions localized around $z \sim 1/\Lambda$ and the (s -channel) photon wavefunctions localized around $z' \sim 1/Q$.

4.2 A pragmatic approach

One can incorporate the procedures in the previous subsection into the worldsheet OPE approach [38, 40] by introducing vertex operators for the external and internal states. However, applied to our problem of polarized DIS, it appears difficult in this approach to obtain the expected tensor structure (namely, the epsilon tensor $\epsilon^{\mu\nu\alpha\beta}$ in (3.2)) and determine the normalization factor essential to discuss the sum rule. We therefore take a different and more pragmatic tack. The idea is to try to reconstruct the imaginary part from the first term in the OPE (3.6). Let us focus on the factor

$$\frac{1}{x} \langle PS | J_c^+(0) | PS \rangle. \quad (4.18)$$

The insertion of the operator $J_c^+(0)$ excites a non-normalizable mode of the bulk gauge field

$$A_+(z, y) = \frac{2i}{\pi^2} \int d^4 y' \frac{z^2}{(z^2 - (y - y')^2 + i\epsilon)^3} A_+(y'), \quad (4.19)$$

where $A_+(x')$ is the boundary source which, for a local insertion $J_c^+(y' = 0)$, is given by a delta function $A_\mu(y') = \delta_\mu^+ \delta^{(4)}(y')$. The integral kernel in (4.19) is the bulk-to-boundary propagator. The matrix element (4.18) may be evaluated as

$$\frac{1}{x} \langle PS | J_c^+(0) | PS \rangle = \frac{\mathcal{Q}_c}{x} \int d^4 y \int dz \sqrt{G} A_+(z, y) \bar{\psi} \gamma^+ \psi(z, y). \quad (4.20)$$

The y -dependence of the target wavefunctions is simply the plane wave $e^{iP \cdot y}$ which cancels between ψ and $\bar{\psi}$ (for the forward scattering). The $d^4 y$ integral can then be done

$$\int d^4 y A_+(z, y) = \frac{2i}{\pi^2} \int d^4 y d^4 y' \frac{z^2}{(z^2 - (y - y')^2 + i\epsilon)^3} A_+(y') = \int d^4 y' A_+(y') = 1, \quad (4.21)$$

so that (4.20) reduces to

$$\frac{1}{x} \langle PS | J_c^+(0) | PS \rangle = \frac{\mathcal{Q}_c}{x} \int dz \sqrt{G} \bar{\psi} \gamma^+ \psi(z) = \frac{\mathcal{Q}_c}{x} (AS^+ + BP^+). \quad (4.22)$$

Alternatively, one can introduce an effective bulk current J_{bulk} which sources (4.19) via the bulk-to-bulk propagator.

$$A_+(z, y) = \int d^4 y' dz' \sqrt{G'} D_{+-}(u, u') J_{bulk}^-(z', y'). \quad (4.23)$$

Naturally, J_{bulk} has a support at small $z < 1/Q$ reflecting the uncertainty $|\Delta y^2| \sim 1/Q^2$ in the location of the operator insertion (4.18). Using the asymptotic form of the propagator [42] (adapted to the Minkowski metric)

$$D_{+-}(u, u') \approx \frac{i}{\pi^2 z z'} \left(\frac{z z'}{z^2 - (y - y')^2 + i\epsilon} \right)^3, \quad (z' \rightarrow 0) \quad (4.24)$$

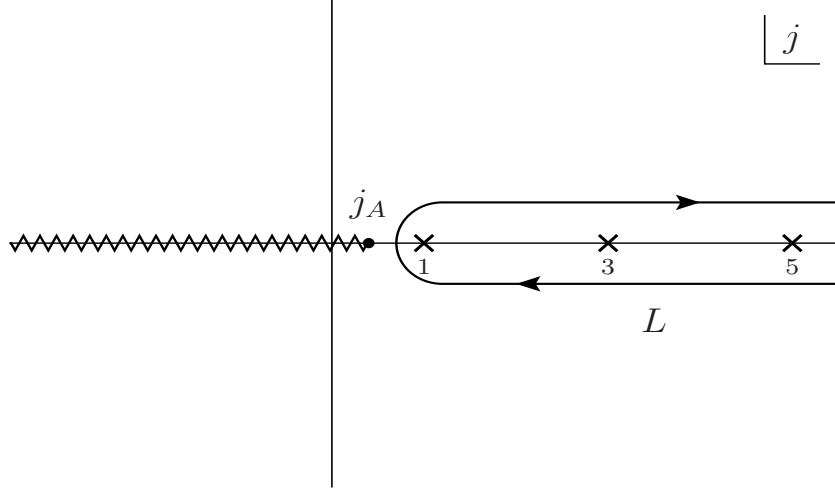


Figure 1: The integration contour in the complex j -plane. A branch cut appears after integrating over ν .

one finds the following relation between $J_+^{bulk}(z, y)$ and $A_+(y)$

$$A_+(y') = \frac{1}{2} \int \frac{dz'}{z'} J_+^{bulk}(z', y'). \quad (4.25)$$

[Note that $J^- = z'^2 J_+$.] The propagator D_{+-} is identical to the one defined in (4.14) with $j = 1$. (4.23) can be rewritten as

$$A_+(z, y) = \int d^5 u' \frac{1}{\Delta_1 - 3} \delta^{(5)}(u - u') J_+^{bulk}(u'). \quad (4.26)$$

So far we have dealt with only a single term in the JJ OPE. However, our experience with QCD tells us that one should sum over the twist-two operators with odd j values as in (2.6). We do this in the form of a contour integral in the complex j -plane

$$\sum_j^{odd} = \int_L \frac{dj}{4i} \frac{1 - e^{-i\pi j}}{\sin \pi j}, \quad (4.27)$$

where the contour L is shown in fig. 1. Accordingly, all the factors in (4.20) should be analytically continued away from $j = 1$. As for the propagator, it suffices to use (4.14) in anticipation that in the end the important values of j will be close to $j = 1$. (4.26) is then replaced by

$$A_{j+}(z, y) = \int d^5 u' \frac{1}{\Delta_j - 3 + 2(j-1)/\alpha'} \delta^{(5)}(u - u') J_{j+}^{bulk}(u'). \quad (4.28)$$

On the other hand, the factor $1/x$ obviously generalizes to

$$\frac{1}{x} \rightarrow \left(\frac{1}{x} \right)^j. \quad (4.29)$$

All in all, (4.20) becomes⁵

$$I \equiv \mathcal{Q}_c \int \frac{dj}{4i} \frac{1 - e^{-i\pi j}}{\sin \pi j} \left(\frac{1}{x}\right)^j \int d^4 y dz \sqrt{G} \int d^4 y' dz' \\ \times \frac{1}{\Delta_j - 3 + 2(j-1)/\alpha'} \delta^{(5)}(u - u') J_{j+}^{bulk}(u') \bar{\psi} \gamma^+ (\partial^+)^{j-1} \psi(z). \quad (4.30)$$

Let us do the j -integral first. Deforming the contour to pick up the pole from the propagator (see, fig. 1), we get⁶

$$I = \mathcal{Q}_c \frac{\pi}{4\sqrt{\lambda}} \int d^4 y dz \sqrt{G} \int d^4 y' dz' \frac{1 - e^{-i\pi j}}{\sin \pi j} \left(\frac{1}{x}\right)^{1-\alpha' \frac{\Delta_j-3}{2}} \\ \times \delta^{(5)}(u - u') J_{j+}^{bulk}(u') \bar{\psi} \gamma^+ \psi(z). \quad (4.31)$$

In order to diagonalize the Laplacian operator, we notice that

$$\left(\frac{1}{x}\right)^{1-\alpha' \frac{\Delta_j-3}{2}} = z^{2-j} \left(\frac{1}{x}\right)^{1-\alpha' \frac{\Delta_2-3}{2}} z^{j-2}, \quad (4.32)$$

where

$$-\Delta_2 + 3 = z^2 \partial_z^2 + z \partial_z + z^2 \partial_\perp^2 - 1 = 4\partial_\rho^2 + z^2 t - 1. \quad (4.33)$$

In the case of forward scattering $t = 0$, the eigenfunction of the operator Δ_2 is simply a plane wave in ρ , so the following representation of the delta function is useful

$$\delta(z - z') = \int \frac{d\nu}{\pi} \left(\frac{1}{z}\right)^{j-2+2i\nu} z'^{j-3+2i\nu} = \int \frac{d\nu}{\pi} \left(\frac{1}{z}\right)^{j-2} z'^{j-3} e^{i\nu(\rho-\rho')}. \quad (4.34)$$

(4.31) can then be evaluated as

$$I \approx \mathcal{Q}_c \frac{\pi}{4\sqrt{\lambda}} \int d^4 y dz \sqrt{G} \int dz' \sqrt{G'} (G'^{+-})^{j_A} (zz')^{2-j_A} \\ \times \frac{1 - e^{-i\pi j_A}}{\sin \pi j_A} \int \frac{d\nu}{\pi} \left(\frac{1}{x}\right)^{j_A - \frac{2\nu^2}{\sqrt{\lambda}}} e^{i\nu(\rho-\rho')} J_{j+}^{bulk}(z', y) \bar{\psi} \gamma^+ \psi(z) \quad (4.35) \\ \approx \mathcal{Q}_c \left(1 + i \frac{\pi}{4\sqrt{\lambda}}\right) \int d^4 y dz \sqrt{G} \int dz' \sqrt{G'} (G'^{+-})^{j_A} (zz')^{2-j_A} \\ \times \left(\frac{1}{x}\right)^{1 - \frac{1}{2\sqrt{\lambda}}} \frac{e^{-(\rho-\rho')^2/4D\tau}}{\sqrt{\pi D\tau}} J_{j+}^{bulk}(z', y) \bar{\psi} \gamma^+ \psi(z),$$

where $j_A = 1 - 1/2\sqrt{\lambda}$ is as in (4.17) and $\tau = \ln 1/x$. We remind the reader that the calculation is valid in the regime $x \sim e^{-\sqrt{\lambda}}$, or $(1/x)^{1/\sqrt{\lambda}} \sim \mathcal{O}(1)$. In (4.35), one recognizes

⁵There may be an unknown, multiplicative factor $c(j)$ in the integrand of (4.30) such that $c(1) = 1$. However, the j -integral will be dominated by the region $j \approx 1$ where the function $c(j)$ must be slowly varying. [Physically, we do not see any source of rapid variation in j near $j = 1$ other than (4.28) and (4.29).] Therefore we may set $c(j) = 1$ from the beginning.

⁶To avoid cumbersome expressions, we retain the letter j in (4.31) as representing the pole of the propagator: $j = 1 - \alpha'(\Delta_j - 3)/2$.

an imaginary part which can be identified with the g_1 structure function. Working out the prefactor, we arrive at⁷

$$g_1(x, Q^2)S^+ + \frac{F_3(x, Q^2)}{2}P^+ = \frac{d^{33c}\mathcal{Q}_c}{6\pi} \frac{\pi}{4\sqrt{\lambda}} \int d^4y dz \sqrt{G} \int dz' \sqrt{G'} G'^{+-} z z' \\ \times \left(\frac{1}{x}\right)^{1-\frac{1}{2\sqrt{\lambda}}} \frac{e^{-(\rho-\rho')^2/4D\tau}}{\sqrt{\pi D\tau}} J_+^{bulk}(z', y) \bar{\psi} \gamma^+ \psi(z). \quad (4.36)$$

This is the main result of this paper. The Q^2 -dependence is implicit in J_+^{bulk} . It is important to notice that the Regge intercept j_A is independent of Q^2 .

Let us check that (4.36) satisfies the sum rule. For this purpose it is convenient to undo the ν -integral and integrate over x (or equivalently, τ) first. Using

$$\int \frac{d\nu}{\pi} e^{i\nu(\rho-\rho')} \int_0^\infty d\tau e^{-\tau+(1-1/2\sqrt{\lambda})\tau-2\nu^2\tau/\sqrt{\lambda}} \\ = \frac{\sqrt{\lambda}}{2} \int_{-\infty}^\infty \frac{d\nu}{\pi} e^{i\nu(\rho-\rho')} \frac{1}{\nu^2 + \frac{1}{4}} \\ = \sqrt{\lambda} \frac{z'}{z}, \quad (4.37)$$

where $z' \sim 1/Q < z \sim 1/\Lambda$, we get

$$\int_0^1 dx g_1(x, Q^2) = \frac{d^{33c}\mathcal{Q}_c}{6\pi S^+} \frac{\pi}{4\sqrt{\lambda}} \sqrt{\lambda} \int d^4y dz \sqrt{G} \int \frac{dz'}{z'} J_+^{bulk}(z', y) \bar{\psi} \gamma^+ \psi(z) \\ = \frac{d^{33c}\mathcal{Q}_c}{12S^+} \int dz \sqrt{G} \bar{\psi} \gamma^+ \psi(z) \\ = \frac{d^{33c}\mathcal{Q}_c}{12} A, \quad (4.38)$$

where we take only the term proportional to S^+ in the integral. In the second equality, we used (4.25). This is exactly (3.8).

It is also instructive to identify the region of x which dominantly contributes to the sum rule. Doing the ν integral first, we encounter the following integral

$$\int_0^1 dx g_1(x, Q^2) \sim \int_0^\infty d\tau e^{-\frac{\tau}{2\sqrt{\lambda}} - \frac{\ln^2 z^2/z'^2}{4D\tau}}. \quad (4.39)$$

The integrand is strongly peaked at

$$\tau = \frac{\sqrt{\lambda}}{2} \ln \frac{z^2}{z'^2} \sim \frac{\sqrt{\lambda}}{2} \ln \frac{Q^2}{\Lambda^2}. \quad (4.40)$$

According to our interpretation of the g_1 structure function, (4.40) is the value around which the \mathcal{R} -charge of the target is concentrated. Incidentally, we note that the energy-momentum sum rule is saturated around the value [17]

$$\tau = \frac{\sqrt{\lambda}}{4} \ln \frac{Q^2}{\Lambda^2}. \quad (4.41)$$

⁷Since we keep a small imaginary part of order $1/\sqrt{\lambda}$, consistency requires that one should neglect the $\mathcal{O}(1/\sqrt{\lambda})$ terms in $(G'^{+-})^{j_A}(zz')^{2-j_A}$.

Finally, consider the anomalous dimension of twist-two operators. This can be read off from the z' dependence of the integrand. In (4.35) one recognizes the factor

$$\left(\frac{1}{z'}\right)^{j-2} e^{-i\nu\rho'} = \left(\frac{1}{z'}\right)^{j-2-2i\nu} = \left(\frac{1}{z'^2}\right)^{\gamma(j)-1} \sim (Q^2)^{\gamma(j)-1}, \quad (4.42)$$

where [17, 38]

$$\gamma(j) = \frac{j}{2} - i\nu. \quad (4.43)$$

This means that

$$\left(\frac{j}{2} - \gamma\right)^2 = -\nu^2 = \frac{1}{D}(j - j_A), \quad (4.44)$$

or equivalently,

$$\gamma(j) = \frac{j}{2} - \sqrt{\frac{\sqrt{\lambda}}{2} \left(j - 1 + \frac{1}{2\sqrt{\lambda}}\right)}. \quad (4.45)$$

The total dimension is then

$$\Delta(j) = j + 2 - 2\gamma(j) = 2 + 2\sqrt{\frac{\sqrt{\lambda}}{2} \left(j - 1 + \frac{1}{2\sqrt{\lambda}}\right)}. \quad (4.46)$$

When $j = 1$, $\Delta(1) = 3$ and $\gamma(1) = 0$. This is the \mathcal{R} -current operator which is protected. The above formula gives an analytic continuation to $j > 1$. Inverting (4.46), one gets an expression

$$j = j_A + \frac{(\Delta - 2)^2}{2\sqrt{\lambda}}, \quad (4.47)$$

which clearly shows the the known symmetry $\Delta \leftrightarrow 4 - \Delta$. Also, it is interesting to note that one can write

$$\Delta = 2 \pm \sqrt{1 + m^2}, \quad (4.48)$$

with

$$m^2 = \frac{2(j-1)}{\alpha'}. \quad (4.49)$$

(4.48) is the formula which relates the dimension of field theory operators and the mass of vector fields in AdS_5 , and (4.49) is the mass of the spin $j = 2n + 1$ ($n = 0, 1, 2, \dots$) state in the n -th excited level $m^2 = 4n/\alpha'$ of type IIB superstring theory.

5. Discussions

5.1 ‘Asymptotic’ spin decomposition

It was observed by Ji [43] that in QCD the partition of the proton spin among quarks and gluons approaches the ratio $3N_f : 16$ in the asymptotic limit $Q^2 \rightarrow \infty$. Here the numbers include both the helicity and orbital contributions. The result has been obtained quite independently of the parton dynamics in DIS, and is solely based on the renormalization of the angular momentum operator which turns out to be identical to that of the energy-momentum tensor. In $\mathcal{N} = 4$ SYM one can repeat the same analysis. The energy-momentum operator is given as a sum of contributions from gluons, Weyl fermions and scalars

$$T_{\mu\nu} = T_{\mu\nu}^g + T_{\mu\nu}^\psi + T_{\mu\nu}^\phi. \quad (5.1)$$

There are two other operators which are eigenvectors of the twist-two anomalous dimension matrix with spin $j = 2$

$$\begin{aligned} T' &= -2T^g + T^\psi + 2T^\phi, \\ T'' &= -T^g + 4T^\psi - 6T^\phi. \end{aligned} \quad (5.2)$$

The coefficients are actually exact, i.e., they do not receive radiative corrections [37, 44]. [See, also, Appendix C of [18].] Therefore, for all values of the coupling one can decompose as

$$\begin{aligned} T^g &= \frac{2}{5}T - \frac{2}{7}T' - \frac{1}{35}T'', \\ T^\psi &= \frac{2}{5}T + \frac{1}{7}T' + \frac{4}{35}T'', \\ T^\phi &= \frac{1}{5}T + \frac{1}{7}T' - \frac{3}{35}T''. \end{aligned} \quad (5.3)$$

This implies that both the total energy and the total spin are partitioned among gluons, fermions and scalars with the ratio $2 : 2 : 1$. A novel feature at strong coupling is that T' and T'' acquire a large anomalous dimension of order $\lambda^{1/4}$. Therefore, the ratio is realized not only asymptotically, but in practice for all values of Q^2 .

Let us comment on a further decomposition into the helicity and orbital parts. Of course the scalars ϕ have only orbital angular momentum. As we have argued, the contribution of the Weyl fermions purely consists of orbital angular momentum. Although ΔG remains undetermined in our approach,⁸ we are inclined to think that the gluons give only orbital angular momentum as well, i.e., $C = 0$ in (3.20), since we see no reason why the gluons are special. Thus we conclude that in strongly coupled theories that have a gravity dual the helicity contribution tends to be suppressed, and in the limit of large coupling the entire hadron spin is of orbital origin.

⁸Note that in Ji’s approach the gluon spin cannot be further decomposed into helicity and orbital parts in a gauge invariant way. The gluon helicity contribution ΔG we have been discussing is the first moment of a certain nonlocal gauge-invariant operator. Then the orbital part is defined as the rest.

5.2 Small- x behavior

Let us compare our result in $\mathcal{N} = 4$ SYM at small- x

$$g_1^S(x) \approx 0, \quad g_1^{NS}(x) \sim \left(\frac{1}{x}\right)^{1-1/2\sqrt{\lambda}}, \quad (5.4)$$

for the singlet and non-singlet parts, respectively, with the corresponding behavior in QCD. Experimentally, within the currently accessible range of x the flavor non-singlet contribution to g_1 rises as $x^{-\alpha}$ with $\alpha \gtrsim 0.5$ [14, 45] whereas the flavor singlet contribution is almost zero at small- x [3]. This is qualitatively similar to (5.4), though of course such a comparison should be taken with caution. On the other hand, predictions from perturbative QCD vary [31–33] depending on which terms in higher order graphs are resummed. Including the so-called double logarithmic contributions, Refs. [32, 33] obtained

$$g_1^S(x) \sim \left(\frac{1}{x}\right)^{k\frac{\sqrt{\lambda}}{2\pi}}, \quad g_1^{NS}(x) \sim \left(\frac{1}{x}\right)^{\frac{\sqrt{\lambda}}{2\pi}}, \quad (5.5)$$

with $k \approx 2.5$, which clearly means $|g_1^S| \gg |g_1^{NS}|$ at low- x . Whether this opposite behavior can be observed in a yet unexplored region of x in future experiments is not clear at the moment. In any case, it is interesting to notice that the exponent of the non-singlet part appears to have a smooth interpolation from 0 to 1 as the coupling λ is varied from zero to infinity. We finally mention that in the conventional Regge theory, the singlet and non-singlet parts are governed by the f_1 and a_1 meson Regge trajectories with *negative* intercepts $-0.18 \gtrsim \alpha_{a_1}$, $\alpha_{f_1} \gtrsim -0.4$. Therefore, $g_1(x)$ goes to zero as $x \rightarrow 0$ in this framework.

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A. OPE at weak coupling

Since (3.5) is an exact result for any value of the coupling, one should be able to check it in the free theory. The \mathcal{R} -current operator is

$$\begin{aligned} J_a^\mu &= \bar{\psi}_i \bar{\sigma}^\mu t_{ij}^a \psi_j + i \phi_{ij}^\dagger (\partial^\mu - \overleftarrow{\partial}^\mu) T_{ij,kl}^a \phi_{kl} \\ &= \bar{\psi} \gamma^\mu \frac{1 + \gamma_5}{2} t^a \psi + i \phi_{ij}^\dagger (\partial^\mu - \overleftarrow{\partial}^\mu) T_{ij,kl}^a \phi_{kl}, \end{aligned} \quad (A.1)$$

where $i, j = 1, 2, 3, 4$ and in the second line we use the Majorana four-component representation. For $a = 3$,

$$J_3^\mu = \frac{1}{2} (\bar{\psi}^1 \bar{\sigma}^\mu \psi^1 - \bar{\psi}^2 \bar{\sigma}^\mu \psi^2) + \frac{i}{2} \left(\phi_{13}^\dagger \partial^\mu \phi_{13} - \partial^\mu \phi_{13}^\dagger \phi_{13} \right) + \frac{i}{2} \left(\phi_{14}^\dagger \partial^\mu \phi_{13} - \partial^\mu \phi_{14}^\dagger \phi_{14} \right). \quad (\text{A.2})$$

In the JJ OPE, the antisymmetric tensor comes solely from the fermionic term. One finds

$$J_a^\mu(y) J_b^\nu(0) = \epsilon^{\mu\nu\alpha\beta} d^{abc} \frac{y_\alpha}{2\pi^2 y^4} \bar{\psi} \gamma_\beta \frac{1 + \gamma_5}{2} t^c \psi + \dots \quad (\text{A.3})$$

Naively, the coefficient does not match (3.5). However one has to take into account the operator mixing. This means that one should decompose in the following way

$$\begin{aligned} \bar{\psi} \gamma_\beta \frac{1 + \gamma_5}{2} t^c \psi &= \frac{2}{3} \left(\bar{\psi} \gamma_\beta \frac{1 + \gamma_5}{2} t^c \psi + i \phi^\dagger (\partial_\beta - \overleftarrow{\partial}_\beta) T^c \phi \right) \\ &+ \frac{1}{3} \left(\bar{\psi} \gamma_\beta \frac{1 + \gamma_5}{2} t^c \psi - 2i \phi^\dagger (\partial_\beta - \overleftarrow{\partial}_\beta) T^c \phi \right) \equiv \frac{2}{3} J_\beta^c + \frac{1}{3} \tilde{J}_\beta^c, \end{aligned} \quad (\text{A.4})$$

which gives

$$J_a^\mu(y) J_b^\nu(0) = \epsilon^{\mu\nu\alpha\beta} d^{abc} \frac{y_\alpha}{3\pi^2 y^4} \left(J_\beta^c(0) + \frac{1}{2} \tilde{J}_\beta^c(0) \right) + \dots \quad (\text{A.5})$$

The prefactor in front of the \mathcal{R} -current precisely reproduces the coefficient in (3.5). The second term is the orthogonal operator to the \mathcal{R} -current operator in the sense that

$$\langle J_a^\alpha(y) \tilde{J}_b^\beta(0) \rangle = 0. \quad (\text{A.6})$$

To see this, we only need to note that the coefficient in (3.4) decomposes as $\frac{3}{8} = \frac{1}{4} + \frac{1}{8}$ where $\frac{1}{4}$ and $\frac{1}{8}$ are the fermionic and bosonic contributions, respectively. At strong coupling, \tilde{J} acquires a large anomalous dimension, so it is irrelevant.

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